Bach’s Numbers

In eighteenth-century Germany the universal harmony of God’s creation and the perfection of its proportions still held philosophical, moral and devotional significance. Reproducing proportions close to the unity (1 : 1) across compositions could render them beautiful, perfect and even eternal. Using the principles of her ground-breaking theory of proportional parallelism and the latest source research, Ruth Tatlow reveals how Bach used the number of bars to create numerical perfection across his published collections, and explains why he did so. The first part of the book illustrates the wide-ranging application of belief in the unity, showing how planning a well-proportioned structure was a normal compositional procedure in Bach’s time. In the second part Tatlow presents practical demonstrations of this in Bach’s works, illustrating the layers of proportion that appear within a movement, within a work, between two works in a collection, across a collection and between collections.

British-Swedish musicologist Ruth Tatlow is an independent scholar based in Stockholm. Her research into Bach’s use of numbers led from her classic monograph *Bach and the Riddle of the Number Alphabet* (Cambridge, 1991) to this sequel, *Bach’s Numbers*, through publications on methodology, inventive techniques and the theory of proportional parallelism. In 2004 she co-founded Bach Network UK (BNUK), establishing its open-access web-journal *Understanding Bach* in 2006. She is currently Chair of the BNUK Council, joint editor of *Understanding Bach*, and a member of the Editorial Board of the American Bach Society. Her research has attracted awards and grants from numerous sources including the Swedish Research Council, the Royal Swedish Academy of Music, the Society of Authors of Great Britain, the British Council, The Hinrichsen Foundation, The Leverhulme Trust and the Royal Swedish Academy of Letters.
Bach’s Numbers

Compositional Proportion and Significance

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To
Mark
Louisa, Emily
Benjamin and Anita
with love
Contents

List of figures   [page ix]
List of tables   [x]
Acknowledgements [xv]
List of abbreviations [xvii]

PART I FOUNDATIONS   [1]
1 Bach’s numbers   [3]
2 Symmetry, proportion and parallels   [36]
3 Unity, proportions and universal Harmony in Bach’s world   [73]
4 Bars, compositional planning and proportional parallelism   [102]

PART II DEMONSTRATIONS   [131]
5 Three collections for strings   [133]
6 Four in two collections for keyboard   [159]
7 Two further collections for keyboard   [182]
8 Two small late collections   [204]
9 Two large late collections   [224]
10 Collections of concertos   [255]
11 Collections of organ works   [275]
12 Great passions and cantatas   [294]
13 Festive cut-and-paste projects: masses and oratorios   [326]
CONTENTS

14 Lost compositional blueprints [354]

Appendix: A theology of musical proportions and Harmony in Bach’s time [370]

Bibliography [383]
Index of sources [396]
Index [400]
Figures

Proportion by situation. Courtesy of Sterling Library, Senate House Library, University of London [page 40]

Proportion in figure. Courtesy of Sterling Library, Senate House Library, University of London [41]


3.2 Mattheson, *Das Beschützte Orchestre* (Hamburg, 1717), 478. Courtesy of Sibley Music Library, Eastman School of Music, University of Rochester [82]


