Part I

Introduction

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Universality and Bose-Einstein Condensation: Perspectives on Recent Work

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The study of Bose-Einstein condensation has undergone a remarkable expansion during the last twenty years. Observations of this phenomenon have been reported in a number of diverse atomic, optical, and condensed matter systems, facilitated by remarkable experimental advances. The synergy of experimental and theoretical work in this broad research area is unique, leading to the establishment of Bose-Einstein condensation as a universal interdisciplinary area of modern physics. This chapter reviews the broad expansion of Bose-Einstein condensation physics in the past two decades.

1.1 Introduction

The field of Bose-Einstein condensation (BEC) has undergone an explosive expansion during the past twenty years. Newcomers to this field are now often introduced to this as a universal phenomenon, which nonetheless exhibits diverse (and sometimes strikingly different) manifestations. Despite such differences, the common underlying theme creates a unique identity across many different energy and length scales.

The study of BEC as a universal phenomenon was highlighted in a focused conference[1] in 1993, leading to the publication of the well-known "green book" [1], which surveyed the breadth of the field of condensate physics at that time. The success of the conference led to a second meeting[2] in 1995, at which Eric Cornell and Carl Wieman announced the achievement of Bose-Einstein condensation of ultracold ^{87}Rb atoms in a harmonic trap. That work began an explosion of new research in the field of cold atoms, which has continued to this day, and this very success inevitably led many of those studying cold atoms to pay less attention to other types of condensates. Wolfgang Ketterle gives a historical overview of this exciting period of time in Chapter 3. Recently, various scientific meetings have worked to re-establish the physical connections across different BEC systems, and in 2013 a workshop[3] was held with the focused goal of improving communications across disciplines. This present book is an outgrowth of that meeting.

1.2 The Situation Before the Revolution

Because of the great success of the cold atom BEC and other BEC systems in the past twenty years, it may be hard for young scientists to understand the climate of BEC research in the early 1990s. At that time, there was only one known example of BEC, namely liquid helium-4, and there was a small but vocal minority of scientists who questioned whether BEC was established even in that system. One reason for these objections was that there was no clear-cut demonstration in liquid helium of the canonical property of a condensate, namely macroscopic occupation of the ground state. Neutron scattering data (reviewed by Paul Sokol in the 1995 green book [1]) were consistent with macroscopic occupation of the ground state, but because liquid helium is strongly interacting, there was no obvious condensate peak at zero momentum.

At the same time, up to the early 1990s, some scientists still argued that BEC had little or nothing to do with Bardeen–Cooper–Schrieffer (BCS) superconductivity or the BCS state of liquid helium-3, following John Bardeen's own original objections to such a connection, although J. M. Blatt, [2] Tony Leggett, [3] and others had argued for a deep connection between the two very early on; Leggett's recent book, *Quantum Fluids* [4], elegantly draws the connection between the two. Mohit Randeria's chapter on BEC-BCS crossover was considered groundbreaking

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[1] First International Conference on Bose-Einstein Condensation, Levico Terme, Italy, 1993; organized by D. Snoke, A. Griffin, S. Stringari, and A. Mysyrowicz.
[2] Second International Conference on Bose-Einstein Condensation, Mt. Ste. Odile, France, 1995; organized by J. T. M. Walraven, J. Treiner, and M. W. Reynolds.
[3] "Universal Themes of Bose-Einstein Condensation," Lorentz Center, Leiden, The Netherlands; organized by K. Burnett, P. B. Littlewood, N. P. Proukakis, D. W. Snoke, and H. T. C. Stoof.

to include in the green book [1]. Indeed, this was one of the universal themes explored at the 1993 meeting, with the general theory of BEC-BCS crossover originally explored in the context of excitons in semiconductors [5, 6] (see also the chapter by L. V. Keldysh in the green book [7]), and later extended to superconductors [8]). BEC-BCS crossover has been an active field of cold atom research in the past decade.

