HOLOGRAPHIC DUALITY IN CONDENSED MATTER PHYSICS

This pioneering treatise presents how the new mathematical techniques of holographic duality unify seemingly unrelated fields of physics. Morphing quantum field theory, general relativity and the renormalisation group into a single computational framework, this book is the first to bring together a wide range of research in this rapidly developing field. Set within the context of condensed matter physics and using boxes highlighting the specific techniques required, it examines the holographic description of thermal properties of matter, Fermi liquids and superconductors, and hitherto unknown forms of macroscopically entangled quantum matter in terms of general relativity, stars and black holes.

Showing that holographic duality can succeed where classic mathematical approaches fail, this text provides a thorough overview of this major breakthrough at the heart of modern physics. The inclusion of extensive introductory material using non-technical language and online Mathematica notebooks ensures the appeal to students and researchers alike.

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Holographic duality in condensed matter physics

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## Contents

**Preface**

| Preface          | page ix |

### 1 Introduction

| 1.1 A tour guide of holographic matter | 1 |
| 1.2 The AdS/CFT correspondence: unifying the theories of physics | 3 |
| 1.3 AdS/CFT, the geometrisation of the renormalisation group and the quantum critical state | 6 |
| 1.4 Holographic duality and the nature of matter | 18 |
| 1.5 Holography, condensed matter physics and quantum information | 27 |

### 2 Condensed matter: the charted territory

| 2.1 Quantum field theory in condensed matter: the fluctuating order of bosons | 34 |
| 2.2 Quantum matter: when entanglement becomes macroscopic | 51 |
| 2.3 The remarkable Fermi liquid | 57 |
| 2.4 The mean-field instabilities of the fermion system: BCS and beyond | 64 |
| 2.5 The Hertz–Millis model and the critical Fermi surface | 69 |

### 3 Condensed matter: the challenges

| 3.1 The repertoire of strongly correlated electron models | 76 |
| 3.2 Mottness, non-Fermi liquids and RVB superconductivity | 79 |
| 3.3 Unconventional superconductivity by weak repulsive interactions | 88 |
| 3.4 Numerical machinery I: the density-matrix renormalisation group and tensor-product states | 90 |
| 3.5 Numerical machinery II: infinitely many dimensions and the dynamical mean-field theory | 92 |
| 3.6 Quantum matter in the laboratory | 94 |
### Contents

#### Chapter 4: Large-\(N\) field theories for holography and condensed matter

4.1 A short history of the holographic principle, black holes, string theory and the origins of the \(\text{AdS/CFT} \) correspondence  
4.2 Yang–Mills as a matrix field theory and the ultimate mean field at large-\(N\)  
4.3 The master formulation of large-\(N\) matrix models and string theory  
4.4 The \(\text{AdS/CFT} \) correspondence is a field-theory/\(\text{gravity}\) duality

#### Chapter 5: The \(\text{AdS/CFT} \) correspondence as computational device: the dictionary

5.1 The GKPW rule in action: computing correlation functions  
5.2 Correlations, scaling and RG flows  
5.3 The identification of symmetries and the dictionary table

#### Chapter 6: Finite-temperature magic: black holes and holographic thermodynamics

6.1 Black holes in the bulk and finite temperature in the boundary  
6.2 Holographic thermodynamics: the Hawking–Page transition  
6.3 A brief introduction to \(\text{AdS/QCD} \)  
6.4 The GKPW rule at finite temperature: thermal correlation functions and the Schwinger–Keldysh formalism

#### Chapter 7: Holographic hydrodynamics

7.1 Quantum criticality and the minimal viscosity  
7.2 Deriving the Navier–Stokes fluid from the bulk dynamical gravity  
7.3 The conductivity: conserved currents as photons in the bulk  
7.4 Hydrodynamics and quantum anomalies

#### Chapter 8: Finite density: the Reissner–Nordström black hole and strange metals

8.1 The Reissner–Nordström strange metal  
8.2 The \(\text{AdS}_2 \) near-horizon geometry and the emergent local quantum criticality  
8.3 The zero sound and conductivity of the RN metal in the Galilean continuum  
8.4 The scaling atlas of emergent holographic quantum critical phases

