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CONTINUUM MECHANICS

Foundations and Applications of Mechanics

Volume I, Third Edition

C. S. Jog

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To

My Parents and IISc

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Preface

Apart from the usual goal of presenting a unified treatment of seemingly diverse branches such as solid or fluid mechanics, it is also our goal in this book to discuss some important topics, which have come into prominence in the latter half of the twentieth century, such as material symmetry, frame-indifference and thermomechanics. The use of these principles in tandem (of course, in addition to the usual axioms) in many cases delivers the most powerful and beautiful results.

Chapter 1 presents the necessary mathematical background for the following chapters in the form of a brief introduction to tensor analysis. Almost throughout this chapter, the assumed three-dimensional nature of the vector space is exploited to yield shorter proofs of results which in most (but not all) cases also hold for dimensions other than three. In keeping with modern practice, we emphasize the use of direct as opposed to indicial notation, although in many cases we present the proofs using indicial notation also. Presenting the equations in direct notation reinforces the idea that the governing equations are valid with respect to any set of coordinate axes in a given reference frame. It is beneficial, however, to be adept at the use of both, direct and indicial notation. When presenting equations in indicial form, we use only Cartesian components of tensors since we feel that they are sufficient to present the basic principles of the subject. Although other types of components are useful, especially when dealing with curvilinear coordinate systems, we believe that a detailed discussion of say covariant and contravariant components of tensors right at the outset can digress from the purpose of presenting the main physical concepts, and hence, we present them in an Appendix.

Chapter 2 discusses the branch of continuum mechanics known as kinematics, which deals with purely geometrical notions such as strain, rate of deformation, etc.

Chapter 3 discusses the basic axioms of continuum mechanics, namely the principles of conservation of mass, the principles of balance of linear and angular momentum, and the laws of thermodynamics. The principles of balance of linear and angular momentum in the form that are used today are due to Euler. The fascinating history of his struggle to derive the governing equations for a continuum using the linear balance momentum principle alone, before finally realizing the need for accepting the angular momentum balance as a separate axiom, and how using these two axioms simultaneously yielded a rich cornucopia of results, is outlined in [336].

Euler's axioms are formulated directly for a 'large' or 'massy' body. By large or massy, we mean bodies that are large in comparison to atomic dimensions; thus, a speck of dust, for example, would also constitute a large body. In the classical mechanics literature, one often finds 'derivations' of Euler's axioms by considering a continuum as a collection of particles, and by assuming Newton's laws to hold good for each particle. This approach might seem logical, but is, in fact, completely fallacious [336]. To begin with, Newton's laws become poorer and poorer approximations as the size of the body is reduced so that one needs to use quantum mechanics to predict the behavior of a particle. Thus, it does not make sense to deduce laws for a large body by using Newton's laws for a particle, which in fact do not hold. Worse still, Newton himself never meant a particle when he used the Latin word *particula*–he used it to mean a small body or an element of integration. On the other hand, Euler's axioms which are formulated directly for large bodies are known to be very accurate. *Thus, it seems that the particle-based approach can be avoided altogether*. As demonstrated in Chapter 8, Euler's axioms yield all the classical equations for rigid body motion (and much more). Of course, in cases where the motion of a body with respect to its center of mass can be neglected, it might be convenient to *approximate* the body as a particle with its entire mass lumped at a point–this approach, for example, is usually used in deducing the motion of planets around the sun.

Another anachronism in the view of this author, particularly in this post-Einstein era, is the use of inertial and non-inertial frames of reference. In this book we treat all reference frames on an equal footing in the sense that we *require* that the governing equations have the same form in all reference frames. This requirement yields a relation for the way body forces should transform under a change of frame (see Eqn. (4.18)), which, quite happily, also agrees with physical experience.

A reader may wonder as to what exact role Newton's laws, which one generally learns during highschool, and which are proclaimed to be basic, play in continuum mechanics. The answer is that although one does not use Newton's laws directly, one does obtain statements close in spirit to Newton's laws from Euler's axioms. To see this, let us consider Newton's laws. The first law states that 'A body continues in a state of rest or of uniform motion in a straight line unless acted upon by a net external force'. The part about the 'straight line motion' is not strictly true for bodies, since a body which is rotating in the absence of any forces would continue to rotate. Hence, modern treatments modify this statement and replace 'body' by 'particle'. Since our purpose in continuum mechanics is to study large bodies and not particles, and since we have already indicated that classical laws may no longer hold when one considers particles, this law is not directly used in our formulation. The second law which is again claimed to hold good for a particle, and which states that 'force is equal to mass times acceleration of a particle' is now obtained in a form which holds directly for the body; the acceleration is that of the center of mass (see Eqn. (3.16)). The third law which states that 'action and reaction are equal and opposite', is obtained in the form given by Eqn. (3.25).

