

Cambridge University Press

978-1-107-10232-3 - Random Wireless Networks: An Information Theoretic Perspective

Rahul Vaze

Frontmatter

[More information](#)

Random Wireless Networks

An Information Theoretic Perspective

Rahul Vaze



CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE
UNIVERSITY PRESS

Cambridge House, 4381/4 Ansari Road, Daryaganj, Delhi 110002, India

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org

Information on this title: www.cambridge.org/9781107102323

© Rahul Vaze 2015

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 2015

Printed in India

A catalogue record for this publication is available from the British Library

Library of Congress Cataloguing-in-Publication data

Vaze, Rahul.

Random wireless networks : an information theoretic perspective / Rahul Vaze.

pages cm

Includes bibliographical references and index.

Summary: "Provides detailed discussion on single hop and multi hop model, feedback constraints and modern communication techniques such as multiple antenna nodes and cognitive radios"– Provided by publisher.

ISBN 978-1-107-10232-3 (hardback)

1. Wireless communication systems. I. Title.

TK5103.2.V39 2015

621.382'1–dc23

2014044738

ISBN 978-1-107-10232-3 Hardback

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication, and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.

Cambridge University Press

978-1-107-10232-3 - Random Wireless Networks: An Information Theoretic Perspective

Rahul Vaze

Frontmatter

[More information](#)

To my son Niraad;
this book was written while babysitting him.

Cambridge University Press

978-1-107-10232-3 - Random Wireless Networks: An Information Theoretic Perspective

Rahul Vaze

Frontmatter

[More information](#)

Contents

List of Figures	vii
Preface	xi
Acknowledgments	xiv
Notation	xv
1 Introduction	1
1.1 Introduction	1
1.2 Point-to-Point Wireless Signal Propagation Model	3
1.3 Shannon Capacity	5
1.4 Outage Capacity	6
1.5 Wireless Network Signal Model	7
1.6 Connectivity in Wireless Networks	10
Bibliography	11
2 Transmission Capacity of ad hoc Networks	12
2.1 Introduction	12
2.2 Transmission Capacity Formulation	13
2.3 Basics of Stochastic Geometry	16
2.4 Rayleigh Fading Model	19
2.5 Path-Loss Model	22
2.6 Optimal ALOHA Transmission Probability	26
2.7 Correlations with ALOHA Protocol	27
2.8 Transmission Capacity with Scheduling in Wireless Networks	31
2.9 Reference Notes	41
Bibliography	42
3 Multiple Antennas	43
3.1 Introduction	43
3.2 Role of Multiple Antennas in ad hoc Networks	44
3.3 Channel State Information Only at Receiver	44
3.4 Channel State Information at Both Transmitter and Receiver	58
3.5 Spectrum-Sharing/Cognitive Radios	66
3.6 Reference Notes	77
Bibliography	77

4	Two-Way Networks	80
4.1	Introduction	80
4.2	Two-Way Communication	80
4.3	Effect of Limited Feedback on Two-Way Transmission Capacity with Beamforming	90
4.4	Reference Notes	94
	Bibliography	94
5	Performance Analysis of Cellular Networks	95
5.1	Introduction	95
5.2	Random Cellular Network	96
5.3	Distance-Dependent Shadowing Model	103
5.4	Reference Notes	113
	Bibliography	114
6	Delay Normalized Transmission Capacity	116
6.1	Introduction	116
6.2	Delay Normalized Transmission Capacity	117
6.3	Fixed Distance Dedicated Relays Multi-Hop Model with ARQ	127
6.4	Shared Relays Multi-Hop Communication Model	137
6.5	Reference Notes	144
	Bibliography	144
7	Percolation Theory	146
7.1	Introduction	146
7.2	Discrete Percolation	147
7.3	Continuum Percolation	154
7.4	Reference Notes	175
	Bibliography	175
8	Percolation and Connectivity in Wireless Networks	176
8.1	Introduction	176
8.2	SINR Graph	177
8.3	Percolation on the PSG	177
8.4	Connectivity on the SINR Graph	185
8.5	Information Theoretic Secure SINR Graph	192
8.6	Reference Notes	200
	Bibliography	200
9	Throughput Capacity	202
9.1	Introduction	202
9.2	Throughput Capacity Formulation	203
9.3	Information Theoretic Upper Bound on the Throughput Capacity	214
9.4	Extended Networks	221
9.5	Reference Notes	223
9.A	Hierarchical Cooperation	223
9.B	Mutual Information of Multiple Antenna Channel with Quantization	224
	Bibliography	228
	Index	229

