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More Information

1

Introduction

In a broad brush, the grand metamorphosis that has created the astrophysical and cosmic structures arise from an interplay of (a) Wigner's work on unitary representations of the inhomogeneous Lorentz group including reflections, (b) Yang-Mills-Higgs framework for understanding interactions, and (c) an expanding universe governed by Einstein's theory of general relativity. The wood is provided by the spacetime symmetries. These tell us whatever matter exists, it must be one representation or the other of the extended Lorentz symmetries (Wigner, 1939, 1964). The fire and the glue is provided by the principle of local gauge symmetries *à la* Yang and Mills (1954) and by general relativistic gravity. The Wigner-Yang-Mills framework, as implemented in the standard model of high energy physics, then provides the right-hand side of the Einstein's field equations to determine the evolution of that very spacetime which these matter and gauge fields give birth to in a mutuality yet not completely formulated to its quantum completion. Dark energy thought to be needed for the accelerated expansion of the universe and dark matter required by data on the velocities of the stars in galaxies, the motion of galaxies in galactic clusters, and cosmic structure formation keeps roughly ninety five percent of the universe dark, and of yet-to-be-understood origin.

It is nothing less than the most sublime poetry and primal magic that this picture can explain the rise of mountains and flow of water in the rivers, and go as deep as to invoke metamorphosis of light into the water and the mountains, the stars and galaxies. Where a scientist knows where and how the water first came to be (Hogerheijde et al., 2011; Podio et al., 2013), and a poet asks if there was thirst when the water first rose. The origin of biological structures remains an inspiring open subject.

It is within this framework that we wish to add a new chapter and show how to construct dark matter and understand its darkness from first principles. It is thus that in this monograph we present an unexpected theoretical discovery of new fermions of spin one half. The fermions of the standard model, be they leptons or quarks, carry mass dimension three halves. The mass dimension of the new fermions is one. Their quartic self-interaction, despite being fermions, is a mass dimension four operator as is their interaction with the Higgs. Their interaction with the standard model fermions is suppressed by one power of the Planck scale and because they couple to Higgs, quantum corrections can bring about tiny magnetic moments for the new fermions. These aspects make them natural dark matter candidate and can provide tiny interaction between matter and gauge fields of the standard model – something that is already suggested observationally (Barkana, 2018). Studies in cosmology hint that the quantum field associated with the new particles may also play an important role in inflation and accelerated expansion of the universe (Böhmer, 2007b; Böhmer et al., 2010; Basak et al., 2013; Pereira et al., 2014, 2017a,b; Basak and Shankaranarayanan, 2015; Bueno Rogerio et al., 2018).

And as the monograph was nearing its final form, we discovered that the new spinors carried an unexpected behaviour under spatial rotations with unique implications for cosmic evolution (Ahluwalia and Sarmah, 2019). This has resulted in Chapter 12.

We thus weave a story of how the non-locality of the first effort evaporated (Ahluwalia and Grumiller, 2005a,b; Ahluwalia, 2017a,c). We tell of the evaporation of the violation of Lorentz symmetry. In the process, we construct a quantum field that is local and fermionic. It finds not its description in the Dirac formalism, but in a new formalism appropriate for its own nature. Beyond the immediate focus it makes explicit many insights, otherwise hidden in the work of Weinberg (2005).

The meandering path from non-locality to locality, from Lorentz symmetry violation to preserving Lorentz symmetry, owes its existence to certain widespread errors and misconceptions in most textbook presentations of quantum field theory (and we had to learn, and correct these), and the eventual breakthrough to certain phases that affect locality and to a construction of a theory of duals and adjoints.

In Chapters 2–5 of Weinberg (2005) Steven Weinberg proves what may be called a No-Go theorem: a Lorentz and parity covariant local theory of spin half fermions must be based on a field expanded in terms of the eigenspinors of the parity operator, that is Dirac spinors – and nothing else. Furthermore, these expansion coefficients must come with certain relative phases. And in addition, there must be a specific pairing between the expansion coefficients and the annihilation and creation operators satisfying fermionic statistics.

A reader who finds these remarks mysterious, may undertake the exercise of comparing 'coefficient functions at zero momentum' which Weinberg arrives at in his equations (5.5.35) and (5.5.36) with their counterparts written by some of the other popular authors, for example (Ryder, 1986, 1996; Folland, 2008; Schwartz, 2014a). Srednicki's book avoids these errors with profound consequences for the consistency of the theory with Lorentz symmetry and locality (Srednicki, 2007).

