Cambridge University Press 978-1-107-10012-1 - Random Processes for Engineers Bruce Hajek Table of Contents More information

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