The state of the field in 1993, therefore, was that of having one physical example of Bose-Einstein condensation, liquid helium-4, and a large number of proposed systems, with various degrees of theoretical and experimental work. The main candidates at that time were cold atoms, excitons in semiconductors, and spin-polarized hydrogen. (BEC of positronium was also proposed at that time, as discussed in a short chapter by P. M. Platzmann and A. P. Mills in the green book [9], and still remains a possibility.) Condensation of spin-polarized hydrogen was obtained in 1998 [10]; Thomas Greytak and Daniel Kleppner review the history of that work in Chapter 2. The remarkable early work on spin-polarized hydrogen contributed to the eventual success of BEC in cold atoms, through the development of evaporative cooling methods, a feat recognized by the scientific community through the Senior BEC Award in 2015 to Thomas Greytak, Harald Hess, and Daniel Kleppner for "inventing the technique of evaporative cooling, and for the observation of Bose-Einstein condensation in a dilute gas of atomic hydrogen."

1.3 Early Work on Excitons in Semiconductors

In the early 1990s, excitons in semiconductors (especially in bulk Cu_2O and GaAs coupled quantum wells) seemed to be the next most promising system. Good fits of the Bose-Einstein distribution to the experimental exciton energy distribution in Cu_2O had been observed over a wide range of density and temperature. However, questions about this interpretation arose already in 1996 [11], and by 2000 it was known [12] that the experiments on excitons in Cu_2O did not show BEC. The spatial distribution was found to be much more inhomogeneous than initially believed. The fits of the exciton kinetic-energy distribution to the Bose-Einstein distribution were surprisingly good given the accidental nature of how they arose.

A review of work on Cu_2O in the past two decades is given in Ref. [13]. Until very recently, the behavior of the excitons at high density in Cu_2O was believed to be dominated by a density-dependent, nonradiative recombination process known as Auger recombination. Recently, indications have emerged [14] that there may be a strongly bound biexciton state in Cu_2O. (The biexciton, or excitonic molecule, is comprised of two excitons, and is well known to exist in other semiconductors, but was widely believed to be nonexistent or very weakly bound in Cu_2O, based on theoretical calculations [15, 16].) Weak biexciton light emission could be masked

by a nearby, brighter single-exciton emission line. If a stable biexciton state exists, then it would be the ground state which would undergo condensation, not the single exciton states. It may also play a role in assisting nonradiative recombination [17].

Similarly, reports of evidence for BEC of dipolar excitons in coupled semiconductor quantum wells, reported at the 1995 meeting, did not play out as hoped. Most of the anomalous results at low temperature in this system have found other explanations (for a review, see Ref. [18]). Recently, two groups [19, 20] have reported evidence for increased coherence in the light emission from this system at low temperature. Objections have been raised [21, 22] that this coherence is an artifact of the way the experiments were done at the limit of the spatial resolution of the imaging lenses – a reduction in the spatial extent of the exciton cloud could give a similar effect. Although these objections have been addressed in part [23], the interpretation of the dipolar exciton experiments as BEC remains controversial. Spectral narrowing with increase of particle density, considered a key test of coherence in optical systems, and a tell-tale for excitonic BEC, has not been observed for Cu_2O or coupled quantum well excitons.

Although the early work on Cu_2O had to be reinterpreted, experiments pursuing BEC of excitons in this unique semiconductor continue to this day. Two groups [24, 25, 26], in particular, have pursued studies of excitons in this material at millikelvin temperatures. The low temperature allows the possibility of BEC at very low exciton density, so that nonradiative and biexciton effects become much less important.

1.4 The "Atomic Revolution"

As mentioned above, the year 1995 was a turning point for ultracold atoms and for the broader field of weakly interacting Bose-Einstein condensation. Within a period of only a few weeks, three different US groups reported the observation of Bose-Einstein condensation in trapped, dilute weakly interacting atomic gases of ^{87}Rb [27], ^7Li [28], and ^{23}Na [29]. The observations and subsequent investigations led to the 2001 Nobel Prize in Physics being jointly awarded to Eric Cornell, Wolfgang Ketterle, and Carl Wieman "for the achievement of Bose-Einstein condensation in dilute gases of alkali atoms, and for early fundamental studies of the properties of the condensates." Randy Hulet's groundbreaking work with effectively attractive atoms of ^7Li turned out to have some issues with imaging resolution, with the definitive realization of BEC in ^7Li reported by the same group (as an *erratum* to the original manuscript) in 1997 [30].

