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Excerpt
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PART I

Foundations

1 Bach's numbers

Sonnet
Du edle Musica, die du das Hertz bewegst,
Du schönes Himmel-Kind, wer wolte doch nicht lieben;
Die sind von guter Art, die dich rechtmässig üben,
Die du im innern Grund Zahl, Maaß, Gewichte hegst.
Und die Proportion von Erd' und Himmel trägst:
Dein Werck besteht in sechs, und deine Ruh in sieben;
Du bist mit Heimligkeit und Kunst durchaus beschrieben,
Die du des Himmels Bild in deine Wercke prägst.
Du must dich zwarten auch offt übel zerren lassen;
Der Mißbrauch lässet dich in deinen Würden nicht;
Und ob Apollo dich mit allem Ernst verficht;
So finden sich doch die, die deine Schöne hassen.
Du aber bleibest wol: Ob sie nicht achten dein;
So wirstu doch das Spiel der Frommen ewig seyn.

Henr. Georg. Neuss,
Past. Gvelpherbyt., 1691

The dangers of playing with numbers are many and legendary. The humiliation of the sixteenth-century mathematician and pastor Michael Stiefel, whose calculations on biblical verses enabled him to predict that Christ would return at 8 a.m. on 18 October 1533,[1] was matched by the shame of his cold and hungry parishioners after their disappointing vigil. The embarrassment of theologian and musicologist Friedrich Smend four hundred years later was less public, but it still had a significant influence on the inhibition of number research. His interpretation of the number '84' epitomises the problems:

> Bach noted a symbolic number in the autograph score of the B-minor Mass. At the end of the 'Patrem Omnipotentem' he writes the bar number of the movements '84' (7 × 12) . . . [This] chorus is about creation ('factorem coeli et terrae') . . . Earth and heaven are contained in 7 [3 symbolises heaven and 4 earth] . . . We hear

[1] Zedler, *Lexicon*, s.v. 'Stiefel, Michael'.

the word ‘Credo’ forty-nine times, and its continuation ‘in unum Deum’ eighty-four times. Yet again numbers with symbolic content appear.[2]

Eighty-four quickly became a buzzword for the folly of symbolic numbers in music when it was discovered that the annotation had been written into the score by one of C. P. E. Bach’s copyists[3] decades after J. S. Bach’s death. In spite of the inherent risks, this book is nonetheless devoted to compositional numbers, tackling their form, purpose and meaning in Bach’s music.

I How numbers became associated with Bach

Bach left no description of his methods of composition, or of whether or not he used numbers when he composed. One of the earliest allusions to the mathematical bases of his music dates to the early 1740s. It can be read in a published collection of musicians’ autobiographies,[4] in which Lorenz Mizler (1711–78) wrote that he had been influenced by ‘reading good books, listening to good music, perusing many scores by good masters and also in his association with Capellmeister Bach’.[5] This entry enraged the editor of the compilation, Johann Mattheson (1681–1764), causing him to add that ‘Bach no more taught Mizler the mathematical bases of music than I did myself’,[6] referring to an earlier discussion of the value of mathematics for music in which he had named Mizler’s training in mathematics, philosophy and music.[7] Mattheson’s comments should have killed any later rumours that Bach was interested in the use of mathematics in music, but they did not. A century later the great music historian Philipp Spitta unwittingly revived the topic when he drew attention to the Mizler–Mattheson dialogue. Although intended to demonstrate that Bach had no interest in the mathematical basis of composition,[8] it had the opposite effect.

In the 1920s Arnold Schering (1877–1941) further raised the profile of numbers in music when he unearthed traditions of permutation in compositional

2 F. Smend, *J. S. Bach Kirchen-Kantaten* (Berlin, 1947; rev. edn 1966), vol. IV, 14 and 19.

3 Score, P 180, on page 105 in Bach’s pagination. The same scribe wrote the figure 84 in the corresponding place in the soprano solo part, St 118/2 in C. P. E. Bach’s 1786 copies of the parts of the Credo, St 118, thus ruling out the possibility that J. S. Bach wrote the figure.

4 J. Mattheson, *Grundlage einer Ehrenpforte* (Hamburg, 1740).

5 *BD II*, Doc. 470, 380.

