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The Homotopy Theory of $(\infty, 1)$-Categories

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CAMBRIDGE
UNIVERSITY PRESS

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University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre,
New Delhi – 110025, India

79 Anson Road, #06–04/06, Singapore 079906

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www.cambridge.org
Information on this title: www.cambridge.org/9781107101364
DOI: 10.1017/9781316181874

First published 2018

Printed in the United Kingdom by Clays, St Ives plc

A catalogue record for this publication is available from the British Library.

ISBN 978-1-107-10136-4 Hardback
ISBN 978-1-107-49902-7 Paperback

Ad Majorem Dei Gloriam

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Preface

The starting point for this book was the mini-workshop on "The Homotopy Theory of Homotopy Theories", held in Caesarea, Israel, in May 2010, and the lecture notes from the talks given there. I was asked by a number of people if those notes might be turned into a more formal manuscript, and this book is the result. In the end, I have omitted some of the topics that were addressed at that workshop, simply because to have included them properly would have greatly increased the length. The topics included in the last chapter of this book were chosen to be in line with some of the applications that were discussed there.

Since 2010, our understanding of $(\infty, 1)$-categories has only increased, and they are being used in a wide range of applications. There are many directions I could have taken with this book, but I have chosen here to give a balanced treatment of the different models and the comparisons between them. I also look at these structures primarily from the viewpoint of homotopy theory, considering model structures and Quillen equivalences. In particular, I do not go into much detail on the treatment of $(\infty, 1)$-categories as generalizations of categories, nor to the development of standard categorical notions in this new context. Joyal and Lurie have treated this topic extensively, extending many categorical notions into the context of quasi-categories in particular [73, 88]. Much less has been done in other models with weak composition, but some work has been done in complete Segal spaces, for example [76].

A number of choices have been made in the presentation given here. While the goal is to give a thorough treatment of the different models, experts on the subject know that just about every model has some technicalities which are intuitively sensible but exceptionally messy to prove. In many cases, I have chosen to omit these technical points and simply refer the reader to the original reference. While this decision makes our book less comprehensive, the hope is that it allows the reader to get the big ideas and most of the details of the proofs without getting sidetracked into often unenlightening, if necessary, combinatorial arguments. I have also deliberately suppressed most set-theoretic points, but have tried to point out where they arise.

Finally, I should point out that our treatment does not treat all possible approaches to the subject. Most notably absent is Toën's axiomatic treatment [116], but recently there have also been more geometric models, for example by Ayala, Francis, and Rozenblyum [5, 6], as well as a formal categorical treatment by Riehl and Verity [105, 106, 107].

Acknowledgments

I would like to extend my gratitude to David Blanc, Emmanuel Farjoun, and David Kazhdan for organizing the workshop whose notes were the starting point for this book, and inviting me to be the primary speaker that year. I'd also like to thank Ilan Barnea, David Blanc, Boris Chorny, Emmanuel Farjoun, Yonatan Harpaz, Vladimir Hinich, Matan Prasma, and Tomer Schlanck for their talks during the workshop; while not everything that was discussed there ended up in this book, I am thankful for their contribution to the ideas that led to this work.

Converting 40 pages of lecture notes to a full-length book was a nontrivial task. I want to thank the people who have shared their enthusiasm for the project over the years, including John Greenlees, Mike Hill, Nick Kuhn, and Angélica Osorno. I would also like to thank Matthew Barber, Christina Osborne, Viktoriya Ozornova, Alex Sherbetjian, and Jacob West for their comments on various stages of this manuscript, and the many minor errors they helped me to identify. I'd also like to thank the staff at Cambridge University Press for their patience and help along the way.

Along the way, I was able to correct some mistakes and hopefully clarify some confusing points in some of my earlier work. Some of these difficulties were discovered in conversations with Matthew Barber, Clark Barwick, Christina Osborne, Luis Pereira, and Chris Schommer-Pries, and I thank each of them for bringing these issues to my attention. Discussions with Charles Rezk and Ieke Moerdijk, mostly in the context of ongoing collaborative work, have led to some improved versions of proofs, and I thank both of them for their helpful insights. I'd also like to thank Emily Riehl for bringing my attention to the description of cofibrant simplicial categories.

Finally, much of my own work that appears here had its origins in my PhD thesis. I'd like to thank Bill Dwyer for getting me started in this direction of research and for his continuing support and enthusiasm.

During the time I worked on this book, I was supported by NSF grants DMS-1105766 and DMS-1352298. Some of this work was done while I was a participant at the MSRI program on Algebraic Topology in Spring 2014, which was supported by NSF grant 0932078 000, and while I was a visitor at the Hausdorff Institute for Mathematics in Summer 2015.