Page

105
106
118
130
139
151
154
165
172
176
178
193
205
213
222
226
238
243
253
259
261
267
271
281
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>Holographic photoemission and the RN metal: the fermions as probes</td>
<td></td>
</tr>
<tr>
<td>9.1</td>
<td>The holographic encoding of fermions</td>
<td>294</td>
</tr>
<tr>
<td>9.2</td>
<td>The discovery of the holographic Fermi surfaces</td>
<td>304</td>
</tr>
<tr>
<td>9.3</td>
<td>Computing fermion spectral functions: Schrödinger potentials and the matching method</td>
<td>312</td>
</tr>
<tr>
<td>9.4</td>
<td>The physics of the holographic fermions: confinement, semi-holography and black-hole stability</td>
<td>323</td>
</tr>
<tr>
<td>10</td>
<td>Holographic superconductivity</td>
<td></td>
</tr>
<tr>
<td>10.1</td>
<td>The black hole in AdS with scalar hair</td>
<td>330</td>
</tr>
<tr>
<td>10.2</td>
<td>The phenomenology of holographic superconductivity</td>
<td>340</td>
</tr>
<tr>
<td>10.3</td>
<td>Observing the origin of $T_c$: the pair susceptibility of the strange metal</td>
<td>355</td>
</tr>
<tr>
<td>10.4</td>
<td>The phase diagram of holographic superconductivity</td>
<td>362</td>
</tr>
<tr>
<td>10.5</td>
<td>The zero-temperature states of holographic superconductors</td>
<td>368</td>
</tr>
<tr>
<td>11</td>
<td>Holographic Fermi liquids: the stable Fermi liquid and the electron star as holographic dual</td>
<td></td>
</tr>
<tr>
<td>11.1</td>
<td>The cohesive Landau Fermi liquid from hard-wall holography</td>
<td>381</td>
</tr>
<tr>
<td>11.2</td>
<td>The electron star as the dual of holographic fermions</td>
<td>390</td>
</tr>
<tr>
<td>11.3</td>
<td>The landscape of holographic Fermi liquids: radial re-quantisation and instabilities</td>
<td>403</td>
</tr>
<tr>
<td>12</td>
<td>Breaking translational invariance</td>
<td></td>
</tr>
<tr>
<td>12.1</td>
<td>Transport and un-particle physics: the memory-matrix formalism</td>
<td>420</td>
</tr>
<tr>
<td>12.2</td>
<td>Periodic potentials in holographic superconductors and the optical conductivity</td>
<td>432</td>
</tr>
<tr>
<td>12.3</td>
<td>Lattice potentials and the fermion spectral functions of the Reissner–Nordström metal</td>
<td>437</td>
</tr>
<tr>
<td>12.4</td>
<td>Unidirectional potentials becoming strong: Bianchi VII geometry and the quantum smectic</td>
<td>442</td>
</tr>
<tr>
<td>12.5</td>
<td>The dual of translational-symmetry breaking as gravity with a mass</td>
<td>452</td>
</tr>
<tr>
<td>12.6</td>
<td>Holographic crystallisation: the spontaneous breaking of translational symmetry</td>
<td>463</td>
</tr>
<tr>
<td>13</td>
<td>AdS/CMT from the top down</td>
<td></td>
</tr>
<tr>
<td>13.1</td>
<td>Top-down AdS/CMT models from supergravity</td>
<td>472</td>
</tr>
<tr>
<td>13.2</td>
<td>Probe-brane holography from $Dp/Dq$-brane intersections</td>
<td>483</td>
</tr>
</tbody>
</table>
14 Outlook: holography and quantum matter

14.1 The UHOs: the unidentified holographic objects
14.2 Is holographic matter extreme quantum matter?
14.3 The final message for condensed matter

References

Index
Preface

Not so long ago, two large and quite old fields in physics, string theory and condensed matter physics, were more or less at the opposite ends of the physics building. During the 40 or so years of its history, string theory has developed into a high art of “mathematical machine building”, propelled forwards by the internal powers of mathematics as inspired by physics. Yet, it has suffered greatly for the shortcoming that its theoretical answers are always beyond the reach of experimental machinery. Modern condensed matter physics is in the opposite corner. It has been propelled forwards by continuously improving experiments, which have delivered one serendipitous discovery after another during the last few decades. However, its interpretational framework rests by and large on equations developed 40 years or so ago. There has been an increasing sense that it is these that fall short in trying to explain the strongly interacting quantum many-body systems as realised by electrons in high-$T_c$ superconductors and other unconventional materials.