List of Figures

2.1	Transmission capacity with Rayleigh fading and path-loss model with ALOHA protocol.	26
2.2	Network goodput G with Rayleigh fading as a function of ALOHA access probability p .	27
2.3	Dots represent transmitters and squares represent receivers. Only those transmitters (squares) are allowed to transmit that lie outside the discs of radius d_{gz} centered at all the receivers.	32
2.4	Transmission capacity with Rayleigh fading as a function of guard zone distance d_{gz} .	34
2.5	A pictorial description of PPP Φ_h in comparison to original process Φ , where the density increases with increasing distance from the origin.	36
2.6	Network goodput with Rayleigh fading as a function of CSMA transmitter channel access threshold of τ_h for neighbourhood contention threshold of $\tau_c = 1$.	37
2.7	Network goodput with Rayleigh fading as a function of CSMA neighbourhood contention threshold τ_c with channel access threshold of $\tau_h = 1$.	38
2.8	Spatial model for CSMA with packet arrivals.	39
2.9	Outage probability comparison of ALOHA and SINR-based CSMA with Rayleigh fading.	41
3.1	Transmit-receive strategy with no CSI at the transmitter.	45
3.2	Squares represent the N_{canc} nearest canceled interferers with dashed lines, solid circles represent the r nearest uncanceled interferers whose interference contribution will be used to derive the lower bound on the outage probability, and unfilled circles are all the other uncanceled interferers.	51
3.3	Transmission capacity versus N with CSIR while canceling the nearest interferers for $k = 1$, $d = 1$ m, $\beta = 1$ bits, $\alpha = 3$, $\epsilon = 0.1$.	56
3.4	Transmit-receive strategy with beamforming at the transmitter.	60
3.5	Empirical expected value of the reciprocal of the largest eigenvalue of $\mathbf{H}_{00}\mathbf{H}_{00}^\dagger$.	63
3.6	Transmission capacity versus the number of antennas N with multimode beamforming and canceling the nearest interferers with single stream data transmission $k = 1$, $d = 5$ m, $\beta = 1$ ($B = 1$ bits/sec/Hz), $\alpha = 4$, $\epsilon = 0.1$.	64
3.7	Transmission capacity versus the number of transmitted data streams k with multimode beamforming and canceling the nearest interferers with $d = 5$ m, $\beta = 1$, $\alpha = 4$, $\epsilon = 0.1$, total number of antennas $N = 8$.	65

- 3.8 Transmit-receive strategy of secondary transmitters and receivers (dots) and primary transmitters and receivers (squares), where each secondary transmitter suppresses its interference toward its $N_t - 1$ nearest primary receivers. 68
- 3.9 Each dot (secondary transmitter) suppresses its interference toward its 3 nearest squares (primary receivers) denoted by dashed lines, but still a square can receive interference from one of its 3 nearest dots. 69
- 3.10 Density of the secondary network with respect to number of transmit and receive antennas N_t, N_r at the secondary nodes. 72
- 4.1 Schematic for wireless network with two-way communication, where black dots represent nodes of Φ_T and gray dots represent nodes of Φ_R . 81
- 4.2 Schematic of two-way communication with two pairs of nodes. 85
- 4.3 Comparison of one-way and two-way transmission capacity with $d = 5$ m, $\alpha = 4$, $B_{TR} = 1$ Mbits, $B_{RT} = 0.03$ Mbits, $F = 1.1$ MHz, and $F_{TR} = 1$ MHz. 87
- 4.4 Two-way transmission capacity as a function of bandwidth allocation. 88
- 4.5 Comparison of transmission capacity performance of beamforming with genie-aided and practical feedback as a function of number of transmit antennas N . 93
- 5.1 (a) Multi-tier wireless network with macro (dots), femto (circles), and pico basestations (squares) versus (b) the random cellular network deployment with identical density. Fig. (a) shows the Voronoi regions of macro basestations, while (b) shows the the Voronoi regions of all basestations combined. 96
- 5.2 Connection probability $P_c(\beta)$ as a function of the basestation density λ . For the no noise case, $P_c(\beta)$ is invariant to λ . With additive noise, $P_c(\beta)$ does depend on λ , but the dependence is very minimal, and no noise assumption is fairly accurate. 100
- 5.3 Connection probability $P_c(\beta)$ as a function of SINR threshold β for $\lambda = 1$ and path-loss exponent $\alpha = 4$. 101
- 5.4 Comparing the connection probability $P_c(\beta)$ as a function of SINR threshold β for $\lambda = 1$ and path-loss exponent $\alpha = 4$ for the random wireless network and a square grid network. 103
- 5.5 Circle nodes are basestations and the square node is the receiver. The blockage process is described by randomly oriented rectangles, and the thickness of the line between basestations and receiver indicates the relative signal strength at the receiver that is inversely proportional to the number of blockages crossing the link. 105
- 5.6 Link \mathcal{L} of length d between basestation T and mobile user at o . Any rectangle of $\Sigma(\mathbf{C}_k, \ell, w, \theta)$ intersects \mathcal{L}_n only if its center lies in the region defined by vertices $ABCDEF$, where each vertex is the center of the six rectangles of length ℓ and width w . 107
- 5.7 Connection probability as a function of the basestation density for $\mathbb{E}[L] = \mathbb{E}[W] = 15$ m, $\mu_0 = 4.5 \times 10^{-4}/\text{m}^2$, $\lambda_0 = 3.5 \times 10^{-5}/\text{m}^2$, $\alpha = 4$, and $P = 1$. 112
- 6.1 Retransmission strategy where in any slot, retransmission (shaded square) is made if $\mathbf{1}_{T_s(t)} = 1$ and no attempt is made (empty square) otherwise. 119
- 6.2 Success probability as a function of D for $N_h = 1$. 126
- 6.3 Schematic of the system model where connected lines depict a path between a source and its destination. 128

