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Introduction

1.1 Introduction

Fracture mechanics provides the basis for designing machine and structural components with materials containing defects such as crack, gives rational approach for assessing degree of safety or reliability of an in-service degraded machine component, and helps to calculate the life of a component with crack subjected to cyclically fluctuating load, corrosion, creep, or a combination of all these. A crack is a discontinuity, internal or external (Figs 1.1 and 1.2), in the material with zero tip radius. The development of the subject has been driven by the stringent safety requirements of the aerospace industry, nuclear power plants and other safety-critical applications. The advancement in the understanding of the subject coupled with developments in material science, experimental methods, and numerical techniques such as finite element, boundary element, and meshless methods, has facilitated optimum design and minimization of material usage for an application.

This book presents the gradual development in the fundamental understanding of the subject and in numerical methods that have facilitated its applications. Though the subject can be studied from the viewpoint of material science and mechanics, the focus here is on the latter.

1.2 Linear Elastic Fracture Mechanics

Development of the subject originated with the work of Griffith (1921), who propounded the condition of unstable extension of an existing crack in a brittle material within the framework of global energy balance or the First Law of Thermodynamics. The shortcomings of the approach were eliminated by Irwin (1948), who classified the three fundamental modes of crack extension and presented the condition of fracture in terms of a parameter associated with the stress–strain field in the close neighbourhood of the crack-tip. He also showed the link between the crack-tip field parameter, the stress intensity factor (SIF), and the energy release rate parameter introduced by Griffith. These parameters have proved useful in characterizing the fracture of brittle materials and have helped in practical design applications. Brittle materials fracture without showing any plastic deformation before the onset of crack extension or during crack propagation. This type of failure is distinguished by the fact that the fractured parts can be put together to get the original geometry almost reconstructed (Fig. 1.1).

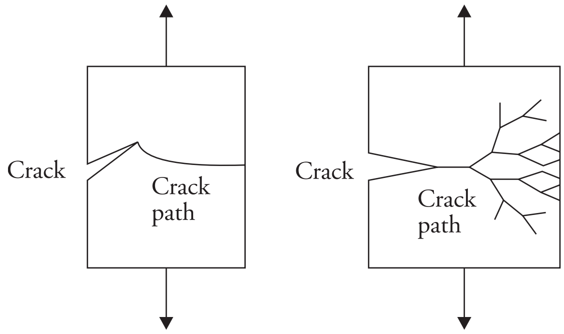

Figure 1.1 Brittle fractures of plates.

1.3 Elastic Plastic or Yielding Fracture Mechanics

Most materials that are used in engineering constructions and machine building are metallic and show plastic deformation around the crack-tip prior to crack extension and during crack extension (Fig. 1.2). They fracture showing features, combined to a varying degree, of normal stress driven brittle and shear stress driven ductile fractures. The plastic deformation preceding fracture was brought within the scope of energy balance approach and SIF through some minor amendments. A substantial increase in the scope was possible through an introduction of crack-tip field characterizing parameters such as the crack opening displacement (COD) by Wells (1961) and J integral by Rice (1968). While the energy release rate and the SIF concepts helped to lay the foundation of the linear elastic fracture mechanics (LEFM), the latter two provided much of the basis for yielding or elastic plastic fracture mechanics (YFM or EPFM).

1.4 Mixed Mode Fracture

Although Irwin classified the crack extensions into three fundamental modes, practical problems very often involve mixed mode of crack extensions (Fig. 1.3).

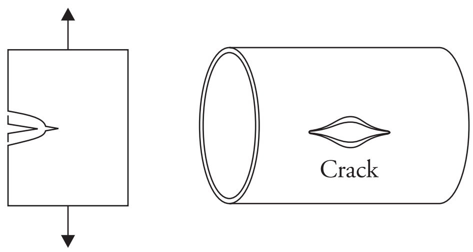

Figure 1.2 Crack extensions attendant with elastic plastic deformation.

Independent developments have subsequently taken place which explain such fractures within the scope of LEFM.