6 Mattheson, *Ehrenpforte*, 231.

7 Ibid., 230, where Mattheson issued a lengthy diatribe against mathematics in music.

8 P. Spitta, *Johann Sebastian Bach: His Works and Influence on the Music of Germany (1685–1750)*, trans. A. C. Bell and J. A. Fuller-Maitland. 3 vols. (London, 1884; reprint edn New York, 1951), vol. III, 24.

invention.[9] He steered the number discussion towards Bach studies when he demonstrated the presence of four types of symbolism in Bach's cantata 'Du sollt Gott, deinen Herren, lieben' (BWV 77), spicing the commentary with provocative phrases such as 'the esotericism of Bach's vocal canons' and 'holy and mysterious numbers'.[10] Many musicologists were eager to run with Schering's ideas before the ground had been fully prepared. Friedrich Smend (1893–1980) was at the forefront, introducing the term 'cabbalism' and using number alphabets to interpret patterns he had found in Bach's music.[11] A frisson rippled through the musical world. Amateurs and less discerning scholars lovingly nurtured the ideas, while professional musicologists openly voiced their disdain.

The paragram,[12] which used one of more than thirty different alphabets to substitute numbers for letters, was Smend's major stumbling block. As a widely read theologian with a particular interest in church history, he had met similar numero-alphabetical techniques in Jewish mysticism, which duped him into making an association between Bach's numbers and religious symbolism. It was a fabulous premise with which to work, as it promised to reveal the unseen depths of Bach's spiritual motivation, although, like the allure of the sirens' call, it proved treacherous. Nonetheless, numerical readings of Bach's compositions continued to be published. Many contained fanciful and fallacious interpretations, some were downright illogical,[13] and the majority fell short of their promise not only because of weak methodology, but because of their lack of solid historical or documentary evidence.[14] The shaky historical foundations on which number and interpretation structures were built made their collapse inevitable. What could have become a valuable scientific discipline of numbers within musicology became known as 'numerology', in all its notoriety.

And this is where my research enters the history. An examination of Smend's work led to a study of number alphabets; from their origins, through the quagmires of mystical cabbalism, black and white magic and two centuries of Lutheran exegesis, to the poetical paragram. The results,

[9] A. Schering, 'Geschichtliches zur *ars inveniendi* in der Musik', in *Jahrbuch der Musikbibliothek Peters für 1925*, ed. Rudolf Schwartz (Leipzig: Peters, 1926), 25–34. Schering cites Glareanus (1547), Kircher (1650), Heinichen (1738), Mattheson (1739) and Sulzer (1778).

[10] Schering, 'Bach und das Symbol', *BJ* (1925), 44.

[11] Ruth Tatlow, *Bach and the Riddle of the Number Alphabet* (Cambridge University Press, 1991), 130–8.

[12] See Chapter 2, §III.

[13] Malcolm Boyd, *Bach*, The Master Musicians (London: Dent, 1983), 223.

[14] Notable exceptions include the number research of Ulrich Siegele and Don O. Franklin, to whom I am enormously grateful for their generosity in sharing their expertise, and lending their support as I pursued my research paths.

given first in my doctoral thesis[15] and later revised in the monograph *Bach and the Riddle of the Number Alphabet*, prove conclusively that numbers and numerical interpretations were an integral part of Bach's heritage. Word and number conceits, including the anagram, chronogram, acrostic and paragram, were popular drawing room pastimes as well as useful tools for the more serious poet or orator in need of creative inspiration.

Research for this book began where *Bach and the Riddle* left off, addressing the question of whether Bach and his contemporaries actually used numbers and number alphabets when they composed. It was originally designed to be a comprehensive survey of theoretical evidence showing where numbers and numerical constructions fitted into compositional theory in Bach's time, with the anticipated conclusion that composers made little or no use of numbers in practice. The structure and contents changed radically, however, with the unexpected discovery of proportional parallelism in all the collections and multi-movement works that Bach revised for publication.[16] At its most basic, the theory of proportional parallelism shows that Bach created layers of 1 : 1 and 1 : 2 proportions, using the numbers of bars in the parts and sections of compositions. The original theoretical survey is included in Part I of this book, while the demonstration of Bach's use of numbers, including evidence of the changes Bach made as he transformed early works into perfectly proportioned collections, forms Part II.

