Introduction to the AdS/CFT Correspondence

Providing a pedagogical introduction to the rapidly developing field of AdS/CFT correspondence, this is one of the first texts to provide an accessible introduction to all the necessary concepts needed to engage with the methods, tools, and applications of AdS/CFT. Without assuming anything beyond an introductory course in quantum field theory, it begins by guiding the reader through the basic concepts of field theory and gauge theory, general relativity, supersymmetry, supergravity, string theory, and conformal field theory, before moving on to give a clear and rigorous account of AdS/CFT correspondence. The final section discusses the more specialized applications, including QCD, quark–gluon plasma, and condensed matter. This book is self-contained and learner-focused, featuring numerous exercises and examples. It is essential reading for both students and researchers across the fields of particle, nuclear, and condensed matter physics.

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To the memory of my mother,
who inspired me to become a physicist

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Preface

This book is intended as a pedagogical introduction to the rapidly developing field of the AdS/CFT correspondence. This subject has grown to the point where graduate students, as well as researchers, from fields outside string theory or even particle theory, in particular nuclear physics and condensed matter physics, want to learn about it. With this in mind, the book endeavours to introduce AdS/CFT without assuming anything beyond an introductory course in quantum field theory. Some familiarity with the principles of general relativity, supersymmetry or string theory would help the reader follow more easily, but is not necessary, as I introduce all the necessary concepts. I do not overload the book with unnecessary details about these fields, only what I need to give a simple, yet completely rigorous, account of all the basic methods, tools, and applications of AdS/CFT. For more details on these subjects, one can consult a number of good textbooks available for each, which I suggest at the end of the corresponding chapters. When explaining AdS/CFT, I try to give a simple introduction to each method, tool, or application, without aiming for an in-depth or exhaustive treatment. The goal is to introduce most of the AdS/CFT methods, but for an in-depth treatment one should refer to research articles instead. Part I of the book deals with the necessary background material, so someone familiar with this can skip it. Part II describes the basics of AdS/CFT in the context of its best understood example, $\mathcal{N} = 4$ SYM vs. string theory in $AdS_5 \times S^5$. Part III deals with more specialized applications and other dualities, generalizing to the gauge–gravity dualities.

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This book originated from a course I first gave at Tokyo Tech, then at Tokyo Metropolitan University, and finally at the IFT in São Paulo, so I would like to thank all the students that participated in the classes for their input about the material.

In writing this book, I benefited from encouragement and comments on the text from David Berenstein, Aki Hashimoto, and Jeff Murugan. My students Thiago Araujo, Prieslei Goulart, and Renato Costa helped me get rid of errors and typos in an earlier version of the book, and Thiago Araujo and his wife Aline Lima helped me create the figures and make them intelligible. I would like to thank my collaborators, students, and postdocs for their understanding about having less time for interacting with them while I wrote this book.

Introduction

This book gives an introduction to the Anti-de Sitter/Conformal Field Theory correspondence, or AdS/CFT, so it would be useful to first understand what it is about.

From the name, we see that it is a relation between a quantum field theory with conformal invariance (which is a generalization of scaling invariance), living in our flat 4-dimensional space, and string theory, which is a quantum theory of gravity and other fields, living in the background solution of $AdS_5 \times S^5$ (5-dimensional Anti-de Sitter space times a 5-sphere), a curved space with the property that a light signal sent to infinity comes back in a finite time.

The flat 4-dimensional space containing the field theory lives at the boundary (situated at infinity) of the $AdS_5 \times S^5$, thus the correspondence, or equivalence, is said to be an example of *holography*, since it is similar to the way a 2-dimensional hologram encodes the information about a 3-dimensional object. The background $AdS_5 \times S^5$ solution is itself a solution of string theory, as the relevant theory of quantum gravity.

From this description, it is obvious that before we describe AdS/CFT, we must first introduce a number of topics, which is done in Part I of the book. First, we review some relevant notions of quantum field theory, though I assume that the reader has a working knowledge of quantum field theory. Then I describe some basic concepts of general relativity, supersymmetry, and supergravity, since string theory is a supersymmetric theory, whose low energy limit is supergravity. After that, I introduce black holes and p-branes, since the $AdS_5 \times S^5$ string theory background appears as a limit of them. Finally, I introduce string theory, elements of conformal field theory (4-dimensional flat space theories with conformal invariance), and D-branes, which are objects in string theory on which the relevant quantum field theories can be defined.

The AdS/CFT correspondence was put forward by Juan Maldacena in 1997, as a conjectured duality based on a heuristic derivation which will be explained, and until now there is no exact proof for it. However, there is an enormous amount of evidence in its favor in the form of calculations matching on the two sides of the correspondence, turning it into a virtual certainty, so while technically we should append the name “conjecture” to it, this would be a pedantic point, and I shall refrain from doing so.

However, while this is true for all dualities which can be derived in the manner of Maldacena, there are now applications to real-world physics, which I call “phenomenological AdS/CFT,” where one uses some general lessons learned from AdS/CFT to engineer a description in terms of quantum field theory that has the right properties to be relevant for systems of interest, but without a microscopic derivation. In this category fall some applications to QCD, quark–gluon plasma, and condensed matter, which are described in detail in Part III of the book. In these cases it is therefore important to realize the conjectural nature of the correspondence.

Another question that we should ask is why is the AdS/CFT correspondence interesting? The reason is that it relates perturbative (weak coupling) string theory calculations in a gravitational theory to nonperturbative (strong coupling) gauge theory calculations, which would otherwise be very difficult to obtain. Of course, the reverse is also true, namely nonperturbative (strong coupling) string theory in a gravitational background is related to perturbative (weak coupling) gauge theory, allowing in principle an (otherwise unknown) definition of the former through the latter, but the rules in this case are much less clear. The strong–weak coupling relation means that AdS/CFT is an example of duality, in the sense of the electric–magnetic duality of Maxwell theory.

The applications to QCD and condensed matter are, however, hampered by the fact that the AdS/CFT duality becomes calculable in the limit of large rank of the gauge group, or "number of colours" on the field theory side, $N_c \to \infty$. Also, the best understood example of $\mathcal{N} = 4$ SYM is very far from the real world, having both supersymmetry and conformal invariance. When we move away from supersymmetry and conformal invariance, the rules are less clear and we can calculate less, as we will see. Nevertheless, AdS/CFT is a developing field, and we have already obtained many useful results and insights, so we can hope that these methods will lead to solving interesting problems that cannot be solved otherwise.