It should be recognized that Euler's axioms are nontrivial generalizations of Newton's laws. In particular, the axioms are required to hold for any arbitrary sub-part of a body. It is precisely this feature (along with the Cauchy principle) that yields the governing differential equations from the integral form of the equations.

A new era in continuum mechanics began around 1955 with the works of Walter Noll. He proposed an axiomatization for forces, the principle of material-frame indifference, and group-theoretic definitions for simple solids and fluids, among numerous other things. Along with Bernard Coleman, he was also instrumental in using the second law of thermodynamics to deduce restrictions on constitutive relations. Constitutive relations had been treated in a very empirical way until their seminal work in which they showed how the use of material frame-indifference, material symmetry and the second law of thermodynamics can help restrict (and in some cases, even yield the final form of) constitutive relations. Chapter 4 discusses the concepts of material frame-indifference and material symmetry, and shows how these concepts can be applied to elastic materials when thermal effects are ignored. In Chapters 5, the nonlinear theory of elasticity is presented, while in Chapter 6, the classical theory of linear elasticity is obtained by linearizing the governing equations of nonlinear elasticity.

In Chapter 7, we discuss the very important topic of thermomechanics. To those seeking a rigorous treatment of thermodynamics, the treatment in standard works, with their liberal use of d's, Δ's and δ's, has always seemed obscure [335]. In particular, the usual treatment of the second law of thermodynamics has come in for severe criticism, with the definition of the entropy function itself in terms of reversible processes, while the processes that one is trying to model are usually highly irreversible. A lot of work has been done in the past few decades to remedy this unsatisfactory state of affairs–the modern theory may be found, for example, in [295]. Although the mathematics used in proving the various theorems is nontrivial, those concerned with applications may effectively assume (as we do) the Clausius–Duhem inequality as a statement of the second law of thermodynamics. We discuss the restrictions imposed by this inequality on solids and fluids. Quite amazingly, all the classical results, which had historically been obtained either empirically or through ad hoc means, are now recovered using rigorous arguments. In particular, in the case of fluids, we recover the entire classical theory as a special case, including the Gibbs relation, the fact that pressure is a state function of density and temperature, the constitutive relation for the viscous stress, the relation between the entropy and the free energy, and so on.

A point of elementary logic that we wish to stress is that the statement "A implies B" is logically equivalent to the statement "not B implies not A". Thus, we shall often prove that A implies B by assuming 'not B' and deducing 'not A'.

Continuum mechanics and fluid mechanics are classical subjects with a huge amount of literature, and this author has naturally benefitted by reading a part of this literature. The main works that have been consulted in writing this book are stated in the bibliographies in the two volumes; the author is greatly indebted to their authors. The author is also indebted to his teachers at the Theoretical and Applied Mechanics Department (now merged with Mechanical Engineering), University of Illinois at Urbana-Champaign, in particular, Professor D. E. Carlson, R. B. Haber, R. E. Johnson and R. T. Shield. Professor D. E. Carlson was kind enough to give the author a copy of his unpublished notes; Sections 1.12, 4.6 and 4.7 are based on these notes, and the author thanks him for granting permission to use them[1]. The author is thankful to the Mechanical Engineering Department at the Indian Institute of Science for having provided a stimulating environment during the development of this book. The author gratefully acknowledges the tremendous effort put in by Professor Annem Narayana Reddy of IIT(G) in designing and developing a web-based NPTEL course based primarily on this book. Students who have taken the continuum mechanics course offered by this author or worked on projects offered by him have made many valuable suggestions and comments which have resulted in a vast improvement in the book, and the author wishes to thank all of them. The author expresses his thanks to Cambridge University Press for agreeing to publish this book, and for their kind cooperation. Special thanks go to the Commissioning Editor Gauravjeet Singh Reen, Assistant Editors (Academic) Sana Bhanot and Shikha Vats, and their team for their tireless efforts in bringing this project to fruition. The entire book was typeset in the Linux environment using LaTeX–the author expresses his gratitude to the creators of these fine pieces of software. He also expresses his

[1]Professor Carlson passed away in August 2010. His notes are now available online at http://imechanica.org/node/15845, courtesy of Professor Amit Acharya.

gratitude to the creators of that jewel in the crown, Mathematica [353], which he used extensively in writing Chapter 6 and parts of Volume II. Finally, the author wishes to thank his family for having supported him through the writing of this book.

