PICTURING QUANTUM PROCESSES

The unique features of the quantum world are explained in this book through the language of diagrams, setting out an innovative visual method for presenting complex theories. Requiring only basic mathematical literacy this book employs a unique formalism that builds an intuitive understanding of quantum features while eliminating the need for complex calculations. This entirely diagrammatic presentation of quantum theory represents the culmination of 10 years of research, uniting classical techniques in linear algebra and Hilbert spaces with cutting-edge developments in quantum computation and foundations.

Written in an entertaining and user-friendly style and including more than 100 exercises, this book is an ideal first course in quantum theory, foundations, and computation for students from undergraduate to PhD level, as well as an opportunity for researchers from a broad range of fields, from physics to biology, linguistics, and cognitive science, to discover a new set of tools for studying processes and interaction.

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A First Course in Quantum Theory and Diagrammatic Reasoning

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Preface

Glad you made it here! This book is about telling the story of quantum theory entirely in terms of pictures. Before we get into telling the story itself, it's worth saying a few words about how it came about. On the one hand, this is a very new story, in that it is closely tied to the past 10 years of research by us and our colleagues. On the other hand, one could say that it traces back some 80 years when the amazing John von Neumann denounced his own quantum formalism and embarked on a quest for something better. One could also say it began when Erwin Schrödinger addressed Albert Einstein's concerns about 'spooky action at a distance' by identifying the structure of composed systems (and in particular, their non-separability) as the beating heart of quantum theory.

From a complementary perspective, it traces back some 40 years when an undergraduate student named Roger Penrose noticed that pictures out-classed symbolic reasoning when working with the tensor calculus.

But 80 years ago the authors weren't around yet, at least not in human form, and 40 years ago there wasn't really that much of us either, so this preface will provide an egocentric take on the birth of this book. This also allows us to wholeheartedly acknowledge all of those without whom this book would never have existed (as well as some who nearly succeeded in killing it).

Things started out pretty badly for Bob, with a PhD in the 1990s on a then completely irrelevant topic of contextual 'hidden variable representations' of quantum theory – which recently have been diplomatically renamed to *ontological models* (Harrigan and Spekkens, 2010; Pusey et al., 2012). After a period of unemployment and a failed attempt to become a rock star, Bob ventured into the then even more irrelevant topic of von Neumann's quantum logic (Birkhoff and von Neumann, 1936) in the vicinity of the eccentric iconoclast Constantin Piron (1976).

It was there that category theory entered the picture, as well as serious considerations on the fundamental status of composition in quantum systems – something that went hand-in-hand with bringing quantum processes (rather than quantum states) to the forefront …

⚠ In the case you are suffering from some kind of category theory phobia, do not stop reading here! Though it has influenced many of the ideas in this book, this is by no means a book about category theory!

… these considerations would ultimately provide the formal and the conceptual backbone for a pictorial approach to quantum theory. The categorical push in quantum foundations came initially from David Moore (1995), a very gifted researcher who suffered his academic end in the late 1990s, an era in which conceptually oriented physics suffered from widespread prohibition. In collaboration with Moore and Isar Stubbe, Bob made some early attempts towards a categorical reformulation of quantum theory (Coecke et al., 2001), which unfortunately inherited too many deficiencies from old-fashioned quantum logic. The main problem with quantum logic was its implicit assumption that the physical systems under consideration are always: 'some part of the ostensibly external phenomenal world, supposed separated from its surroundings in the sense that its interactions with the environment can either be ignored or effectively modelled in a simple way' (Moore, 1999). However, interactions with the environment happen to be something one should really care about!

After being kicked out of his university (cf. bureaucrats, village politics, and lots of hypocrisy), a second failed attempt in the arts, and looming unemployment, a bit of a miracle happened to Bob when two complete strangers, Prakash Panangaden and Samson Abramsky, arranged a 'trial' postdoc in the Computing Laboratory at Oxford, which was back then affectionately known as the Comlab. Despite knowing nothing about computer science and thinking of computer scientists as a bunch of nerds staring at screens all day, Bob found a home in this department and quickly discovered that, unlike quantum logicians, computer scientists had for a long time already studied the structure of interacting systems and were able to describe such systems elegantly in the language of category theory. In fact, in this particular computer science department, category theory was even taught at undergraduate level.