Proportional parallelism would have seemed a self-evident practice to any composer living in Bach's time and locality, which is not to say that all composers used it. Symmetrical organisation, parallel techniques, perfect proportions and unity were all commonplace, were found in everyday life, in every academic discipline and creative pursuit, and were also described by music theorists in books about how to compose. Numerous observations of the symmetrical organisation found in Bach's multi-movement works have been accepted into the canon of Bach scholarship. For example, in recent years Christoph Wolff has been a long-term champion for Bach's architectural designs; their symmetry, order, organisation, connection and proportion.[17] Many of these observations can now be confirmed empirically by proportional parallelism. Furthermore, since Smend's work, there has been a widely held assumption that numbers in Bach's music would be symbolic. Proportional parallelism shows something subtly, but significantly, different: it is the proportions, rather than the specific numbers, that hold the meaning.

[15] Tatlow, 'Lusus Musicus vel Poeticus'. King's College, University of London, 1987.
[16] Tatlow, 'Collections, bars and numbers', *Understanding Bach* 2 (2007), 37–58.
[17] Many examples in Wolff, *Essays*.

Several types of evidence combine to demonstrate Bach's use of proportional parallelism. There is the numerical evidence found in the scores. By comparing the numbers of bars in his early and later versions, or by tracing the changes he made as he compiled a new collection from pre-existing movements, one can see how Bach introduced the layers of perfect proportion. There is documentary evidence to demonstrate the specific role that numbers, unity, symmetry, proportion and Harmony[18] played in compositional organisation and planning in Bach's time. And there is documentary evidence to show how these numerical concepts would have been viewed and understood at the time. There is also a body of evidence hidden by the eighteenth-century language that has been lost in translation, both literally and culturally. At all times I have aimed to incorporate results from the most up-to-date diplomatic evidence and source studies. The majority of results shown in Part II confirm the conclusions drawn by these source studies, but occasionally my demonstrations and numerical reconstructions suggest a new interpretation.

The discovery of proportional parallelism in Bach's collections raises the fundamental and challenging question of why he spent time striving to create proportional order within his compositions. The answer lies in the philosophical and theological understanding of Harmony and harmonic proportions, which had been an essential element in philosophy, science and the arts since classical times, and was still prevalent in Lutheran Germany.[19] In the late seventeenth and early eighteenth centuries thought-patterns associated with the Enlightenment began to spread across Europe. Although the ancient proportional world view survived longer in some areas, its final rejection in the early nineteenth century caused an intellectual paradigm shift that would have a profound and lasting effect on the formation of twentieth-century European culture. Bach was living, working and using proportions at this tumultuous time of philosophical change.

II Parallels, proportions and Harmony

It was the first four numbers, the perfect *tetrachys*, expressed as the ratios 1 : 1, 1 : 2, 2 : 3 and 3 : 4 and as the proportion 6 : 8 : 9 : 12,[20] that the ancient

[18] Harmony, with a capital 'H', will be used throughout as a translation of *harmonia* and *Harmonie*.
[19] See Chapter 3, and the Sonnet by Neuss on the first page of this chapter.
[20] Nicomachus' tenth proportion. See Tatlow, 'The Use and Abuse of Fibonacci Numbers and the Golden Section in Musicology Today', *Understanding Bach* 1 (2006), 77–9.

Greeks esteemed as the most perfect for music and Harmony.[21] The first six sounding numbers, or the *senarius*, would later be considered perfect because, through the ratios 1 : 1, 1 : 2, 2 : 3, 3 : 4, 4 : 5 and 5 : 6, they were the source of the musical scale, and in sequence formed the unison, the octave, the fifth, the fourth and the major and minor thirds respectively, which is one reason why the Guidonian mnemonic *Ut*, *re*, *mi*, *fa*, *sol*, *la* for the six degrees of the hexachord was also considered perfect.[22] Later still the seven 'harmonic' numbers 1, 2, 3, 4, 5, 6, 8, called the *septenarius*,[23] or *numeri harmonici*,[24] became popular, the sequence omitting the number seven, or the 'Ruh-Zahl'.[25] In Bach's time Euclid's demonstration of the number six as the first perfect number was used to endorse the universal perfection of the *senarius*;[26] those who found the *septenarius* more perfect also came up with numerous reasons.[27] A theology of creation based on proportions and *harmonia* gradually evolved. Harmonic proportions in the cosmos, in the world and in the measurement of the human being were understood to be a reflection of the 'indescribable wisdom and perfection' of the Creator God.[28] The proportional perfection of musical intervals gave rise to the terms 'perfect unison', 'perfect octave', 'perfect fifth' and 'perfect fourth'. The term *trias harmonica* was coined to describe the consonant triad, because its perfection reflected the perfect Harmony of the Holy Trinity,[29] and because its intervals, the fifth and the major and minor thirds, expressed in the

[21] Pythagoras considered it the perfect number as 1+2+3+4 equals 10, from which numbers the proportions 1 : 1, 1 : 2, 2 : 3 and 3 : 4 form the unison, octave, fifth and fourth.