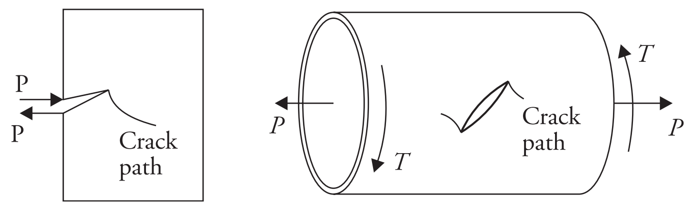

Figure 1.3 Mixed mode crack extensions in plate and hollow shaft.

1.5 Fatigue Crack Growth

A majority of machine and structural components (Fig. 1.4) are subjected to cyclically fluctuating load (Fig. 1.5).

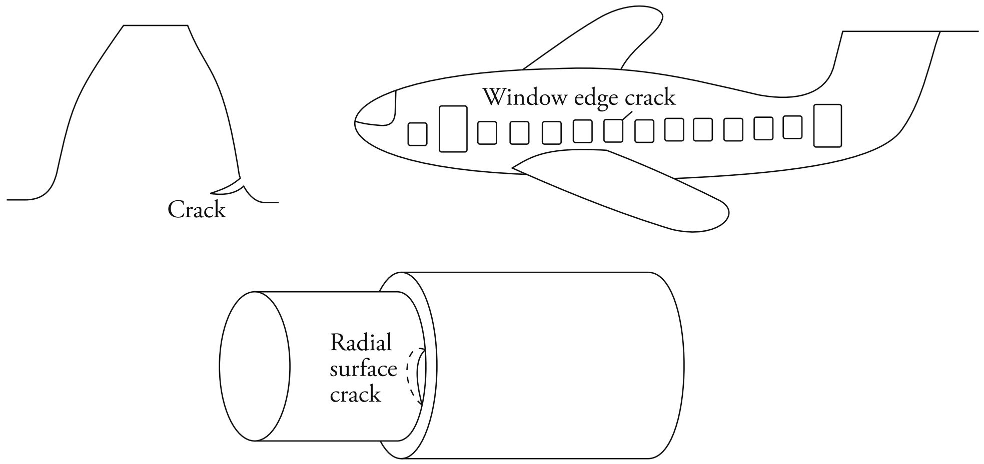

Figure 1.4 Cracks resulting from fatigue loading at the root of the gear tooth, window corner of an aircraft, and step of a shaft.

Based on an experimental study, Paris (1963) showed that the cyclic crack growth rate under fatigue loading can be characterized in terms of cyclic SIF range. This paved the way for an estimation of cyclic life of components. Paris law was subsequently enlarged in scope to accommodate variable and random amplitude cyclic loading, effects of occasional overloads, and so on.

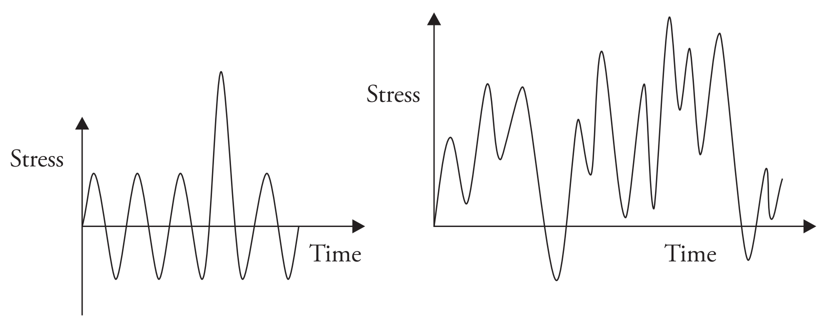

Figure 1.5 Constant amplitude fatigue loading with overload cycle and random cyclic fatigue loading.

Gradually the scope of LEFM has enlarged to take care of crack growth under stress corrosion, creep, and their combinations.

1.6 Computational Fracture Mechanics

The development of fracture mechanics has been driven by safety critical applications in defence, aerospace, power plants, oil industry, transport, etc. The applications of the principles of fracture mechanics in these areas have been facilitated by developments in numerical methods such as finite element method (FEM), boundary element method (BEM), and meshless method. Although many problems can be solved analytically and practical geometries can be handled by FEM or experimental methods, but problems of EPFM could rarely be handled by the analytical methods; solutions were obtained only through numerical methods such as FEM. Thus, computational methods have become part and parcel of fracture mechanics.