A unique attraction of the cold atom experiments was that they provided a "smoking gun" of BEC: although BEC refers to condensation in *momentum* space (for a homogeneous gas), the harmonic traps used in cold atom experiments also

enabled the phase transition to be observed in *position* space.[4] The traps used in liquid helium experiments had nominally flat potential energy profile, and therefore the density inhomogeneity in the cold atoms experiments added both excitement and a challenge to identify (both experimentally and theoretically) the precise nature of the observable effects and novel features present in such systems due to the zero-point motion and the modified density of states. Atomic BECs were realized at remarkably low temperatures of the order of 100 nK. The low temperatures reflect the required condition to reach quantum degeneracy for sufficiently dilute atomic gases. At higher densities, which nominally would allow higher critical temperature for BEC, three-body collisions severely limit the lifetime of the atoms in the traps. The extremely low temperatures are impressive experimental achievements, but pose a challenge to any technological applications, as they require an extremely good vacuum.

The experimental observation of BEC in cold atoms sparked a significant interest in the broader public domain, with coverage reaching mainstream newspaper headlines, nonscientific journals,[5] and national news sources. Bose-Einstein condensation, recently identified as one of the top five physics achievements of the past twenty-five years,[6] was no longer an obscure physical phenomenon.

Atomic BEC experiments during the first few years focused primarily on equilibrium properties, elementary and nonlinear excitations (vortices, solitons), low-order correlation functions, and the effects of thermal fluctuations (see, e.g., the 1999 review by Dalfovo et al. [31]). Soon atomic BEC experiments, which were by now starting to be done in numerous labs worldwide, moved toward enhanced control (e.g., of interaction strength [32] or dimensionality [33]), characterization of dynamics, and manifestation of distinct regimes mimicking the solid state. These latter include periodic "optical lattice" potentials (reviewed in Chapter 13 by Immanuel Bloch) and spinor condensates/quantum magnetism (reviewed in Chapter 18 by Masahito Ueda). The achievement of quantum degeneracy of fermionic gases [34] has paved the way for studies of Fermi superfluids and a controlled characterization of the BEC-BCS crossover.

By the late 1990s, then, we had three good experimental examples of BEC: liquid helium-4, trapped cold alkali atoms, and spin-polarized hydrogen experiments, with cold atoms already demonstrating an immense potential for characterizing and harnessing the properties of quantum gases. Superconductors and helium-3 in

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[4] Actually, the initial experiments measured the density distribution after expansion, thus corresponding to information on the velocity distributions, with subsequent experiments able to directly image density profiles "in situ."

[5] See, e.g., "Bose knows," in the Science & Technology section, *The Economist*, 1 July 1995, pages 93–94.

[6] *Physics World*, October 2013 Special Issue (Institute of Physics); see also www.bbc.co.uk/news/science-environment-24282059.

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the BCS limit provided alternative, more involved, manifestations of macroscopic quantum coherence.

1.5 BEC Physics in the New Millennium: Excitons, Polaritons, Photons, Magnons, and Triplons

The decade of the 2000s turned another important corner for controlled quantum gas studies, with numerous credible examples of BEC or quasi-BEC in different systems in the optical and condensed matter regimes.

Two variations of the excitonic systems made it much easier to observe BEC than in the early types of experiments discussed above. In one variation, magnetic field and electronic gating was used to control the density of electrons and holes in a coupled quantum well system. The pairing of electrons and holes in this system can be described as thermodynamically stable excitons. (For a review, see Ref. [35].) These experiments have much in common with earlier experiments on the integer and fractional quantum Hall effects, but show a different physics, with transport through the bulk of the two-dimensional (2D) gas [36] instead of just along the edges.

The other variation which exploded on the scene in the early to mid 2000s was to place the excitons in a high-Q optical cavity, creating strong coupling between an optical cavity mode and the exciton state. The eigenstates of this system are known as polaritons. This system can be visualized in either of two ways. Starting with excitons, one can think of the polariton effect as giving the exciton much lighter mass (typically by three orders of magnitude), so that the excitons have much longer wavelength, and therefore much greater quantum effects, at typical densities and temperatures. The light mass also means that disorder plays much less of a role, since the polaritons average over disorder on length scales less than their wavelength. On the other hand, starting with photons, one can think of the polariton mixing as giving the photons much stronger interactions than they would normally have in a solid, so that they can thermalize. (Recent work [37] indicates that these interactions may even be much stronger than theoretically expected.)

