

MASS DIMENSION ONE FERMIONS

In 2005, Dharam Ahluwalia and Daniel Grumiller reported an unexpected theoretical discovery of mass dimension one fermions. These are an entirely new class of spin one half particles, and because of their mass dimensionality mismatch with the standard model fermions they are a first-principle dark matter candidate.

Written by one of the physicists involved in the discovery, this is the first book to outline the discovery of mass dimension one fermions. Using a foundation of Lorentz algebra, it provides a detailed construction of the eigenspinors of the charge conjugation operator (Elko) and their properties. The theory of dual spaces is then covered, before mass dimension one fermions are discussed in detail. With mass dimension one fermions having applications to cosmology and high energy physics, this book is essential for graduate students and researchers in quantum field theory, mathematical physics and particle theory.

DHARAM AHLUWALIA is a theoretical physicist known internationally for his research on neutrino oscillations, interface of the gravitational and quantum realms and new constructs in quantum field theory. In the early 1990s he introduced what later came to be known as Elko, and in 2005, together with Daniel Grumiller, reported on an unexpected theoretical discovery of mass dimension one fermions. This work has opened the field of Elko cosmology and has attracted the attention of physicists working on the localisation problem in five dimensional branes. In 1996, he was awarded the First Prize of the Gravity Research Foundation, jointly with Christoph Burgard.

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to the Journeyers to the East
in the tradition of Hermann Hesse’s The Glass Bead Game

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Preface

We report an unexpected theoretical discovery of a spin one half matter field with mass dimension one.

Ahluwalia and Grumiller (2005a)

We provide the first details on the unexpected theoretical discovery of a spin-one-half matter field with mass dimension one.

Ahluwalia and Grumiller (2005b)

With these opening lines, Daniel Grumiller and I introduced an entirely new class of fermions. They carry mass dimension one. That is, they do not satisfy the Dirac equation, but only the spinorial Klein–Gordon equation for spin one half. In the intervening decade and a half, the issues of non-locality, and Lorentz-symmetry violation, have been completely resolved. But a self-contained and an *ab initio* treatment of such an unexpected theoretical discovery is missing. It is therefore necessary to lift a logical version of this development from the pages of various journals to a monograph.

I present here what we know of the subject at the present moment (late 2018). In making this selection, I have strictly confined, with a minor exception, to that part of the existing literature which has passed through my own pen and paper. This is not to negate the contributions of my collaborators, and many others who have worked on the subject, but to take full personal responsibility for the presented formalism.¹

Not unexpectedly, there is a group of physicists who have seized upon the new construct and based much of their careers on exploiting the physical consequences and studying the underlying mathematical structure of the new theoretical discovery. This is evident from some one hundred papers, and several doctoral theses, that are entirely devoted to the new spinors and the associated fermions.

Then there is a group of physicists who simply dismiss the subject as an impossibility. For the latter, I can only suggest that they first construct the eigenspinors

¹ Of the reader it is assumed that they are at home with the theory of special relativity, and first few chapters of books on the theory of quantum fields. With that background they would be ready for the journey through this monograph.

of the $(1/2, 0) \oplus (0, 1/2)$ charge conjugation operator – with eigenvalues ± 1 – and show that neither $(\gamma_\mu p^\mu + m\mathbb{I}_4)$ nor $(\gamma_\mu p^\mu - m\mathbb{I}_4)$ annihilates these spinors. Having done this preliminary exercise carefully, without falling into the temptation of Grassmann-isaton of the new spinors, they may start with Chapter 5 – returning to the earlier chapters only for notational details – and come to the end of Chapter 13. At that stage, they would have enough information to develop their own theory and to see if their calculations produce something similar to what follows in the remainder of the monograph.

The rest of the preface provides a brief scientific journey of the author. It provides a context in which the reported results were obtained. What follows may thus be seen as a brief scientific autobiography that excludes, with the exception of the next paragraph, large parts of my work on the interface of the gravitational and quantum realms, and on neutrino oscillations.