### Tables

<table>
<thead>
<tr>
<th>Table Number</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>B/H-A-C key pattern across <em>Clavier Übung I</em> and <em>II</em>. Original print [page 64]</td>
</tr>
<tr>
<td>2.2</td>
<td>Statistical survey of Bach’s signatures [66]</td>
</tr>
<tr>
<td>2.3</td>
<td>Bach’s signatures in three common number alphabets [67]</td>
</tr>
<tr>
<td>4.1</td>
<td>Symmetry of intervals in the hexachord [103]</td>
</tr>
<tr>
<td>4.2</td>
<td>Symmetry of semitones in B-A-C-H [103]</td>
</tr>
<tr>
<td>5.1</td>
<td>Six Solos. Autograph score, P 967 [135]</td>
</tr>
<tr>
<td>5.2</td>
<td>Proportioned pair: Sonata 1 and Partita 1. Autograph Score, P 967 [137]</td>
</tr>
<tr>
<td>5.3</td>
<td>Proportion in Partita 3 in E major. Autograph score, P 967 [138]</td>
</tr>
<tr>
<td>5.4</td>
<td>Six Solos. Autograph score, P 967. Six Sonatas. Copyist Altnickol, P 229 [140]</td>
</tr>
<tr>
<td>5.5</td>
<td>Large-scale movements. Fuga and Ciaccona. Autograph score, P 967 [142]</td>
</tr>
<tr>
<td>5.6</td>
<td>Five Solos for violin. Copyist Kellner, P 804 [144]</td>
</tr>
<tr>
<td>5.7</td>
<td>Revisions to the Six Solos. P 804, and P 967 [145]</td>
</tr>
<tr>
<td>5.8</td>
<td>Six Sonatas for violin and harpsichord. Copyist Altnickol, P 229 [147]</td>
</tr>
<tr>
<td>5.9</td>
<td>Six Sonatas. Early and final versions compared. St 162 and P 229 [148]</td>
</tr>
<tr>
<td>5.10</td>
<td>Changes to Sonata 6. St 162 and P 229 [148]</td>
</tr>
<tr>
<td>5.11</td>
<td>Parallels between two violin collections. P 967, St 162 and P 229 [149]</td>
</tr>
<tr>
<td>5.12</td>
<td>Six Cello Suites. Copyist Anna Magdalena Bach, P 269 [152]</td>
</tr>
<tr>
<td>5.13</td>
<td>D-major Cello Suite. Copyist Anna Magdalena Bach, P 269 [153]</td>
</tr>
<tr>
<td>5.14</td>
<td>Proportion formed between movements. Cello Suites, P 269 [154]</td>
</tr>
<tr>
<td>5.15</td>
<td>A second large-scale proportion. Cello Suites, P 269 [154]</td>
</tr>
<tr>
<td>5.16</td>
<td>Further evidence of proportioning. Cello Suites, P 269 [155]</td>
</tr>
<tr>
<td>5.17</td>
<td>An earlier version of the Cello Suites? P 269 and autograph score of BWV 998 [156]</td>
</tr>
<tr>
<td>5.18</td>
<td>Proportioning in an earlier version of the Cello Suites [157]</td>
</tr>
</tbody>
</table>
6.2 Early preludes expanded. *Clavier-Büchlein*, Kayser’s copy, P 401 and P 415 [163]
6.3 *Aufrichtige Anleitung*. Autograph score, P 610 [165]
6.4 Earlier names and orders. *Clavier-Büchlein*, 1722/3 and copy by Kayser, P 219 [167]
6.5 Two additional collections. Autograph score, P 415 and copy by Kayser, P 219 [169]
6.6 *WTC I*. Autograph score, P 415 with *Aufrichtige Anleitung*, P 610 and P 219 [170]
6.7 A chronological overview. *WTC I* and *Aufrichtige Anleitung* [171]
6.8 *Clavier Übung I* and II. Original print [173]
6.9 Proportional integration of two sets of two keyboard collections [174]
6.10 Partitas in A and E minors. Autograph score, P 225, and print (BWV 827, 830) [175]
6.11 Partitas in A and E minor. Autograph score, P 225, 1725 [176]
6.12 Evolving plans for *Clavier Übung I*. Print. Autograph score, P 225, P 226 [177]
6.13 Early ordering of *Clavier Übung I* and II. Print. Autograph score, P 226 [179]
6.14 *Clavier Übung* Parts 1-IV. Original prints [180]
7.1 Earliest fifteen movements. *Clavier Übung III*. Original print [185]
7.2 Twelve later movements. *Clavier Übung III*. Original print [187]
7.3 *Clavier Übung III*. Original print [187]
7.4 Twelve consecutive thematic groups. *Clavier Übung III*. Original print [188]
7.5 Perfect proportioning in consecutive movements. *Clavier Übung III*. Original print [189]
7.8 Time and keyboard attributions. Goldberg Variations, *Clavier Übung IV*. Original print [196]
7.9 1742 plans. *Clavier Übung III* and *Clavier Übung IV*. Original prints [198]
7.10 Canons, BWV 1087. Autograph score, F-Pn Mus. Ms. 17669 [200]
7.11 Large-scale *Clavier Übung* series. Original prints [202]
8.1 Canonic Variations. Engraving of three original variations [206]
8.2 Canonic variations. Numerical value of engraved title page [207]
List of Tables