6.4	Success probability as a function of number of retransmissions D for a two-hop network $N_h = 2$ with $d_1 = d_2 = 1m$ and equally dividing the retransmissions constraint over two hops, $D_1 = D_2 = D/2$.	132
6.5	Delay normalized transmission capacity as a function of retransmissions used on first hop D_1 with total retransmissions $D = 4$, for equidistant and non-equidistant two hops $N_h = 2$.	135
6.6	Delay normalized transmission capacity as a function of number of hops N_h for $\lambda = 0.1$ and $\lambda = 0.5$.	138
6.7	Transmission model for multi-hop communication, where dots are transmitters and squares are receivers, and the spatial progress for T_0 is the largest projection of squares on the x -axis for which $e_{0j} = 1$.	143
7.1	Square lattice \mathbb{Z}^2 with open and closed edges.	148
7.2	Dual lattice \mathbb{D}^2 of the square lattice \mathbb{Z}^2 is represented with dashed lines, where an edge is open/closed in \mathbb{D}^2 if the edge of \mathbb{Z}^2 intersecting it is open/closed.	149
7.3	Depiction of a closed circuit of dual lattice \mathbb{D}^2 surrounding the origin.	150
7.4	Counting the maximum number of closed circuits of length n surrounding the origin.	151
7.5	Left-right crossings of box $B_{n/2}$ by connected paths of square lattice \mathbb{Z}^2 .	153
7.6	Hexagonal tiling of \mathbb{R}^2 with each face open (shaded)/closed independently.	154
7.7	Gilbert's disc model where each node has a radio range $r/2$ and any two nodes are connected that are at a distance of less than r .	155
7.8	Invariance property of the Gilbert's disc model under fixed λr^2 , where scaling radio range by $1/r$ and scaling distance between any two nodes also by $\frac{1}{r}$ has no effect on the connection model, where the two-sided arrow depicts an edge.	156
7.9	Mapping Gilbert's disc model on a hexagonal tiling of \mathbb{R}^2 , where a face is open if it contains at least one node of Φ .	157
7.10	The largest region for finding new neighbors of x_1 that are not neighbors of the origin.	158
7.11	A realization of the Gilbert's random disc model, where each node x_i has radius r_i and two nodes are defined to be connected if their corresponding discs overlap.	166
7.12	Depiction of scenario considered for obtaining Proposition 7.3.20, where the farthest node lies outside of B_{10r} and event $G(r)$.	168
7.13	Covering of $B_{10} \setminus B_9$ by discrete points (black dots) lying on the boundary of B_{10} using boxes B_1 .	169
7.14	Depiction of scenario when both events $E(o, 10r)$ and $A_{B_{100r}}(r)$ occur simultaneously, giving rise to two smaller events that are i.i.d. with $E(o, r)$ around B_{10r} and B_{80r} .	171
8.1	Definition of event A_e for any edge e in \mathbf{S} .	179
8.2	Two adjacent open edges of \mathbf{S} imply a connected component of $G_P(\lambda, r)$ crossing rectangle \mathbf{R}_e 's corresponding to the open edges of \mathbf{S} . Solid lines are for rectangle \mathbf{R}_{e_1} and dashed lines for \mathbf{R}_{e_2} .	181
8.3	Square grid formed by centers (represented as dots) of edges of \mathbf{S} with side $\frac{s}{\sqrt{2}}$.	184
8.4	Square tiling of the unit square, and pictorial definition of square $s_t(m)$ for each node x_t .	186
8.5	Coloring the square tiling of the unit square with four sets of colors.	188