Suggestions and comments for improving this book are welcome.

Notation

$\boldsymbol{a}$	acceleration vector
$\boldsymbol{b}$	body force vector per unit mass
$\boldsymbol{B}$	left Cauchy–Green strain tensor
c_p	specific heat at constant pressure
c_v	specific heat at constant volume
$\mathbf{C}$	first elasticity tensor
$\mathbb{C}$	second elasticity tensor
$\boldsymbol{C}$	right Cauchy–Green strain tensor
$\boldsymbol{\chi}$	mapping characterizing the deformation
D/Dt	material derivative
$\boldsymbol{D}$	rate-of-deformation tensor
e	specific internal energy
$\boldsymbol{e}_i$	Cartesian basis vectors
$\boldsymbol{E}$	Green strain tensor
$\bar{\boldsymbol{E}}$	Almansi strain tensor
$\boldsymbol{F}$	deformation gradient
$\boldsymbol{g}$	temperature gradient
γ	ratio of specific heats (c_p/c_v)
Γ	circulation
h	specific enthalpy
$\boldsymbol{h}$	angular momentum
η	specific entropy
J	determinant of $\boldsymbol{F}$
$\boldsymbol{J}$	inertia tensor relative to the center of mass
K	kinetic energy
κ	bulk viscosity
$\boldsymbol{l}$	linear momentum
$\boldsymbol{L}$	velocity gradient tensor
λ	dilatational viscosity; also Lame constant

$\boldsymbol{\lambda}$	unit tangent to a contour
μ	shear or dynamic viscosity; also Lame constant
$\boldsymbol{n}$	unit normal to a surface
ν	kinematic viscosity
p	thermodynamic pressure
$\hat{p}$	mean pressure
$\boldsymbol{q}$	heat flux vector
Q_h	heat generated per unit mass per unit time
$\boldsymbol{Q}$	orthogonal tensor
θ	absolute temperature
R	universal gas constant
ρ	density
S	control surface
$S(t)$	surface of material volume
$\boldsymbol{S}$	second Piola–Kirchhoff stress tensor
$\boldsymbol{\sigma}$	viscous stress tensor
t	time
$\boldsymbol{t}$	traction vector
$\boldsymbol{T}$	first Piola–Kirchhoff stress tensor
$\boldsymbol{\tau}$	Cauchy stress tensor
$\boldsymbol{u}$	displacement vector
$\boldsymbol{v}$	velocity vector
v	specific volume
V	vector space
$V(t)$	material volume
V_t	control volume
$\hat{W}$	stored energy function
$\hat{\boldsymbol{W}}$	vorticity tensor
$\boldsymbol{W}$	the skew tensor $\dot{\boldsymbol{Q}}\boldsymbol{Q}^T$
$\boldsymbol{\omega}$	vorticity vector (axial vector of $2\hat{\boldsymbol{W}}$)
$\boldsymbol{w}$	angular velocity (axial vector of $\dot{\boldsymbol{Q}}\boldsymbol{Q}^T$)
$\boldsymbol{\Omega}$	axial vector of $\boldsymbol{Q}^T\dot{\boldsymbol{Q}}$ (equal to $\boldsymbol{Q}^T\boldsymbol{w}$)
$\boldsymbol{x}$	spatial coordinates
$\bar{\boldsymbol{x}}$	center of mass
$\boldsymbol{X}$	material coordinates
ψ	free energy

Sets

Lin $=$ Set of all tensors

Lin$^+$ $=$ Set of all tensors $\boldsymbol{T}$ with $\det \boldsymbol{T} > 0$

Sym $=$ Set of all symmetric tensors

Psym $=$ Set of all symmetric, positive definite tensors

Orth $=$ Set of all orthogonal tensors

Orth$^+$ $=$ Set of all rotations ($\boldsymbol{Q}\boldsymbol{Q}^T = \boldsymbol{I}$ and $\det \boldsymbol{Q} = +1$)

Skw $=$ Set of all skew-symmetric tensors

Unim $=$ Set of all unimodular tensors ($|\det \boldsymbol{H}| = +1$)

Unim$^+$ $=$ Set of all proper unimodular tensors ($\det \boldsymbol{H} = +1$)