8.3 Canonic Variations. Comparison of 1747 print and autograph score, P 271 [210]
8.4 Canonic Variations, Schübler Chorales and Goldberg Canons. Original prints [213]
8.5 Collections of canons, 1739 and 1747. Original prints [214]
8.6 A collection for organ? Autograph score, P 271 [215]
8.7 Schübler Chorales. Sources and proportions. Original print [219]
8.8 Schübler Chorales. Title page. Original print [221]
8.9 Original binding of Bach’s Handexemplar of Clavier Übung III and Schübler Chorales [222]
9.1 Musical Offering. Original print [228]
9.2 Sonata and Canons in Sections C and D. Musical Offering. Original print [231]
9.3 Number values of title words. Musical Offering [232]
9.4 Parallel values. Musical Offering. Original print [233]
9.5 Title page. Musical Offering. Original print [235]
9.7 The Art of Fugue. Original print [244]
9.8 Contrapunctus 1–10. The Art of Fugue. Original print [245]
9.9 Fourteen movements. The Art of Fugue. Original print [246]
9.10 Possible ground plans. The Art of Fugue. Original print [247]
9.11 A possible ground plan. The Art of Fugue. Original print [248]
9.12 The Art of Fugue. Autograph score, P 200 [249]
9.13 Transformation process. The Art of Fugue, P 200 and original print [249]
9.14 Transformation process. The Art of Fugue, P 200 and original print [250]
9.15 Title page. The Art of Fugue. Copyist, Altnickol, P 200, and original print [251]
9.16 Schübler Chorales; Fuga a 3 Soggetti. Original prints [253]
10.1 Early Keyboard Transcriptions. Mixed copyists, P 804 [258]
10.2 Twelve concerto transcriptions. Copyist, Johann Bernhard Bach, P 280 [260]
10.3 Two sets of six transcriptions. Copyist, Johann Bernhard Bach, P 280 [262]
10.4 A proportionally related collection of twelve. Copyist, Johann Bernhard Bach, P 180 [262]
10.5 Brandenburg Concertos. Autograph score, Am. B. 78 [269]
<table>
<thead>
<tr>
<th>Number</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.6</td>
<td>Reconstruction of Bach’s numerical plans. Am. B. 78 [271]</td>
</tr>
<tr>
<td>10.7</td>
<td>Rastration of BWV 1050/1. Autograph score, Am. B. 78 [272]</td>
</tr>
<tr>
<td>11.1</td>
<td>‘Great Eighteen’ organ chorale preludes. Autograph score, P 271, 56–99 [278]</td>
</tr>
<tr>
<td>11.2</td>
<td>The ‘Great Fifteen’, P 271, and Fifteen mass settings from Clavier Übung III in 1739 [283]</td>
</tr>
<tr>
<td>11.3</td>
<td>Additions to P 271. Organ chorales and Canonic Variations [283]</td>
</tr>
<tr>
<td>11.4</td>
<td>Six Trio Sonatas. Autograph score, P 271, 2–55 [286]</td>
</tr>
<tr>
<td>11.5</td>
<td>Two Trio Sonatas in C minor: coincidental parallelism? [289]</td>
</tr>
<tr>
<td>11.6</td>
<td>Large-scale parallels between five collections [290]</td>
</tr>
<tr>
<td>11.7</td>
<td>Parallel proportions between keyboard collections: Clavier Übung III, The Art of Fugue, Schübler Chorales and P 271 [292]</td>
</tr>
<tr>
<td>11.8</td>
<td>The Great Eighteen, P 271, and Fifteen mass settings from Clavier Übung III in 1739 [283]</td>
</tr>
<tr>
<td>11.9</td>
<td>Additions to P 271. Organ chorales and Canonic Variations [283]</td>
</tr>
<tr>
<td>11.10</td>
<td>Six Trio Sonatas. Autograph score, P 271, 2–55 [286]</td>
</tr>
<tr>
<td>11.11</td>
<td>Two Trio Sonatas in C minor: coincidental parallelism? [289]</td>
</tr>
<tr>
<td>11.12</td>
<td>Large-scale parallels between five collections [290]</td>
</tr>
</tbody>
</table>