- 8.6 Interference for node x_u with respect to node x_t only comes from at most one node lying in the shaded squares, where distance from x_u to nodes in the shaded squares at level z is at least $\left(2z \left(\sqrt{\frac{\eta \ln n}{n}} - \sqrt{\frac{m \ln n}{n}}\right)\right)$. 189
- 8.7 Distance-based secure graph model, where dots are legitimate nodes and crosses are eavesdropper nodes, and x_i is connected to x_j if x_j lies in the disc of x_i with radius equal to the nearest eavesdropper distance. 192
- 8.8 Open edge definition on a square lattice for super critical regime. 196
- 8.9 Open edge definition on a square lattice for sub critical regime. 198
- 9.1 Shaded region is the guard-zone based exclusion region around the receiver x_j , where no transmitter (squares) other than the intended transmitter x_i is allowed to lie. 206
- 9.2 Shaded discs around the two receivers (dots) x_j and x_ℓ are not allowed to overlap for successful reception at both x_j and x_ℓ from x_i and x_k , respectively. 206
- 9.3 Overlap of $\mathbf{B}(x_i, |x_i - x_j| = r)$ with \mathbf{S}_1 when $x_i, x_j \in \mathbf{S}_1$. 207
- 9.4 Left figure defines a tiling of \mathbf{S}_1 by smaller squares of side $\frac{\tau}{\sqrt{n}}$. On the right figure we join the opposite sides of square by an edge (dashed line) and define it to be open (solid line) if the corresponding square contains at least one node in it. 209
- 9.5 Partitioning \mathbf{S}_1 into rectangles of size $\frac{\sqrt{2}\tau}{\sqrt{n}} \ln \frac{\sqrt{n}}{\sqrt{2}\tau} \times 1$, where each rectangle contains at least $\delta \ln \frac{\sqrt{n}}{\sqrt{2}\tau}$ disjoint left-right crossings of the square grid defined over \mathbf{S}_1 . 210
- 9.6 Time-sharing by relay nodes using K^2 different time slots, where at any time relays lying in shaded squares transmit. 211
- 9.7 Each node (black dots) connects to its nearest relay (hollow circle) and the distance between any node and its nearest relay is no more than the width of the rectangle $\frac{c\sqrt{2} \ln \frac{\sqrt{n}}{\sqrt{2}\tau}}{\sqrt{n}}$. 213
- 9.8 Hierarchical layered strategy for achieving almost linear scaling of the throughput capacity. 216
- 9.9 In phase 1 all nodes in a cluster exchange their bits, where only the clusters in shaded squares are active at any time. 218
- 9.10 Sequential transmission of information between all M nodes of two clusters over long-range multiple antenna communication. 219
- 9.11 Three-phase protocol for source (hollow dot)-destination (black square) pairs lying in adjacent clusters. 220
- 9.12 Shaded squares are active source-destination clusters in Phase 2. 224

Preface

In addition to the traditional cellular wireless networks, in recent past, many other wireless networks have gained widespread popularity, such as sensor networks, military networks, and vehicular networks. In a sensor network, a large number of sensors are deployed in a geographical area for monitoring physical parameters (temperature, rainfall), intrusion detection, animal census, etc., while in a military network, heterogeneous military hardware interconnects to form a network in a battlefield, and vehicular networks are being deployed today for traffic management, emergency evacuations, and efficient routing. For efficient scalability, these new wireless networks are envisaged to be self-configurable with no centralized control, sometimes referred to as *ad hoc* networks.

The decentralized mode of operation makes it easier to deploy these networks, however, that also presents with several challenges, such as creating large amount of interference, large overheads for finding optimal routes, complicated protocols for cooperation and coordination. Because of these challenges, finding the performance limits, both in terms of the amount of information that can be carried across the network and ensuring connectivity in the wireless network, is a very hard problem and has remained unsolved in its full generality.