1.7 Scope of the Book

The book deals with the fundamentals of LEFM, EPFM, fatigue, computational issues, and mixed mode fracture. It also covers experimental methods and applications of fracture mechanics in design.

Developments in computational methods and experimental techniques have facilitated the study of fracture under impact loading conditions, high strain rates of deformation, large deformation processes such as metal forming, and so on. In addition methods have been developed for the study of fracture of composites. However these issues are beyond the scope of the book.

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Linear Elastic Fracture Mechanics

2.1 Introduction

The foundation for the understanding of brittle fracture originating from a crack in a component was laid by Griffith (1921), who considered the phenomenon to occur within the framework of its global energy balance. He gives the condition for unstable crack extension in terms of a critical strain energy release rate (SERR) per unit crack extension. The next phase of development, which is due to Irwin (1957a and b), is based on the crack-tip local stress–strain field and its characterization in terms of stress intensity factor (SIF). The condition of fracture is given in terms of the SIF reaching a critical value, and the parameter is shown to be related to the critical energy release rate given by Griffith. Later, the scope of the SIF approach was amended to take care of small-scale plastic deformation ahead of the crack-tip. Most of the present applications of the principles of linear elastic fracture mechanics (LEFM) for design or safety analysis have been based on this SIF.

This chapter presents the gradual developments that have taken place to advance the understanding of fracture of brittle materials and other materials that give rise to small-scale plastic deformation before the onset of crack extension. Examples are presented to illustrate the applications of LEFM to design.

2.2 Calculation of Theoretical Strength

A fracture occurs at the atomic level when the bonds between atoms are broken across a fracture plane, giving rise to new surfaces. This can occur by breaking the bonds perpendicular to the fracture plane, a process called cleavage, or by shearing bonds along a fracture plane, a process called shear. The theoretical tensile strength of a material will therefore be associated with the cleavage phenomenon (Tetelman and McEvily 1967; Knott 1973).

Generally, atoms of a body at no load will be at a fixed distance apart, that is, the equilibrium spacing a_0 (Fig. 2.1). When the external forces are applied to break the atomic bonds, the required force/stress (σ) increases with distance (a or x) till the theoretical strength σ_c is reached. Further displacement of the atoms can occur under a decreasing applied stress. The variation can be represented approximately by a sinusoidal variation as follows.

$$\sigma = \sigma_c \sin \frac{2\pi x}{\lambda} \tag{2.1}$$

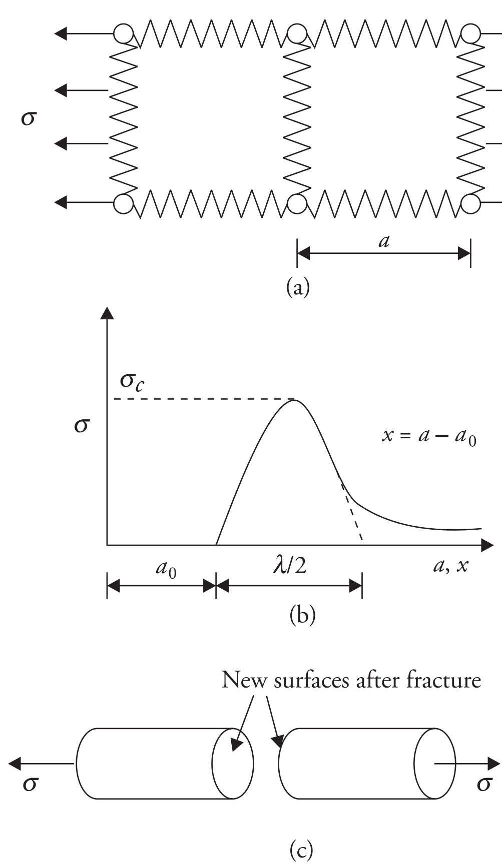

Figure 2.1 Atomic-level modelling of cleavage fracture. (a) Schematic representation of atomic interactions. (b) Variation of inter-atomic forces with spacing. (c) New surfaces created after fracture.