**List of Tables**

<table>
<thead>
<tr>
<th>Number</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.6</td>
<td>Reconstruction of Bach’s numerical plans. Am. B. 78 [271]</td>
</tr>
<tr>
<td>10.7</td>
<td>Rastration of BWV 1050/1. Autograph score, Am. B. 78 [272]</td>
</tr>
<tr>
<td>11.1</td>
<td>‘Great Eighteen’ organ chorale preludes. Autograph score, P 271, 56–99 [278]</td>
</tr>
<tr>
<td>11.2</td>
<td>The ‘Great Fifteen’, P 271, and Fifteen mass settings from Clavier Übung III in 1739 [283]</td>
</tr>
<tr>
<td>11.3</td>
<td>Additions to P 271. Organ chorales and Canonic Variations [283]</td>
</tr>
<tr>
<td>11.4</td>
<td>Six Trio Sonatas. Autograph score, P 271, 2–55 [286]</td>
</tr>
<tr>
<td>11.5</td>
<td>Two Trio Sonatas in C minor: coincidental parallelism? [289]</td>
</tr>
<tr>
<td>11.6</td>
<td>Large-scale parallels between five collections [290]</td>
</tr>
<tr>
<td>11.7</td>
<td>Parallel proportions between keyboard collections: Clavier Übung III, The Art of Fugue, Schübler Chorales and P 271 [292]</td>
</tr>
<tr>
<td>12.1</td>
<td>St Matthew Passion. Autograph score, P 25 [296]</td>
</tr>
<tr>
<td>12.2</td>
<td>St Matthew Passion. Picander’s text and Bach’s autograph score, P 25 [299]</td>
</tr>
<tr>
<td>12.3</td>
<td>St Matthew Passion. Layout of autograph score, P 25 [305]</td>
</tr>
<tr>
<td>12.4</td>
<td>St John Passion. Partial autograph score, P 28 [308]</td>
</tr>
<tr>
<td>12.5</td>
<td>Chorales in St John Passion. Partial autograph score, P 28 [311]</td>
</tr>
<tr>
<td>12.6</td>
<td>Arias, Chorales and Recitatives in St John Passion. Partial autograph score, P 28 [312]</td>
</tr>
<tr>
<td>12.7</td>
<td>St John Passion, layout and proportion. Partial autograph score, P 28 [314]</td>
</tr>
<tr>
<td>12.8</td>
<td>Cantata ‘Wo soll ich fliehen hin’, BWV 5. Autograph score, BL Stefan Zweig Collection Ms. 1 [320]</td>
</tr>
<tr>
<td>12.9</td>
<td>Cantata ‘Gott ist mein König’, BWV 71. Printed text and autograph score, P 45 [322]</td>
</tr>
<tr>
<td>12.10</td>
<td>Psalm 74:12 in BWV 71/1. Printed text and autograph score, P 45 [324]</td>
</tr>
<tr>
<td>13.1</td>
<td>Three Lutheran masses. Copyist Altnickol, P 15/1–3 [328]</td>
</tr>
<tr>
<td>13.2</td>
<td>Mass in A, BWV 234. Autograph score, D-DS Mus. Ms. 971 [329]</td>
</tr>
<tr>
<td>13.3</td>
<td>Missa in B minor. Autograph score, P 180 [331]</td>
</tr>
<tr>
<td>13.4</td>
<td>Symbolum Nicenum–Sanctus–Osanna in B minor. Autograph score, P 180 [332]</td>
</tr>
<tr>
<td>13.5</td>
<td>B-minor Mass. Autograph score, P 180 [334]</td>
</tr>
<tr>
<td>13.6</td>
<td>Christmas Oratorio. Autograph score, P 32 [337]</td>
</tr>
<tr>
<td>13.7</td>
<td>Free-texted material in Christmas Oratorio. Autograph score, P 32 [338]</td>
</tr>
</tbody>
</table>
xv