In arriving at the canonical spin one half fermionic field, Weinberg does not use or invoke Dirac equation, or the Dirac Lagrangian density. These follow by evaluating the vacuum expectation value of the time-ordered product of the field and its adjoint at two spacetime points (x, x'). The resulting Feynman-Dyson propagator determines the mass dimensionality of the field to be three halves.[1]

Thus, information about the mass dimensionality of a quantum field is spread over two objects: the field, and its adjoint. Weinberg first derives the field from general quantum mechanical considerations consistent with spacetime symmetries, cluster decomposition principle, and then as just indicated, uses this field to arrive at the Lagrangian density through evaluating the vacuum expectation value of the time-ordered product of the field and its adjoint at two spacetime points (x, x'). The powers of spacetime derivatives that enter the Lagrangian density is not assumed, but it is determined by the representation space, and the mentioned formalism, in which the field resides. The broad brush lesson is: given a spin, it is naive to propose a Lorentz covariant Lagrangian density. It must be derived *à la* Weinberg. The expansion coefficient, f_α and f'_α, of a quantum field, ψ, are determined by an appropriate finite dimensional representation of the Lorentz algebra and the symmetry of spacetime translation

$$\psi = \sum_\alpha \left[f_\alpha \mathbf{a}_\alpha + f'_\alpha \mathbf{b}^\dagger_\alpha \right]$$

where $\mathbf{a}_\alpha$ and $\mathbf{b}_\alpha$ satisfy canonical fermionic or bosonic commutators or anticommutators. For simplicity of our argument we have suppressed the usual integration on four momentum, and it may be considered absorbed in the summation sign. As is clear from Weinberg's work, though not explicitly stated by him, if f_α and f'_α satisfy a wave equation, so do $u_\alpha = e^{i\zeta_\alpha} f_\alpha$ and $v_\alpha = e^{i\xi_\alpha} f'_\alpha$, with $\zeta_\alpha, \xi_\alpha \in \Re$. If the field ψ has to respect Lorentz covariance, locality, and certain discrete symmetries then the phases $e^{i\zeta_\alpha}$ and $e^{i\xi_\alpha}$ cannot be arbitrary, but must acquire certain values. Up to an overall phase factor, these are determined uniquely in the Weinberg formalism. Furthermore, the pairing of the u_α and v_α with the annihilation and creation operators is also not arbitrary. A concrete example of all this can be found in Ahluwalia (2017a). The second subtle element is: how to define dual of u_α and v_α, and the adjoint of ψ (see below). We develop a general theory of these elements in this monograph suspecting that mathematicians may have already addressed this issue in one form or another – that said, a tourist guide by a mathematician has missed the issues that we point out (Folland, 2008).

Once these observations are taken into account, if one were to envisage a new fermionic field of spin one half and evade Weinberg's no-go theorem then something non-trivial has to be done. Our approach would be to combine elements of

[1] See chapter 12 of Weinberg's cited monograph for a rigorous definition of mass dimensionality of a quantum field.

Weinberg's approach and that of a naive one indicated above. We shall take the f_α and f'_α not to be complete set of eigenspinors of the spinorial parity operator but that of the spin one half charge conjugation operator. We shall fix the phases $e^{i\zeta_\alpha}$ and $e^{i\xi_\alpha}$ to control the covariance under various symmetries, and to satisfy locality. We will find that each of the eigenspinors of the charge conjugation operator has a zero norm under the canonical Dirac dual. This would lead us to an *ab initio* analysis of constructing duals and adjoints. In the process, we find that if the eigenspinors of the parity operators in the Dirac field are replaced by a complete set of the eigenspinors of the charge conjugation operator, and one chooses appropriate relative phases between the 'coefficient functions at zero momentum' and follows a Weinberg analogue of pairing of the expansion coefficients with the annihilation and creation operators, then the resulting field on evaluating the vacuum expectation value of the time ordered product of the field and its adjoint at two spacetime points (x, x') is found to be endowed with mass dimension one, thus giving a fundamentally new fermionic field of spin one half.

One of my younger friends, a physicist in his own right, explains to me the new fermions with the following wisdom (Mishra, 2017), 'Why should parity get all the privilege? Charge conjugation has equal rights.' We will see here that he captures the essence of one of the main results of this monograph.

The monograph may also be seen as chapters envisaged by a referee of a 2006 Marsden Funding Application to the Royal Society of New Zealand. The referee report read, in part (Anonymous Referee, 2006):

> The problem has fueled intense debates in recent years and is generally considered fundamental for the advancement in the field. As for the proposed solution [by Ahluwalia], I find the approach advocated in the project a very solid one, and, remarkably, devoid of speculative excesses common in the field; the whole program is firmly rooted in quantum field theoretic fundamentals, and can potentially contribute to them. If *Elko* and its siblings can be shown to account for dark matter, it will be a major theoretical advancement that will necessitate the rewriting of the first few chapters in any textbook in quantum field theory. If not, the enterprise will still have served its purpose in elucidating the role of all representations of the extended Poincaré group.

Thus this monograph presents the first long chapter envisaged by the referee and contains much that has been discovered since.

From time to time, a junior reader would come across a remark that is not immediately obvious. For example, after (4.20), there suddenly appears a paragraph reading, 'Without the existence of two, in contrast to one, representations for each $\mathfrak{J}$ one would not be able to respect causality in quantum field theoretic formalism respecting Poincaré symmetries, or have antiparticles required to avoid causal paradoxes.' In such an instance our reader may simply go past such matters and continue. The chances are in the course of their studies, they will come to appreciate the insight, or perhaps disagree with it. I hope such liberties shall serve their purpose in the spirit of Hermann Hesse's journeyers to the east, to whom this monograph is dedicated.