where λ is the wavelength of the load variation. The work done over half-cycle or span $\lambda/2$ is given by

$$W = \int_0^{\lambda/2} \sigma dx = \int_0^{\lambda/2} \sigma_c \sin \frac{2\pi x}{\lambda}\, dx = \sigma_c \frac{\lambda}{\pi} \tag{2.2}$$

If a cylinder of unit cross-sectional area breaks upon the application of load variation as shown in Fig. 2.1(b), two new surfaces of unit area each are created. To create each of these areas, specific surface energy γ_s, which is a material property, is needed. The energy stored at the two surfaces W_s is therefore given by

$$W_s = 2\,\gamma_s \tag{2.3}$$

This energy comes from the work done W in deforming the material before the separation. For conservation of energy $W = W_s$. For small displacements x, stress σ can be written using Hooke's law

$$\sigma = E\epsilon = E\,x/a_0 \tag{2.4}$$

Eqs. (2.1) and (2.4) give

$$\lambda = \sigma_c \frac{2\pi a_0}{E} \tag{2.5}$$

Combining Eqs. (2.2), (2.3), and (2.5)

$$\sigma_c^2 = \frac{E\,\gamma_s}{a_0} \quad \text{or} \quad \sigma_c = \sqrt{\frac{E\,\gamma_s}{a_0}} \tag{2.6}$$

For many materials $\gamma_s = \frac{Ea_0}{100}$ (Knott 1973), hence, $\sigma_c = \frac{E}{10}$. Actual strength, the ultimate strength σ_{ult}, measured during the tensile test lies in the range $\frac{E}{1000}$ to $\frac{E}{100}$. For example, for steel, $\sigma_{ult} = 400 - 800$ MPa and $E = 200$ GPa; therefore, $\sigma_c = \frac{E}{500}$ to $\frac{E}{250}$. Similarly, for aluminum, $\sigma_{ult} = 100 - 500$ MPa and $E = 70$ GPa; therefore, $\sigma_c = \frac{E}{700}$ to $\frac{E}{140}$.

The observed ultimate strength for most engineering metals is much below this theoretical prediction $\frac{E}{10}$ for σ_c. The observed strength is, in some cases, of the order of $\frac{E}{1000}$. The atomic model failed to explain the observed reduction in strength of a material.

2.3 Griffith's Explanation Based on Stress Concentration

Griffith(1921) attempted to explain the discrepancy between the theoretical and the actual strengths based on stress concentration. He suggested that a material, although apparently homogeneous, contains small defects such as cracks, which act as stress concentrators. The stress at the tip (A or B in Fig. 2.2) of such a crack reaches very high value σ_{max}, which may be comparable to the theoretical strength σ_c, although the applied stress σ is low. Using Inglis's (1913) solution, he was able to provide some justification for the reduction in the theoretical strength.

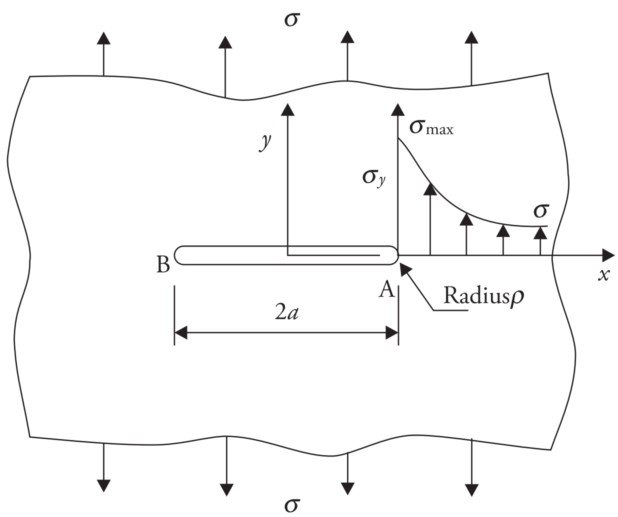

Figure 2.2 Stress concentration at the tip of crack-like defect in plate subjected to tension.