LIST OF TABLES

13.9 Recitatives, Duet and Trio in Christmas Oratorio. Autograph score, P 32 [342]
13.10 Choruses and Chorales in Christmas Oratorio. Autograph score, P 32 [343]
13.11 Layout of Christmas Oratorio. Autograph score, P 32 [345]
13.12 Christmas Oratorio title page. Printed text booklet, Leipzig 1734 [347]
13.13 Easter Oratorio. Autograph score, P 34 [349]
13.14 Ascension Oratorio. Autograph score, P 44 fascicle 5 [351]
13.15 A proportionally related pair: the Easter and Ascension Oratorios [352]
14.1 English Suites. Copyist, Kayser, P 1072 [357]
14.3 The Neumeister Collection, LM 4708 and Bach’s Orgelbüchlein, P 283 [358]
14.4 Five Great Preludes and Fugues. Mixed copyists [360]
14.5 1738/9 plan for three grand organ collections [361]
14.6 Six Great Preludes and Fugues. Unknown Berlin copyist, Am. B. 60 [362]
14.7 Four solo concertos for keyboard. BWV 1052–5. Autograph score, P 234 [364]
14.8 A collection of six concertos for harpsichord. Mixed copyists [365]
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Abbreviations


*Bj*  *Bach-Jahrbuch*, 1904–.

Buttstett, *Ut, Mi, Sol*  Johann Heinrich Buttstett, *Ut, Mi, Sol, Re, Fa, La, Tota Musica et Harmonia Aeterna*, [1716].

*BWV*  *Bach-Werke-Verzeichnis*, 1950; rev. and enlarged, 1990.


*CÜ I–IV*  J. S. Bach, *Clavier Übung*, parts I–IV.

D-B  Staatsbibliothek zu Berlin.


*JP*  J. S. Bach, The Passion according to St John.


*MP*  J. S. Bach, The Passion according to St Matthew.

*NBA KB*  *Neue Bach-Ausgabe: Kritischer Bericht*.

LIST OF ABBREVIATIONS

New Grove  
New Grove Dictionary of Music and Musicians,  

P  
D-B Mus. ms. Bach P[artitur]. Original score  
held at Staatsbibliothek, Berlin.

St  
D-B Mus. ms. Bach St[immen]. Original parts  
held at Staatsbibliothek, Berlin.

Walther, Lexicon  
Johann Gottfried Walther, Musicalisches  
Lexicon, 1732.

Walther, Praecepta  
Johann Gottfried Walther, Praecepta der  
Musicalischen Composition, 1708.

Wolff, Essays  
Christoph Wolff, Bach: Essays on His Life and  

Wolff, Learned  
Christoph Wolff, Johann Sebastian Bach: The  

WTC I–II  
J. S. Bach, Das Wohltemperirte Clavier, books  
1–2.

Zedler, Lexicon  
Johann Heinrich Zedler, Grosses vollständiges  
Universal-Lexicon aller Wissenschaften und  
Künste, 1732–54.