Sometime in the early 1980s, on the banks of the Charles River in Boston, my quantum mechanics teacher was scheduled to give three fifty-minute lectures, thrice a week. Instead, to my pleasure, I was exposed to five lectures a week, each of three hours' duration with a five minutes water break half way through each of the lectures. And these continued for three trimesters. I learned many things, among them, the significance of phases in the quantum description of reality. Years later, it was, in part, for that reason that a day after Christmas of 1995, seeing falling snowflakes on a road trip by car from the Maulbronn Monastery, Germany, to the French border, that I asked myself as to what is the difference between classical snow and quantum snow. I realised that each of the snowflakes had a different mass. Each flake thus picked up a different gravitationally induced phase.² Soon, within minutes, I was thinking of solar neutrinos instead of snowflakes and a back-of-the-envelope calculation rolled through me. It became clear that neutrino oscillations provide a set of flavour oscillation clocks and that these clocks redshift according to the general relativistic expectation, and suffer Zeeman-like splitting in the oscillation frequencies for generalised flavour oscillations clocks. In the process I came to realise that there are instances when the gravitationally induced forces may be zero, but not the gravitationally induced phases. All this, in collaboration with Christoph Burgard, led to a shared 1996 First Prize from the Gravity Research Foundation (GRF), a Fourth Prize for 1997 and a series of other publications that inspired a few hundred papers devoted to the interface of the gravitational and quantum realms and won the third, in 2004, and the fifth in 2000, prizes from GRF.

² I now realise that this inference requires a revision due to the inevitable modification of the wave particle duality in the Planck realm (Kempf, Mangano, and Mann, 1995; Ahluwalia, 2000): going from a snowflake to the neutrino mass eigenstates one goes from the Planck-scale induced gravitational modifications of the wave particle duality to the low energy realm of quantum mechanics where the de Broglie's wave particle duality holds to a great accuracy (Hackermueller et al., 2003). For $C_{60}F_{48}$ molecule, the experiment finds fringe visibility lower than expected. It may be indicative that the modification to the de Broglie wave particle duality may become significant at a much lower energy.

When my quantum mechanics teacher moved from the East Coast to the Midwest, I returned with him to my original host university (which has been utterly flexible and graceful to me); I considered him as a natural adviser for my doctoral thesis. Gradually it dawned on me that my questions in Physics were different from his. For my teacher, one must start a conversation with a Lagrangian, while for me I needed a more systematic approach to arrive at Lagrangians – be they be for the Maxwell field, or the Dirac, or any other matter or gauge field. In fact one cannot even fully formulate gauge covariance without knowing the kinematical description of the matter fields.

I now know that once the Lagrangian is given then the principles of quantum mechanics and inhomogeneous Lorentz symmetries – coupled with the operations of parity, time reversal and charge conjugation – intermingle to make a theory predictable in the resulting S-matrix formalism.³ My aim was to understand the opening chapters of any quantum field theory text better. Given a representation space associated with the Lorentz algebra, and the behaviour under discrete symmetries, my desire was to derive Lagrangians for the objects inhabiting these spaces. I expected nothing more than to arrive at the standard results, but in my own way. This is already done by Steven Weinberg in his classic on quantum fields (Weinberg, 2005, 2013).

During the year I started working on my doctoral thesis, Lewis Ryder's book on quantum field theory arrived on the scene. It provided a derivation of Dirac equation.⁴ I could easily extend his derivation to obtain Maxwell equations. So, I was happy for sometime that I could make progress in obtaining the kinematical structure of matter fields, and understand gauge fields from my own perspective. At this juncture, I arranged for a series of breakfast/lunch conversations with a nuclear physicist who had been my teacher for a wonderfully taught course from Jackson's electrodynamics. As a result of these conversations, I realised that I could be useful to the nuclear physics community by formulating a pragmatic approach to dealing with higher spin baryonic and mesonic resonances. These were copiously produced at the then-new Continuous Electron Beam Accelerator Facility in Virginia.⁵

As soon as I submitted my doctoral work to the thesis clerk, came Christoph Burgard, and declared to me that Ryder's derivation of Dirac equation is 'all wrong'.⁶ And it turned out that Christoph was correct, as always. This is now

³ This is the quantum field theoretic formalism presented in (Weinberg, 2005, 2013). Its origins go back to Weinberg (1964a,b, 1969). Its most celebrated offspring is the standard model of high energy physics.