From an information-theoretic point of view, where we are interested in finding the maximum amount of information that can be carried across the network, one of the major bottlenecks in wireless network is the characterization of interference. To make use of the spatial separation between nodes of the wireless network, multiple transmitters communicate at the same time, creating interference at other receivers. The arbitrary topology of the network further compounds the problem by directly affecting the signal interaction or interference profile. Thus, one of the several trade-offs in wireless networks is the extent of spatial reuse viz-a-viz the interference tolerance. Another important trade-off is the relation between the radio range (distance to which each node can transmit) of sensor nodes and the connectivity of the wireless network. Small radio range leads to isolated nodes, while larger radio ranges result in significant interference at the neighboring receivers affecting connectivity.

Over the last decade and a half, these trade-offs have been addressed in a variety of ways, with exact answers derived for *random* wireless networks, where nodes of the wireless network are located uniformly at random in a given area of interest. The primary reason for assuming random location for nodes is the applicability of rich mathematical tools from stochastic geometry, percolation theory, etc. that provide significant mathematical foundation and allow derivation of concrete results. This book ties up the different ideas introduced for understanding the performance limits of random wireless networks and presents a complete overview on the advances made from an information-theoretic (capacity limits) point of view.

In this book, we focus on two capacity metrics for random wireless networks, namely, the transmission and the throughput capacity, that have been defined to capture the successful number

of bits that can be transported across the network. We present a comprehensive analysis of transmission capacity and throughput capacity of random wireless networks. In addition, using the tools from percolation theory, we also discuss the connectivity and percolation properties of random wireless networks, which impact the routing and large-scale connectivity in wireless networks. The book is presented in a cohesive and easy to follow manner, however, without losing the mathematical rigor. Sufficient background and critical details are provided for the advanced mathematical concepts required for solving these problems.

The book is targeted at graduate students looking for an easy and rigorous introduction to the area of information/communication theory of random wireless networks. The book also quantifies the effects of network layer protocols (e.g., automatic repeat requests (ARQs)), physical layer technologies such as multiple antennas (MIMO), successive interference cancellation, information-theoretic security, on the performance of wireless networks. The book is accessible to anyone with a background in basic calculus, probability theory, and matrix theory.

The book starts with an introduction to the signal processing, information theory, and communication theory fundamentals of a point-to-point wireless communication channel. Specifically, a quick overview of the concept of Shannon capacity, outage formulation, basic information-theoretic channels, basics of multiple antenna communication, etc. is provided that lays sufficient background for the rest of the book.

The book is divided into two parts, the first part exclusively deals with single-hop wireless networks, where each source–destination communicates directly with each other, while in the second part, we focus only on the multi-hop wireless networks, where source–destination pairs are out of each others' communication range and use multiple other nodes (called relays) for communication.

For the first part, we begin by deriving analytical expressions for the transmission capacity for a single-hop model with various scheduling protocols such as ALOHA, CSMA, guard-zone based, etc. Next, we discuss in detail the effect of using multiple antennas on the transmission capacity of a random wireless network and derive the optimal role of multiple antennas. We then extend our setup and present performance analysis of random wireless networks under a two-way communication model that allows for bidirectional communication between two nodes. We close the first part of the book by applying stochastic geometry tools to derive a tractable performance analysis of a cellular wireless network in terms of critical measures such as connection probability, average rate, etc. which is extremely useful for practicing engineers.

The second part of the book starts by extending the transmission capacity framework to a multi-hop wireless network, where we derive the transmission capacity expression and find the optimal value of several key parameters relevant to the multi-hop communication model. Then, we give a brief introduction to percolation theory results for both the discrete and the continuum case. The background on percolation theory sets up the platform for deriving several important results for random wireless networks, such as finding the optimal radio range for connectivity, formation of large connected clusters under different connection models, and most importantly for finding tight scaling bounds on the throughput capacity.

We then present the seminal result of Gupta and Kumar which shows that the throughput capacity of a random wireless network scales as square root of the number of nodes. Finally, we discuss the concept of hierarchical cooperation in a wireless network which is used to show that the throughput capacity can scale linearly with the number of nodes.

This book is an effort to present the several disparate ideas developed for deriving capacity of a random wireless network in a unified framework. For effective understanding, extensive effort is made to explain the physical interpretation of all results. As an attempt to reach out to a wider

Cambridge University Press

978-1-107-10232-3 - Random Wireless Networks: An Information Theoretic Perspective

Rahul Vaze

Frontmatter

[More information](#)

audience, effects of practical communication models, such as cellular networks, two-way communication (downlink/uplink) feedback constraints, modern communication techniques (such as multiple antenna nodes, interference cancelation and avoidance, cognitive radios), are also analyzed and discussed in sufficient detail.