All this changed dramatically in 2007 when physicists started to feed condensed matter questions to the most powerful mathematical machine of string theory: the holographic duality in the title of the book, also known as the “anti-de Sitter/conformal field theory” (AdS/CFT) correspondence. This book introduces the explosion of answers that has followed since then.

The first (Jan) and last (Koenraad) of this book’s authors are from such opposite corners. As soon as the seminal work of Herzog, Kovtun, Sachdev and Son in 2007 showed that these two subjects have dealings with each other, Jan and Koenraad recognised the potential and met up, almost literally half-way up the stairs. As has been characteristic for the development at large, it took us remarkably little effort to get on speaking terms, despite our superficially very different backgrounds. Shrouded by differences in language, string theory and condensed matter had already been on a collision course for a while, meeting each other on the common ground of quantum criticality/conformal field theory. In the years that
followed this dialogue only intensified and the upbeat tone of this book is a testi-
mony of the great time we had together. The largest part of that time was shared
with the two middle authors (Ya-Wen and Yan), who came to Leiden as postdocs
in 2010, freshly graduated from the Chinese Academy of Science in Beijing. The
seeds for this book were planted when Jan received the invitation to become the
2012 Solvay Professor in Brussels, with the request to organise an AdS/condensed-
matter-theory lecture course. What you see before you grew from the notes of this
course, joined together with lecture notes by Koenraad at the 2012 Cargèse and
2013 Crete schools.

This is an incredibly fast-moving field, and many pages had to be added to
describe the developments that happened since the summer of 2012. In January
2014 we stopped playing catch up, and we decided to get it out as quickly as possi-
gle, given the high demand for such a text at this moment. The first nine months of
2014 turned into a writing frenzy for all of us, and the result is lying in front of you.
We are well aware that in certain regards the book therefore has its limitations and
that the text will already be obsolete as soon as it appears. Examples of significant
developments that occurred since our cut-off are a holographically inspired the-
ory of incoherent metals\(^1\) and a holographic solution of the anomalous temperature
scaling of the Hall angle as observed in high-\(T_c\) superconductors.\(^2\) Nor do we claim
this to be a comprehensive review that does justice to all of the papers which have
been published on the subject. What we have done is to provide an introduction to
serve a non-expert readership that wishes to be informed about the main develop-
ments. Our aim has been to catch the mainstream, those developments where one
discerns a consensus in the expert community that these are the most significant
accomplishments. As authors we found it quite obvious how to make this selec-
tion and we sincerely believe that our choices will be approved by the AdS/CMT
experts. We felt that we just had the role of humble narrators working on the chron-
icles of a monumental physics odyssey. We wish to take you on board and we hope
you will enjoy it as much as we do!

We are in the first instance indebted to numerous holographists who contributed
to our understanding of the correspondence. We are particularly grateful to Andrea
Amoretti, Steffen Klug, Richard Davison, Andrey Bagrov, Petter Sæterskog and
Balázs Meszéna for their thorough proofreading of the manuscripts and their many
helpful suggestions, and to Mihael Petač for his help on the figures. Both the Leiden
and Madrid physics departments gave us all the room to concentrate on the writing
of this book. We acknowledge financial support of various funding organisations,
in particular the Solvay Foundation and the Dutch Foundation of Fundamental

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Preface

Research on Matter (FOM) in the initial stages of this project, as well as the Spanish MINECO’s “Centros de Excelencia Severo Ochoa” Programme under grant SEV-2012-0249, the Netherlands Organisation for Scientific Research/Ministry of Science and Education (NWO/OCW), and a grant from the Templeton Foundation: the opinions expressed in this publication are those of the authors and do not necessarily reflect the views of the John Templeton Foundation. Jan Zaanen and Koenraad Schalm acknowledge the hospitality of various institutions during the writing process: the Aspen Center of Physics, supported by the National Science Foundation under Grant No. PHYS-1066293, the Kavli Institute for Theoretical Physics, supported by the National Science Foundation under Grant No. NSF PHY11-25915, and the physics department of Harvard University in particular.

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