For a uniformly loaded tensile panel (Fig. 2.2), the maximum stress due to concentration at the tip A (Timoshenko and Goodier 1970) is given by

$$\sigma_{\text{max}} = \sigma \left(1 + 2\sqrt{\frac{a}{\rho}} \right) \tag{2.7}$$

where $2a$ is the crack size and ρ is the tip radius. Assuming $\rho << a$, and equating σ_{max} with σ_c of Eq. (2.6)

$$\sigma = \frac{1}{2}\sqrt{\frac{E\gamma_s}{a}\frac{\rho}{a_0}} \tag{2.8}$$

Considering $\sigma = E/1000$, $\gamma_s = 0.01Ea_0$, and $\rho \approx a_0$ from Eq. (2.8), $2a = 5000a_0$. This means that the measured strength σ is two orders less than the theoretical strength $E/10$ in the presence of a crack size 5000 times the inter-atomic spacing a_0 at no load.

Such a method of establishing the influence of defects on the theoretical strength suffers from a drawback (Knott 1973). It is based on the correlation of two expressions, which are valid at two different dimension levels; the atomic bond strength model is valid at a level where the dimensions are comparable to an inter-atomic spacing and the Inglis's solution is valid at a macroscopic level.

2.4 Griffith's Theory of Brittle Fracture

Griffith(1921) provided a theory within the framework of thermodynamic energy balance for fracture of brittle materials and to calculate the fracture strength of a material with a crack. He assumed the material to be brittle and linearly elastic till fracture. He argued that when a crack in a stressed body extends, there are two forms of energy at play: the strain energy (W_e) and the surface energy (W_s). Further, at the onset of crack extension, the rate of change of potential energy (π) with crack length, where $\pi = W_e - W_s$, is zero, that is, $\frac{\partial \pi}{\partial a} = \frac{\partial (W_e - W_s)}{\partial a} = 0$. This relation can be interpreted in a different manner. The rate of release of strain energy is equal to the rate of increase of surface energy. Alternatively, the release in strain energy per unit crack extension gets converted into the surface energy of the newly created surfaces. The process of energy conversion is irreversible. In a sense, 'source' for energy supply is the strained body; 'sink' is the newly created surfaces. Hence, the unstable crack propagation takes place if the strain energy released rate associated with a crack extension is more than the corresponding energy absorbed in creating new surfaces.

Griffith followed a rigorous method of calculation for changes in strain energy due to an internal crack symmetrically located in an infinite plate of uniform thickness under equi-biaxial tensions (Fig. 2.3(a)) at its outer boundary using Inglis's solution. Here, an approximate method of calculation is presented to help in understanding, considering a centre crack in an infinite plate of uniform thickness subjected to uniaxial tension in the direction perpendicular to the crack

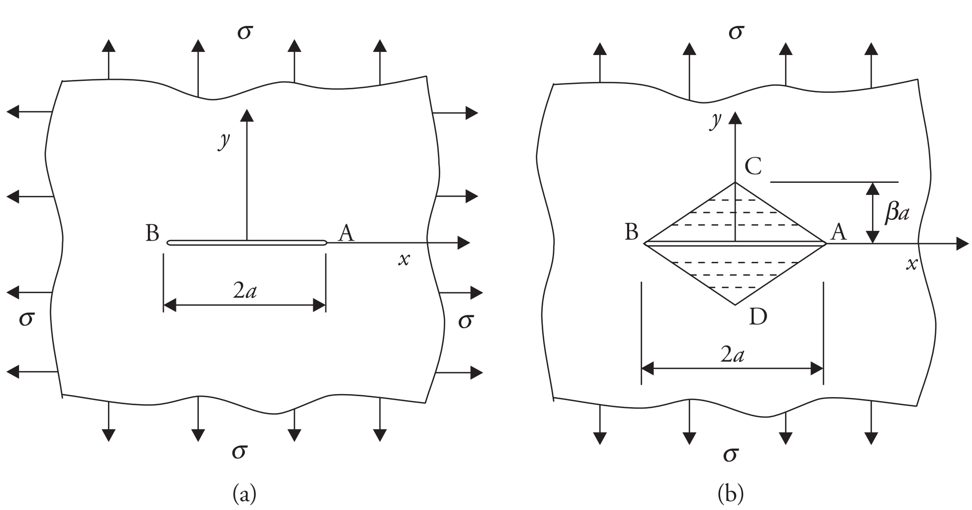

Figure 2.3 (a) Griffith crack. (b) Centre crack under uniaxial tension.