Most of the ideas/results presented in this book are not more than a decade old and have not yet found a consolidated treatment. The presentation is kept short and lucid with sufficient detail and rigor. For clarity, at instances, places simplified proofs of the original results are provided.

Acknowledgments

I would like to thank everyone who have made this book possible. Starting with Srikanth Iyer, D. Yogeshwaran, Aditya Gopalan, Chandra Murthy, Radhakrishna Ganti, Sibiraj Pillai, Siddharth Banerjee, all have made detailed comments on my various drafts, which undoubtedly made the book more readable. Their critical comments have also shaped the structure and content. I would also like to thank Kaibin Huang, who urged me to write this as a textbook that he could use in his course. I do not know yet whether it will serve his purpose. Help from undergraduate internship students Vivek Bagaria, Dheeraj Narasimha, Jainam Doshi, Rushil Nagda, Siddharth Satpathi, Ajay Krishnan, and Navya Prem in proofreading is gratefully acknowledged. Their comments were valuable in making the book accessible to readers with little or no prior background. The review comments by the two referees helped immensely in changing the structure of the book. Their comments especially helped in the reorganization and pruning of the book to keep a clear and sharp focus.

Notation

\mathbf{A}	Matrix A
$\mathbf{A}(i, j)$	(i, j) th entry of matrix A
\mathbf{a}	vector a
$\mathbf{a}(i)$	i th element of vector \mathbf{a}
\mathbf{a}^\dagger	Conjugate transpose of vector \mathbf{a}
\mathbb{R}	Set of real numbers
\mathbb{C}	Set of complex numbers
\mathbb{R}^d	Set of real numbers in d dimensions
$\mathbb{C}^{m \times n}$	Matrices with m rows and n columns over the set of complex numbers
$\mathbb{P}(A)$	Probability of event A
\mathbb{E}	Expectation operator
$\#(A)$	Number of elements in set A
$\nu(A)$	Lebesgue measure of set A
o	Origin in \mathbb{R}^d
d_{xy}	Distance between nodes x and y
α	Path-loss exponent for wireless propagation
$\mathbf{B}(x, r)$	Disc of radius r centered at x
ϕ	Null set
$ a $	Absolute value of a
Φ	A Poisson point process
p	ALOHA access probability
λ	Density of nodes of the network
μ	Density of blockages/obstacles in the network
ϵ	Outage probability constraint
Λ	Density measure of nodes of the network
γ	Interference suppression parameter
ρ	Random variable representing the random radius in the Gilbert's random disc model
$\Gamma(t)$	$\int_0^\infty x^{t-1} \exp^{-x} dx$
\mathbf{J}	$\sqrt{-1}$
$A \propto B$	$A = cB$, where c is a constant
SNR	Signal-to-noise-ratio
SIR	Signal-to-interference-ratio
SINR	Signal-to-interference-plus-noise-ratio

β	Signal-to-interference-plus-ratio threshold for successful packet reception
B	Rate of transmission corresponding to SINR threshold β , $\beta = 2^B - 1$
M_n	Number of retransmissions required on hop n
N_h	Number of hops
M	Number of end-to-end retransmissions required $\sum_{n=1}^{N_h} M_n = M$
$t(n)$	Per-node throughput capacity
$T(n)$	Network wide throughput capacity
C	Transmission capacity
C_{tw}	Two-way transmission capacity
C_d	Delay normalized transmission capacity
C_s	Spatial progress capacity
AWGN	Additive white Gaussian noise
N_0	Variance of the AWGN
$\mathcal{N}(m, \text{var})$	Normal distribution with mean m and variance var
$\mathcal{CN}(m, \text{var})$	Complex normal distribution with mean m and variance var
$\mathbf{1}_n$	Indicator variable for node n
$\chi^2(2m)$	Chi-square distribution with m degrees of freedom
$X \sim Y$	Random variable X has distribution Y
$f(n) = \Omega(g(n))$	If $\exists k > 0, n_0, \forall n > n_0, g(n) k \leq f(n) $
$f(n) = \mathcal{O}(g(n))$	If $\exists k > 0, n_0, \forall n > n_0, f(n) \leq g(n) k$
$f(n) = \Theta(g(n))$	If $\exists k_1, k_2 > 0, n_0, \forall n > n_0, g(n) k_1 \leq f(n) \leq g(n) k_2$
$f(n) = o(g(n))$	If $\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = 0$