[22] T. Christensen, *The Cambridge History of Western Music Theory* (Cambridge University Press, 2002), 253–4, 276–8.

[23] For example in Walther, *Praecepta*, 36 (Benary, 83), and Buttstett, *Ut, mi, sol*, 26–8, citing Andreas Werckmeister, *Musicalische Temperatur* (Quedlinburg, 1691), and Conrad Matthäi, *Kurtzer, doch ausführlicher Bericht von den Modis Musicis* (Königsberg: Matthäi, 1652), 14–15.

[24] Walther, *Praecepta*, 'Musica Poetica', 8 (Benary, 76).

[25] Matthäi, *Modis musicis*, 15, calls seven a 'rest' number because one cannot make any musical interval out of the seventh number on the monochord, and because God rested on the seventh day.

[26] A perfect number is one whose divisors add up exactly to the number itself. The number 6 has the divisors 1, 2, 3 and 1+2+3 equal 6. 28 is the next perfect number because its divisors 1, 2, 4, 7 and 14 add up to 28. 496 is the third, and 8128 is the fourth perfect number. Euclid described this in his Prop. IX 36. Walther, *Lexicon*, s.v. 'numerus perfectus', citing Euclid.

[27] J. F. Riederer, *Gründliche Untersuchung der Zahl Sieben.* (Franckfurt; Nürnberg, 1719).

[28] Appendix, 1691-IV 8, 1691-IV 9. (Sources relevant to the doctrine of music have been assigned a short year/number reference (as here) to the Appendix, where full bibliographic references and full text in parallel English-German translation are given.)

[29] Walther, *Lexicon*, s.v. 'Lippius' does not name Lippius as the author of the term *trias harmonica.*

proportions 4 : 5 : 6, were contained within the God-given *senarius*.[30] The triad thus became a powerful reminder for Lutheran Christians of the centrality of music to God and His created order.

This view of the world was still alive and current in Bach's time. Harmony embraced both silent universal proportions and sounding music. A sonnet by Heinrich Georg Neuss (1654–1716), printed in the preface to a music treatise by Andreas Werckmeister (1645–1706) and also on the first page of this chapter, illustrates the centrality of Harmony to German-speaking Lutherans in 1691. Within a few decades, though, there would be many changes in the understanding of music and philosophy in Europe. Thomas Christensen explains:

> Music theory gradually receded from its Boethian heights through the robust growth of *musica practica* as a discipline. By the eighteenth century, music theory had become only a shell of its former glory. (Rameau felt obliged on numerous occasions to defend the honour and dignity of music theory, while at the same time conceding such knowledge might be of little practical use to musicians.) Yet for every defender of music theory – such as Rameau or Lorenz Mizler (1711–78), the founder of the 'Corresponding Society of Musical Science' – there were critics such as Johann Mattheson (1681–1764), who would lambaste much *theoria* (or, as he preferred to call it, 'musical mathematics') as a discredited remnant of unenlightened prejudice . . . With the weapons of empirical philosophy bequeathed by Locke, writers such as Mattheson could militantly hoist the Aristoxenian flag of *sensus* over that of *ratio*.[31]

Philosophical ideas from France and Britain were gaining popularity within Lutheran Germany, and gradually eroding confidence in the centrality of the unison and proportions. The catastrophe was not unforeseen. In 1728, in a book that Bach knew well,[32] the Dresden Capellmeister Johann David Heinichen (1683–1729) made the following prediction about the effect of this new philosophy upon music:

> The beginning has already been made in our times; no doubt daily progress will be made in our century to this end for those supposedly paradoxical hypotheses, and finally all the remaining weak and partly-worn pillars of the musical past will be torn completely asunder.[33]

30 Walther, *Lexicon*, s.v. 'Trias harmonica oder musica' terms a major triad 'Trias harmonica perfecta', and a minor triad 'Trias harmonica imperfecta'.
31 Christensen, ed., *The Cambridge History of Western Music Theory*, 8.
32 *NBR* Doc. 140, and appendix, 529. Bach was Leipzig sales agent for the treatise, and he owned a copy.
33 J. D. Heinichen, *Der Generalbass in der Composition*, 2nd edn (Dresden, 1728), 5, note (a); translation in Buelow, *Thorough Bass Accompaniment According to Johann David Heinichen*, rev. edn (Ann Arbor: UMI, 1986), appendix B, 310–11.