It was here that the second author entered the picture. While on a two-month exchange to Oxford from his homeland of Tulsa, Oklahoma (Fig. 0.1), Aleks happened to take the aforementioned undergraduate course in category theory, which was at the time taught by Samson. The mind-expanding nature of that course (including a guest lecture about weird-looking pictures of monoidal categories by an equally weird-looking guy) got Aleks interested enough to get involved in the subject. At Samson's prompting, he started coming along to the group's Quantum Lunch seminar. The seminar format consisted of a large pub lunch followed by a talk where a drunk and drowsy speaker addressed an equally drunk and drowsy audience on topics in the newborn subject of categorical quantum mechanics. It was great.

Two months turned into nine years, the Comlab became the 'Department of Computer Science', and though it seemed like nobody could remember when Aleks first starting hanging around, he ended up doing a master's, PhD, and a postdoc.

Figure 0.1 Some typical sights of Tulsa, Oklahoma.

Without the surprising wealth of both mathematical machinery and conceptual thinking in this unique computer science environment, this book simply would not have existed. In sharp contrast to the prohibition in the 1990s of the words 'foundational' and 'conceptual' in physics, in this new environment 'foundational' and 'conceptual' were (and still are) big virtues! This led to the birth of a new research community, in which computer scientists, pure mathematicians, philosophers, and researchers in the now-resurgent area of quantum foundations closely interact. It is probably even fair to say that this unique atmosphere contributed to the resurrection of the quantum foundations community as a whole, and along the way several of its highly respected practitioners have adopted the diagrammatic paradigm, notably Chiribella et al. (2010) and Hardy (2013a).

The conference series Quantum Physics and Logic (QPL), founded by Peter Selinger in 2003 under a different name (but with the same abbreviation!), was a particularly important forum for the development of the key results leading up to this book. In fact, the first paper about diagrammatic reasoning for novel quantum features (Coecke, 2003) was presented at the first QPL. The categorical formalisation of this result (Abramsky and

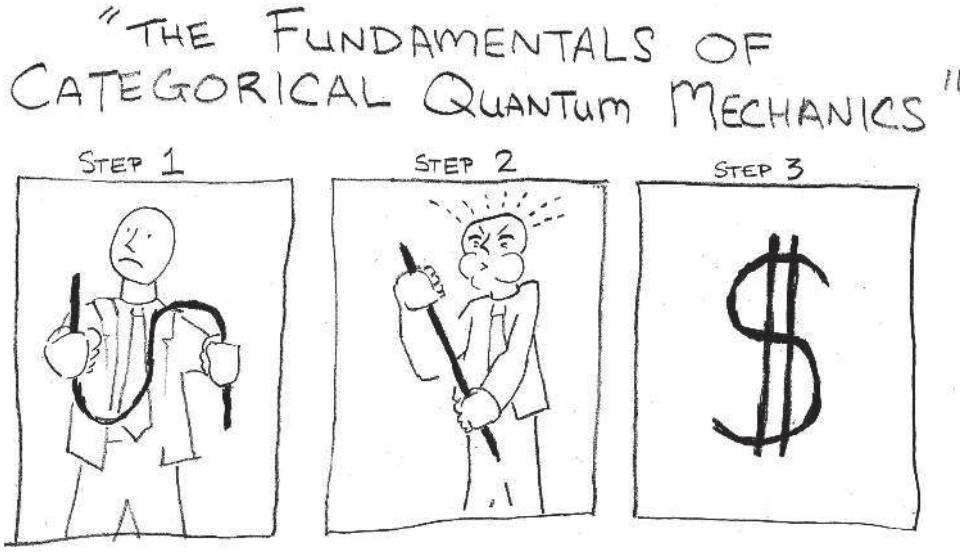

Figure 0.2 Yanking for cash.

