Random Processes for Engineers

This engaging introduction to random processes provides students with the critical tools needed to design and evaluate engineering systems that must operate reliably in uncertain environments.

A brief review of probability theory and real analysis of deterministic functions sets the stage for understanding random processes, while the underlying measure theoretic notions are explained in an intuitive, straightforward style. Students will learn to manage the complexity of randomness through the use of simple classes of random processes, statistical means and correlations, asymptotic analysis, sampling, and effective algorithms. Key topics covered include:

- Calculus of random processes in linear systems
- Kalman and Wiener filtering
- Hidden Markov models for statistical inference
- The estimation maximization (EM) algorithm
- An introduction to martingales and concentration inequalities

Understanding of the key concepts is reinforced through more than 100 worked examples and 300 thoroughly tested homework problems (half of which are solved in detail at the end of the book, with the remaining solutions available online for instructors at *www.cambridge.org/hajek*).

Bruce Hajek has been an avid student, instructor, and user of probability theory for his entire career. He is the Mary Lou and Leonard C. Hoeft Chair of Engineering, Center for Advanced Study Professor of Electrical and Computer Engineering, and Professor in the Coordinated Science Laboratory at the University of Illinois. Among his many awards, he is a member of the US National Academy of Engineering and a recipient of the IEEE Koji Kobayashi Computers and Communications Award. He is co-author, with E. Wong, of the more advanced book, *Stochastic Processes in Engineering Systems*, 2nd edn, 1985.

"A comprehensive exposition of random processes ... Abstract concepts are nicely explained through many examples ... The book will be very helpful for beginning graduate students who want a firm foundational understanding of random processes. It will also serve as a nice reference for the advanced reader."

Anima Anandkumar, The University of California, Irvine

"This is a fantastic book from one of the eminent experts in the field and is the standard text for the graduate class I teach. The material covered is perfect for a first-year graduate class in Probability and Stochastic Processes."

Sanjay Shakkottai, The University of Texas at Austin

"This is an excellent introductory book on random processes and basic estimation theory from the foremost expert and is suitable for advanced undergraduate students and/or first-year graduate students who are interested in stochastic analysis. It covers an extensive set of topics that are very much applicable to a wide range of engineering fields."

Richard La, University of Maryland

"Bruce Hajek has created a textbook for engineering students with interest in control, signal processing, communications, machine learning, amongst other disciplines in electrical engineering and computer science. Anyone who knows Bruce Hajek knows that he cares deeply about the foundations of probability and statistics, and he is equally engaged in applications. Bruce is a dedicated teacher and author in the spirit of Prof. Joe Doob, formally at the statistics department at the University of Illinois.

I was fortunate to have a mature draft of his book when I introduced a stochastic processes course to my department in the spring of 2014. The book provides an entirely accessible introduction to the foundations of stochastic processes. I was surprised to find that the students in my course enjoyed Hajek's introduction to measure theory, and (at least by the end of the course) could appreciate the value of the abstract concepts introduced at the start of the text.

It includes applications of this general theory to many topics that are of tremendous interest to students and practitioners, such as nonlinear filtering, statistical methods such as the EM-algorithm, and stability theory for Markov processes. Because the book establishes strong foundations, in a course it is not difficult to substitute other applications, such as Monte-Carlo methods or reinforcement learning. Graduate students will be thrilled to learn these exciting techniques from an accessible source."

Sean Meyn, University of Florida

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BRUCE HAJEK
University of Illinois

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University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314-321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi - 110025, India

103 Penang Road, #05-06/07, Visioncrest Commercial, Singapore 238467

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www.cambridge.org
Information on this title: www.cambridge.org/9781107100121

First published 2015

A catalogue record for this publication is available from the British Library

Library of Congress Cataloging in Publication data
Hajek, Bruce.
Random processes for engineers / Bruce Hajek, Illinois.
Includes bibliographical references and index.
ISBN 978-1-107-10012-1 (Hardback)
1. Engineering mathematics–Study and teaching (Graduate) 2. Stochastic processes–Study and teaching (Graduate) 3. Numbers, Random–Study and teaching (Graduate) I. Title.
TA340.H325 2015
519.2´3–dc23 2014035900

ISBN 978-1-107-10012-1 Hardback

Additional resources for this publication at www.cambridge.org/hajek

To Beth, for her loving support.

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Preface

From an applications viewpoint, the main reason to study the subject of this book is to help deal with the complexity of describing random, time-varying functions. A random variable can be interpreted as the result of a single measurement. The distribution of a single random variable is fairly simple to describe. It is completely specified by the cumulative distribution function $F(x)$, a function of one variable. It is relatively easy to approximately represent a cumulative distribution function on a computer. The joint distribution of several random variables is much more complex, for in general it is described by a joint cumulative probability distribution function, $F(x_1, x_2, \ldots, x_n)$, which is much more complicated than n functions of one variable. A random process, for example a model of time-varying fading in a communication channel, involves many, possibly infinitely many (one for each time instant t within an observation interval) random variables. Woe the complexity!