Bach grew up in this climate of change, when the proportional world view was under attack and gradually falling out of favour, which makes understanding the presence and significance of proportions in his music even more challenging.[34]

The complexity of this musical and philosophical transition can best be understood from the writings of Mattheson, who was determined to prove that the traditional Lutheran understanding of musical Harmony was erroneous, fighting tooth and nail to prove that true Harmony must be sounding and not silent. 'Mr Organist', he wrote, addressing Johann Heinrich Buttstett (1666–1727), 'why don't you distinguish primarily between what is properly called Harmony (*Harmonia propriè sic dictam)*, and Harmony in Music (*Harmonia in Musicis)*?'[35] His zeal to divide the traditional Lutheran understanding of Harmony explains many of his curious statements about musical mathematics, not least his seemingly illogical charge about Mizler and Bach.[36] From a single united concept that embraced both non-sounding universal harmony and the sounding harmony of pitches and intervals, Mattheson made two distinct harmonies, i.e. Harmony proper (universal harmony) and Harmony in music (the proportions of pitch, intervals and rhythm). In contrast to contemporary theorists such as Mizler and Spiess, Mattheson saw music as a science for the ear alone and not a theory to be studied in terms of proportions and arithmetic. Nonetheless, he continued to believe in many of the philosophical and theological aspects of universal Harmony, sharing some fundamental views about proportions with theorists traditionally understood to be his opponents. Attempting to persuade Buttstett of his erroneous understanding of Harmony, Mattheson demonstrated his personal belief in proportions, writing:

> There is no doubt whatsoever that the Lord God is pleased with proportions, and the universe demonstrates this . . . God is pleased with musical sounds and their proportions: I doubt that as little as I doubt Christ's Birth, because music is His creation, indeed one of His best creations and gifts.[37]

This shows that even Mattheson, reputedly the great opponent of musical mathematics, still profoundly believed that God was pleased with both non-sounding and sounding proportions. It was a belief that motivated him and many other authors and musicians of the period to recommend proportional organisation in musical composition. Mattheson papered

[34] Implications discussed in Chapter 3. [35] Appendix, 1717-I.
[36] Mattheson, *Ehrenpforte*, 231. See also note 7. [37] Appendix, 1717-III.

over the logical cracks of his position, qualifying his recommendation to proportion pieces of music 'for there is nothing more pleasing to the ear'. Theorists who held the traditional view of Harmony as a united entity recommended proportional ordering in compositions without qualification: to them it made no difference if the proportions were heard or not.

III The unison and Harmony applied

A proverb cited frequently in music treatises of Bach's time reads: 'the closer a proportion is to the unity [or equality] the more perfect it is: the further a proportion is from the unity the more imperfect'.[38] This simple formulation holds the essence of much that lies behind the concept of proportional parallelism in Bach's compositions. The unity and the unison of the 1 : 1 proportion had become the ultimate expression of both equality and perfection. Using Christensen's phrase, it was truly a 'generative unison'.[39] Whole lifestyle applications and artistic techniques were based on belief in the unity because of its position in universal Harmony, regardless of the practitioner's stand on the *sensus–ratio* or sounding/non-sounding debate. The generative unison fell within Mattheson's classification of non-sounding *Harmonia propriè*, the proportions of which even he believed pleased God. How this unison generated many structural forms, including symmetry and parallelism, is the subject of Chapter Two. The 1 : 1 kinship between symmetry and parallelism sheds light on the significance of their use in the arts in Bach's time. The dual meaning of emblems has been well researched and documented, but I set their parallel image-meaning into the larger context of belief in the philosophical, theological and aesthetic significance of the unity and Harmony.

Eurythmia was a synonym for symmetry, which, because of its 1 : 1 nature, was considered to be the epitome of beauty and perfection. According to Johann Gottfried Walther (1684–1748) and other theorists, *eurythmia* in music could be demonstrated numerically by *numerus musicus*. There was symmetry in poetry, both in the rhyme scheme and in the metrical organisation, which Morhof described as *numerus poeticus.* Bach was surrounded by a world of symmetry and parallel forms. Appreciating

[38] Appendix, 1708-VI.
[39] T. Christensen, *Rameau and Musical Thought in the Enlightenment* (Cambridge University Press, 1993), 84–90 – although Christensen applied the phrase solely to musical properties.