⁴ Dirac equation, as originally introduced and as now understood are two very different things. The original acted ($i\gamma_\mu\partial^\mu - m\mathbb{I}_4$) on a spinor, while that of the standard model of high energy physics the same very operator acts on a spinorial quantum field.

⁵ Now called Thomas Jefferson National Accelerator Facility.

⁶ To be fair to Lewis Ryder, his derivation, in essence, reproduced the then-existing literature on the subject. For a parallel treatment as Ryder's our reader may consult (Hladik, 1999). It apparently began in the Istanbul lectures in the early 1960s with Feza Gürsey hosting the theoretical physics school.

explained in my review of Ryder’s book (written a few years later) and by Gaioli and Garcia Alvarez in their *American Journal of Physics* paper.⁷ To me, it is again a story that weaves important phases in the analysis: for Dirac spinors the right and left transforming components of the particle and antiparticle rest-spinors have opposite relative phases. This important fact can be read off from Steven Weinberg’s analysis of the Dirac field. The result follows without invoking Dirac equation.

Soon after my arrival at the Los Alamos Meson Physics Facility⁸ in the July of 1992, I started to look at Majorana neutrinos. There was a significant element of confusion on the subject and I requested LAMPF to obtain for me an English translation of the 1937 paper of Majorana. I thus found out that in the 1937 paper there was no notion of Majorana spinors: the Majorana field was still the Dirac field, expanded in terms of the Dirac spinors, but with particle and antiparticle creation operators identified with each other.

On the other hand, I found in Pierre Ramond’s primer a systematic development of Majorana spinors: their origin resided in the fact that if ϕ transformed as a left-handed Weyl spinor then $\sigma_2\phi^*$ transformed as a right-handed Weyl spinor ($\sigma_2 =$ ‘second’ Pauli matrix). But ϕ , and consequently the Majorana spinor, had to be treated as a Grassmann variable. Furthermore, Majorana spinors were looked upon as Weyl spinors in disguise.

I was uncomfortable with both of the mentioned elements. For the Dirac spinors, understood as a direct sum of the right and left transforming Weyl spinors, no such Grassmann-isation was necessary – at least in the operator formalism of quantum field theory. Another matter that concerned me was that for higher spin generalisation of the $\phi\text{-}\sigma_2\phi^*$ argument a replacement of σ_2 by its higher spin counterparts failed to do the magic of Pauli matrices, as Ramond had called it. For a while, this seemed to make a higher spin generalisation of Majorana spinors untenable.

I resolved these problems by taking note of the fact that for spin one half the charge conjugation operator has four, rather than two, independent eigenspinors. And that the $\phi\text{-}\sigma_2\phi^*$ argument works magically well for all spins if σ_2 was instead recognised, up to a phase, as Wigner’s time reversal operator, Θ , for spin one half: if ϕ transforms as an $(0, j)$ object then $\Theta\phi^*$ transforms as a $(j, 0)$ object – with Θ now a spin- j Wigner time reversal operator. As a consequence the $(j, 0) \oplus (0, j)$ object

$$\left(\begin{array}{c} \text{a phase} \times \Theta\phi^* \\ \phi \end{array} \right)$$

⁷ At the time I wrote my book review I was not aware of the analysis of Gaioli and Garcia Alvarez. I only learned of their work when, unexpectedly one day, they walked into my office at the Los Alamos National Laboratory (as by then I was a Director’s Fellow there) and told me about their publication. Lewis Ryder in the second edition of his book does cite Gaioli and Garcia Alvarez, but without attending to the raised concerns.

⁸ Now named Los Alamos Neutron Science Center.

becomes an eigenvector of the spin- j charge conjugation operator if the indicated phases are chosen correctly to satisfy the self/anti-self conjugacy condition. This generalised Majorana spinors to all spins. And such new objects were not $(j, 0)$, or $(0, j)$ objects in disguise: there were $2j$, and not j , independent $(j, 0) \oplus (0, j)$ vectors. Half of them were self-conjugate under charge conjugation, and other half were anti self-conjugate. Later new names were to be invented to avoid confusion with the newly constructed $(j, 0) \oplus (0, j)$ vectors.