Following promising early work [38, 39], the canonical effects expected for BEC, namely onset of coherence, a bimodal momentum distribution with a peak in the ground state, and spatial condensation in the ground state of a trap (effects previously demonstrated for cold atoms), were also shown in the polariton system in 2006–2007 [40, 41]. This book contains a general review of polariton condensation (Chapter 4), along with several chapters reviewing the broad diversity of work now done in such systems (Chapters 10–11 and 20–24). The early work was all done with structures with fairly short polariton lifetime, as they could turn into photons which leaked out of the optical cavity. In the initial experiments, the lifetime was

about 1–2 ps, comparable to the thermalization time. For this reason, polariton condensates have often been referred to as "nonequilibrium condensates." However, the lifetime of polaritons in cavities has been gradually increasing, so that lifetimes of 20–30 ps are now routine and lifetimes up to 300 ps have been obtained [42]. In many cases, it is now more appropriate to say that the polaritons are "nearly equilibrated," and true equilibrium results have recently been reported [43]. In general, there are now many examples of polariton systems that "look like" condensates, such as the ring condensate shown in Fig. 4.4 of Chapter 4 (and on the front cover of this book).

Because the early polariton experimental work arose in systems that were to some extent not in equilibrium, theory in the field addressed at length the universal questions of how dissipation and nonequilibrium affects BEC [44, 45]. Jonathan Keeling and colleagues review some of this work in Chapter 11. Note that the equilibrium of cold atomic BEC is not perfect: three-body recombination gradually converts the gaseous BEC into a solid (for the typical experimental densities this happens on a time scale of many seconds). The decay of a polariton into an external photon by tunneling through a cavity mirror is more analogous to tunneling of an atom through a barrier. While the destruction of BEC by molecular formation is physically distinct from the out-tunneling of a particle, it is still debatable how sharp a distinction can be made between equilibrium and nonequilibrium condensates. Even in the experiments with paired electrons and holes in the quantum-Hall-type experiments discussed above, the population is not truly permanent – the average number of electrons in the two-dimensional plane depends on the balance of electrons scattering into and out of the trapped state from the rest of the semiconductor structure, with rates that depend on the temperature and the applied gate voltages. Only in liquid helium can we say that the number of particles in the system can be truly conserved. Nonequilibrium effects have become a major topic of BEC studies, and are also addressed in the studies of quenches and the onset of coherence, discussed below.

The broad range of work with polariton condensates brings out another general theme in condensate theory, namely BEC-laser crossover, analogous to BEC-BCS crossover (the latter reviewed in Chapter 12 of this book). In general, lasing occurs in a system of photons and electronic transitions when the interaction between the photons is weak or nonexistent, while BEC of polaritons occurs in the strong coupling regime, when the interaction between the photons dominates their behavior. Both emit coherent light and have a coherent medium [46]. In practice, there is not a smooth transition between these two limits; in the experiments, the system "pops" between lasing and polariton condensation [47, 48] at different excitation densities. Alessio Chiocchetta et al. review the connection of lasing and polariton BEC in their chapter in this book (Chapter 20). There is also a crossover between

lasing and photon BEC. The latter was achieved in 2010 [49] by trapping photons in a high-Q cavity containing a solution of dye molecules; Jan Klaers and Martin Weitz review this work in Chapter 19. In this case, the photons are essentially noninteracting, but they can thermalize by repeated absorption into the dye and re-emission, because the high Q of the cavity gives a photon lifetime long compared with the time scale for re-emission. Although the photons have a finite lifetime to leak out of the cavity, a condensate can be established in steady state with an optical pump to replenish the particles. Similar thermalization and condensation behavior has been observed for lasers with a one-dimensional continuum of states [50, 51].

Besides the photonic/excitonic condensates, in the 2000s two more varieties of condensate were demonstrated in magnetic systems, namely magnons created as a nonequilibrium population in a ferromagnet, and "triplons" in antiferromagnets, in which the condensate is the stable ground state at certain values of the magnetic field and temperature. Salman et al. review the former in this book (Chapter 25), while Kollath et al. review the latter (Chapter 28), with the related topic of "spin-tronics" discussed by Duine et al. in Chapter 26.