This book helps prepare the reader to understand and use the following methods for dealing with the complexity of random processes:

- Work with moments, such as means and covariances.
- Use extensively processes with special properties. Most notably, Gaussian processes are characterized entirely by means and covariances, Markov processes are characterized by one-step transition probabilities or transition rates, and initial distributions. Independent increment processes are characterized by the distributions of single increments.
- Appeal to models or approximations based on limit theorems for reduced complexity descriptions, especially in connection with averages of independent, identically distributed random variables. The law of large numbers tells us, in a certain sense, that a probability distribution can be characterized by its mean alone. The central limit theorem similarly tells us that a probability distribution can be characterized by its mean and variance. These limit theorems are analogous to, and in fact examples of, perhaps the most powerful tool ever discovered for dealing with the complexity of functions: Taylor's theorem, in which a function in a small interval can be approximated using its value and a small number of derivatives at a single point.
- Diagonalize. A change of coordinates reduces an arbitrary n-dimensional Gaussian vector into a Gaussian vector with n independent coordinates. In the new coordinates the joint probability distribution is the product of n one-dimensional

distributions, representing a great reduction of complexity. Similarly, a random process on an interval of time is diagonalized by the Karhunen–Loève representation. A periodic random process is diagonalized by a Fourier series representation. Stationary random processes are diagonalized by Fourier transforms.

- Sample. A narrowband continuous time random process can be exactly represented by its samples taken with sampling rate twice the highest frequency of the random process. The samples offer a reduced complexity representation of the original process.
- Work with baseband equivalent. The range of frequencies in a typical wireless transmission is much smaller than the center frequency, or carrier frequency, of the transmission. The signal could be represented directly by sampling at twice the largest frequency component. However, the sampling frequency, and hence the complexity, can be dramatically reduced by sampling a baseband equivalent random process.

This book was written for the first semester graduate course on random processes offered by the Department of Electrical and Computer Engineering at the University of Illinois at Urbana-Champaign. Students in the class are assumed to have had a previous course in probability, which is briefly reviewed in the first chapter. Students are also expected to have some familiarity with real analysis and elementary linear algebra, such as the notions of limits, definitions of derivatives, Riemann integration, and diagonalization of symmetric matrices. These topics are reviewed in the appendix. Finally, students are expected to have some familiarity with transform methods and complex analysis, though the concepts used are reviewed in the relevant chapters.

Each chapter represents roughly two weeks of lectures, and includes homework problems. Solutions to the even numbered problems without stars can be found at the end of the book. Students are encouraged to first read a chapter, then try doing the even numbered problems before looking at the solutions. Problems with stars, for the most part, investigate additional theoretical issues, and solutions are not provided.

Hopefully some students reading this book will find the problems useful for understanding the diverse technical literature on systems engineering, ranging from control systems, signal and image processing, communication theory, and analysis of a variety of networks and algorithms. Hopefully some students will go on to design systems, and define and analyze stochastic models. Hopefully others will be motivated to continue study in probability theory, going on to learn measure theory and its applications to probability and analysis in general.

A brief comment is in order on the level of rigor and generality at which this book is written. Engineers and scientists have great intuition and ingenuity, and routinely use methods that are not typically taught in undergraduate mathematics courses. For example, engineers generally have good experience and intuition about transforms, such as Fourier transforms, Fourier series, and z-transforms, and some associated methods of complex analysis. In addition, they routinely use generalized functions, in particular the delta function is frequently used. The use of these concepts in this book leverages on this knowledge, and it is consistent with mathematical definitions, but full mathematical

Cambridge University Press
978-1-107-10012-1 — Random Processes for Engineers
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justification is not given in every instance. The mathematical background required for a full mathematically rigorous treatment of the material in this book is roughly at the level of a second year graduate course in measure theoretic probability, pursued after a course on measure theory.

The author gratefully acknowledges the many students and faculty members, including Todd Coleman, Christoforos Hadjicostis, Jonathan Ligo, Andrew Singer, R. Srikant, and Venu Veeravalli who gave many helpful comments and suggestions.

Organization

The first four chapters of the book are used heavily in the remaining chapters, so most readers should cover those chapters before moving on.

Chapter 1 is meant primarily as a review of concepts found in a typical first course on probability theory, with an emphasis on axioms and the definition of expectation. Readers desiring a more extensive review of basic probability are referred to the author's notes for ECE 313 at the University of Illinois.

Chapter 2 focuses on various ways in which a sequence of random variables can converge, and the basic limit theorems of probability: law of large numbers, central limit theorem, and the asymptotic behavior of large deviations.

Chapter 3 focuses on minimum mean square error estimation and the orthogonality principle. Kalman filtering is explained from the geometric standpoint based on innovations sequences.

Chapter 4 introduces the notion of a random process, and briefly covers several key examples and classes of random processes. Markov processes and martingales are introduced in this chapter, but are covered in greater depth in later chapters.

After Chapter 4 is covered, the following four topics can be covered independently of each other.

Chapter 5 describes the use of Markov processes for modeling and statistical inference. Applications include natural language processing.

Chapter 6 describes the use of Markov processes for modeling and analysis of dynamical systems. Applications include modeling of queueing systems.

Chapters 7–9 develop calculus for random processes based on mean square convergence, moving to linear filtering, orthogonal expansions, and ending with causal and noncausal Wiener filtering.

Chapter 10 explores martingales with respect to filtrations, with emphasis on elementary concentration inequalities, and on the optional sampling theorem.

In recent one-semester course offerings, the author covered Chapters 1–5, Sections 6.1–6.8, Chapter 7, Sections 8.1–8.4, and Section 9.1. Time did not permit covering the Foster–Lyapunov stability criteria, noncausal Wiener filtering, and the chapter on martingales.

A number of background topics are covered in the appendix, including basic notation.