During this phase, I also began to suspect that the dynamics associated with these new objects would carry unexpected features. It was as true for spin one half, as for higher spins.

All these observations were essentially a mathematical science fiction. In addition, I encountered the same problem as Aitchison and Hey did in their attempt to construct a Lagrangian for the c-number Majorana spinors. With some caveats, without the Lagrangian the story cannot unfold, cannot come to fruition.

Bringing back the focus to spin one half, I expressed some of my frustration in an unpublished e-print where, towards a resolution of the problem, I initiated a work to construct dual space for the new set of four-component spinors. It allowed to define an adjoint for the quantum field with the new spinors as expansion coefficients. At this stage Daniel Grumiller joined my efforts and we calculated the vacuum expectation value of the time ordered product of the field with the newly defined adjoint – that is, the Feynman-Dyson propagator. This gave us a most startling result: the new spin one half fermionic field carried mass dimension one. This we presented as an ‘unexpected theoretical discovery’ in JCAP and PRD – the two 2004 e-prints were published in 2005, almost back to back due to a long, but constructive, refereeing process.

Soon after the publication of these papers, I moved from Zacatecas in Mexico to Canterbury in New Zealand. There, I formed a very active research group until it dismantled in the aftermath of the Christchurch earthquakes. The most active members of this group were my doctoral students: Cheng-Yang Lee, Sebastian Horvath, Dimitri Schrittt and Tom Watson. Though Tom stayed in the group only for a short time, he made an interesting contribution. The most important of these was that Ryder’s definition of the Dirac quantum field, as was the case with many other authors, was not consistent with the construction of quantum fields formulated by Steven Weinberg. This fact, along with what I’ll later describe as the IUCAA breakthrough in Section 14.7 of this monograph, led to evaporating the problems of non-locality and Lorentz-symmetry violation.

After the Canterbury earthquakes, I took a two-year detour to Campinas in Brasil, and returned to India more or less permanently. Thus by 2017, in India I had on my hands a spin one half fermionic field that was local and did not suffer from the violation of the Lorentz symmetry.

With this background, this monograph presents the new theory at a level that should be easily accessible to any good graduate student. An outstanding

question that still remains is to reformulate Weinberg's construction of quantum fields so as to accommodate this new field. My preliminary thoughts on evading the no-go result of Weinberg can be found in my latest paper in Europhysics Letters (EPL) written under the title, 'Evading Weinberg's No-Go Theorem to Construct Mass Dimension One Fermions: Constructing Darkness'.

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Projects like these often evolve over years if not decades. In the process one's scientific style takes birth, often in a merging of one's own genius and an inspiration owed to least one great teacher. The latter for me was Dick Arnowitt. I am immensely grateful to him for teaching me many things of the quantum realm and for his absolute accessibility. Steven Weinberg's books are another source of my inspiration, as is Dirac's classic on quantum mechanics. While for Arnowitt a story begins with the Lagrangian density, for me once the Lagrangian density is given one gives essentially the whole story. And so my quest was for the logical path that leads to Lagrangian densities. This monograph is a reflection of that quest, and that path taken. Arnowitt, in the very first lecture I attended by him, told us all that Dirac's classic was the best book written in one hundred years (Dirac, 1930). Weinberg, in my opinion, does for quantum field theory what Dirac did for quantum mechanics. I am grateful to these scholars. I took Arnowitt's emphasis on the importance of phases in quantum mechanics to heart, and there is perhaps not a single publication of mine where this is not apparent.

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Zacatecas is a beautiful small city in northern Mexico at roughly two thousand and five hundred metres. I very much enjoyed my tenure at Autonomous University of Zacatecas and the city. The papers with Daniel Grumiller were published from there. Gema Mercado, then Director of the Department of Mathematics, not only invited me back from India to Zacatecas, but also provided an inspired scholarly ambience, and a friendship and leadership of unprecedented selflessness. I am utterly grateful to Gema for that and for allowing me to pursue my work without hindrance and with encouragement and support.

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