Coecke, 2004), now referred to as categorical quantum mechanics, became a hit within the computer science semantics community, and ultimately allowed for several young people to establish research careers in this area. Top computer science conferences (e.g. LiCS and ICALP) indeed regularly accept papers on categorical quantum mechanics, and more recently leading physics journals (e.g. PRL and NJP) have started to do so too.

We are very grateful to have received a healthy flow of research funding (cf. Fig. 0.2), by the UK Engineering and Physical Sciences Research Council (EPSRC), the European Commission (FP6 FET Open), the US Office of Naval Research (ONR), the US Air Force Office of Scientific Research (AFOSR), the Foundational Questions Institute (FQXi), and the John Templeton Foundation (JTF). In particular, during the writing of this book both authors were generously supported by the latter. As a result, the Quantum Group at Oxford has grown over the past 10 years from 5 members in 2004 to 50 members now, and several of its former members have even started to build groups elsewhere, spreading the Gospel of pictures and processes. The constant interaction with the Quantum Group (and its numerous diaspora) has of course been absolutely essential in the development of this book.

So where did the idea of actually writing this book come from? Here at Oxford one typically doesn't like to mention 'that other place'. But, in the summer of 2012 Cambridge University Press contacted us to ask whether the diagrammatic language may be ready for a book. This generated the idea of teaching the Quantum Computer Science course that fall entirely with diagrams. The lecture notes would then provide the basis for a book, which would most certainly be ready by spring 2013. A quick glance at the first couple of pages in this book should tell you that this plan failed. The lecture notes were entirely dumped, and in fall 2013 we started again from scratch, leading up to what you are reading now.

Quite a number of things did happen from start to finish, like one of the authors relaunched a music career, met a girl, got married, made a baby, got a baby, and took a baby

Figure 0.3 Aleks' beard growth, as correlated with textbook completion.

to Beijing to watch him play a metal show. Meanwhile, the other author also got married, had a brief foray into stand-up comedy, got a position at Radboud University (amongst Bob's mortal enemies: the Dutch), and grew a humongous beard (Fig. 0.3). Both authors became well known at a local pub for being beaten up outside (rumour has it over a dispute on the interpretations of quantum theory, but neither author remembers too well). They also formed a southern country folk industrial noise band called the Quantum Dagger Orchestra.

The students who have taken this course over the past several years have been an invaluable source of inspiration and practical guidance in terms of what works and what doesn't when teaching this drastically new approach to quantum theory. In particular, we would like to thank students Jiannan Zhang, William Dutton, Jacob Cole, Pak Choy, and Craig Hull in the class of 2013, Tomas Halgas in the class of 2014, and Ernesto Ocampo, Matthew Pickering, Callum Oakley, Ashok Menon, Ignacio Funke Prieto, and Benjamin Dawes in the class of 2015, who have all contributed to this finished product by pointing out typos in those original (and revised) lecture notes.

We would especially like to thank Yaared Al-Mehairi, Daniel Marsden, Katie Casey, John-Mark Allen, Fabrizio Genovese, Maaike Zwart, Hector Miller-Bakewell, Joe Bolt, John van de Wetering, and Adrià Garriga Alonso, who all provided substantial qualities and/or quantities of corrections, and the class tutors Miriam Backens, Vladimir Zamdhiev, Will Zeng, John-Mark Allen, and Ciaran Lee for their valuable overall input, model solutions, and for bearing the brunt of the students' frustration when some of the more 'experimental' exercises went wrong.

Detailed corrections on the final manuscript were provided by booze brothers John Harding and Frank Valckenborgh, and the pictures of angry Bob were all taken by yet another booze brother, Ross Duncan. Consistent reminders for Aleks to eat and avoid head trauma were graciously provided by his wife Claire. Bob failed to take account of these reminders by his wife Selma.

All of the diagrams in this book were created using PGF/TikZ package for LaTeX and the TikZiT software. Grab the latest version from tikzit.github.io.

Finally: Why Cambridge University Press and not Oxford University Press? Because a CUP causes a whole lot of magic in this book while an OUP is ... no clue.