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978-1-107-10347-4 - Multiscale Methods for Fredholm Integral Equations

Zhongying Chen, Charles A. Micchelli and Yuesheng Xu

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