Three of these systems, namely photons in a high-Q cavity, magnons in ferro-magnets, and triplons in antiferromagnets, can undergo BEC at room temperature. Some polariton systems also show characteristics of BEC at room temperature [52, 53, 54, 55]. Polaritons in organic semiconductors may soon become a ubiquitous room temperature condensate used for optical devices.

All of these examples can be called BEC of quasiparticles, but "quasiparticle" here should not be read as "not real." Quasiparticles are simply particles which arise when a simple, diagonal Hamiltonian is constructed out of a more complicated, underlying Hamiltonian. A universal question arising out of the observation of BEC in bosonic quasiparticles, however, is the degree to which we can make a clear demarcation between "quantum" and "classical" effects. It is standard in BEC theory to describe a condensate by a Gross-Pitaevskii equation [56, 57], which is essentially a classical nonlinear wave equation. This is because a coherent macro-scopic condensate acts just the same as a classical wave – this macroscopic coherent behavior is one of the attributes which makes condensates so interesting. But any classical nonlinear system will behave the same way in the low-frequency limit. Jason Fleischer and coworkers [58] have shown that classical optical waves with random phases in a self-defocusing refractive crystal condense in a way fully anal-ogous to that of a BEC. Of course, from the quantum perspective, classical waves exist because the bosonic quantum field relations allow macroscopic occupation and coherence, so that we cannot say that any such classical wave behavior is not at all quantum.

1.6 Recent Experimental Trends

In the meantime, experiments on cold atoms have continued apace, opening up in even more diverse directions, from the very fundamental to the technological. For the purposes of this chapter, we only mention in passing some of the most notable advances over the past two decades. Those include detailed studies in guided, or controlled, geometries (e.g., atom chips [59], ring-traps [60], and box-like potentials [61]), multicomponent and spinor BECs [62] (see also Chapter 18), studies of superfluidity and sound propagation (see also Chapter 16) and nonlinear effects including dark and bright solitons and vortices [63] extending to the realm of quantum turbulence (see Chapter 17), Dirac monopoles [64], observations of Josephson effects [65, 66], and the use of atoms in optical lattices to create strongly correlated systems and as quantum simulators of condensed matter systems [67] (see also Chapter 13). Controlled experiments have further enabled the study of different phase transitions, including the superfluid-Mott insulator transition [68], the BEC-BCS crossover (reviewed in Chapter 12), the BEC phase transition itself (both statically and through dynamic quenches obeying the Kibble-Zurek scaling law – see Chapter 7) and the effect of interactions on it (Chapter 6), the related study of the Berezinskii-Kosterlitz-Thouless (BKT) phase transition in two-dimensional settings and scale invariance (see Chapter 9), studies of integrability in one-dimensional settings [69], and prethermalization (see Chapter 8). Other topics of interest include condensates with controllable [70] or long-range interactions [71], the controlled study of disorder [72] and quasiperiodic potentials [73], creation of artificial magnetic fields (see Chapter 15) and topological band structures (see Chapter 14), and studies of analogue gravity and Hawking radiation [74]. Moreover, the unprecedented experimental control in such systems is setting the scene for technological applications, such as in precision measurements, quantum simulations, or atomtronics.

Most of the above interesting features are now also being routinely studied in polariton condensates, reviewed in Chapter 4 by Peter Littlewood and Alexander Edelman (see also the review article by Carusotto and Ciuti [44]). While by no means a complete survey, this book highlights the study of vortex dynamics (Chapter 21) and other topological excitations (Chapter 24), the role of disorder and phase-locking (Chapter 23), and the versatility of optical control for such systems (Chapter 22). As mentioned above, there is a strong trend toward studies of BEC of polaritons and other quasiparticles at room temperature.

In addition to the laboratory examples discussed above, there are several proposals that BEC may occur at the astronomical and cosmological scales. The theory of BEC of neutron matter is on sound footing; possible condensates in neutron stars are reviewed by Chris Pethick et al. in Chapter 29. There is a long history of