

Cambridge University Press

978-1-107-08008-9 - Holographic Duality in Condensed Matter Physics

Jan Zaanen, Ya-Wen Sun, Yan Liu and Koenraad Schalm

Frontmatter

[More information](#)

HOLOGRAPHIC DUALITY IN CONDENSED MATTER PHYSICS

This pioneering treatise presents how the new mathematical techniques of holographic duality unify seemingly unrelated fields of physics. Morphing quantum field theory, general relativity and the renormalisation group into a single computational framework, this book is the first to bring together a wide range of research in this rapidly developing field. Set within the context of condensed matter physics and using boxes highlighting the specific techniques required, it examines the holographic description of thermal properties of matter, Fermi liquids and superconductors, and hitherto unknown forms of macroscopically entangled quantum matter in terms of general relativity, stars and black holes.

Showing that holographic duality can succeed where classic mathematical approaches fail, this text provides a thorough overview of this major breakthrough at the heart of modern physics. The inclusion of extensive introductory material using non-technical language and online Mathematica notebooks ensures the appeal to students and researchers alike.

JAN ZAAZEN is Professor of Theoretical Physics at the Instituut-Lorentz for Theoretical Physics, Leiden University, the Netherlands where he specialises in the physics of strongly interacting electrons. He is a recipient of the Dutch Spinoza Award and fellow of the Dutch Royal Academy of Sciences and the American Physical Society.

YA-WEN SUN is a Postdoctoral Researcher at the Institute for Theoretical Physics at the Universidad Autónoma de Madrid where she works on applications of AdS/CFT to condensed matter theory, QCD and hydrodynamics as well as other aspects of quantum gravity.

YAN LIU is a Postdoctoral Researcher at the Institute for Theoretical Physics at the Universidad Autónoma de Madrid where he specialises in high-energy theoretical physics including gauge/gravity duality and AdS/CMT.

KOENRAAD SCHALM is Professor of Theoretical Physics at the Instituut-Lorentz for Theoretical Physics, Leiden University, the Netherlands. His research focusses on how string theory may be detected in laboratory experiments or cosmological observations. He is the recipient of Innovative Research Incentives Awards of the Netherlands Organisation for Scientific Research.

Cambridge University Press
978-1-107-08008-9 - Holographic Duality in Condensed Matter Physics
Jan Zaanen, Ya-Wen Sun, Yan Liu and Koenraad Schalm
Frontmatter
[More information](#)

Cambridge University Press
978-1-107-08008-9 - Holographic Duality in Condensed Matter Physics
Jan Zaanen, Ya-Wen Sun, Yan Liu and Koenraad Schalm
Frontmatter
[More information](#)

HOLOGRAPHIC DUALITY IN
CONDENSED MATTER PHYSICS

JAN ZAAANEN

Universiteit Leiden, the Netherlands

YA-WEN SUN

Universidad Autónoma de Madrid, Spain

YAN LIU

Universidad Autónoma de Madrid, Spain

KOENRAAD SCHALM

Universiteit Leiden, the Netherlands



CAMBRIDGE
UNIVERSITY PRESS

Cambridge University Press
978-1-107-08008-9 - Holographic Duality in Condensed Matter Physics
Jan Zaanen, Ya-Wen Sun, Yan Liu and Koenraad Schalm
Frontmatter
[More information](#)

CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

Cambridge University Press is part of the University of Cambridge.

It furthers the University’s mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org

Information on this title: www.cambridge.org/9781107080089

© Cambridge University Press 2015

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 2015

Printed in the United Kingdom by TJ International Ltd. Padstow Cornwall

A catalogue record for this publication is available from the British Library

Library of Congress Cataloguing in Publication data

Zaanen, Jan, 1957– author.

Holographic duality in condensed matter physics / Jan Zaanen (Universiteit Leiden, the Netherlands), Yan Liu (Universidad Autónoma de Madrid, Spain), Ya-Wen Sun (Universidad Autónoma de Madrid, Spain), Koenraad Schalm (Universiteit Leiden, the Netherlands).

pages cm

Includes bibliographical references and index.

ISBN 978-1-107-08008-9 (hbk.)

1. Condensed matter. 2. Holography. I. Liu, Yan (Postdoctoral researcher), author. II. Sun, Ya-Wen, author. III. Schalm, Koenraad, 1971– author. IV. Title. QC173.457.H65Z36 2015

530.4’12–dc23

2015021137

ISBN 978-1-107-08008-9 Hardback

Additional resources for this publication at www.cambridge.org/9781107080089

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication, and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.

Contents

<i>Preface</i>	<i>page ix</i>
1 Introduction	1
1.1 A tour guide of holographic matter	1
1.2 The AdS/CFT correspondence: unifying the theories of physics	3
1.3 AdS/CFT, the geometrisation of the renormalisation group and the quantum critical state	6
1.4 Holographic duality and the nature of matter	18
1.5 Holography, condensed matter physics and quantum information	27
2 Condensed matter: the charted territory	32
2.1 Quantum field theory in condensed matter: the fluctuating order of bosons	34
2.2 Quantum matter: when entanglement becomes macroscopic	51
2.3 The remarkable Fermi liquid	57
2.4 The mean-field instabilities of the fermion system: BCS and beyond	64
2.5 The Hertz–Millis model and the critical Fermi surface	69
3 Condensed matter: the challenges	73
3.1 The repertoire of strongly correlated electron models	76
3.2 Mottness, non-Fermi liquids and RVB superconductivity	79
3.3 Unconventional superconductivity by weak repulsive interactions	88
3.4 Numerical machinery I: the density-matrix renormalisation group and tensor-product states	90
3.5 Numerical machinery II: infinitely many dimensions and the dynamical mean-field theory	92
3.6 Quantum matter in the laboratory	94

vi	<i>Contents</i>	
4	Large-N field theories for holography and condensed matter	105
4.1	A short history of the holographic principle, black holes, string theory and the origins of the AdS/CFT correspondence	106
4.2	Yang–Mills as a matrix field theory and the ultimate mean field at large- N	118
4.3	The master formulation of large- N matrix models and string theory	130
4.4	The AdS/CFT correspondence is a field-theory/gravity duality	139
5	The AdS/CFT correspondence as computational device: the dictionary	151
5.1	The GKPW rule in action: computing correlation functions	154
5.2	Correlations, scaling and RG flows	165
5.3	The identification of symmetries and the dictionary table	172
6	Finite-temperature magic: black holes and holographic thermodynamics	176
6.1	Black holes in the bulk and finite temperature in the boundary	178
6.2	Holographic thermodynamics: the Hawking–Page transition	193
6.3	A brief introduction to AdS/QCD	205
6.4	The GKPW rule at finite temperature: thermal correlation functions and the Schwinger–Keldysh formalism	213
7	Holographic hydrodynamics	222
7.1	Quantum criticality and the minimal viscosity	226
7.2	Deriving the Navier–Stokes fluid from the bulk dynamical gravity	238
7.3	The conductivity: conserved currents as photons in the bulk	243
7.4	Hydrodynamics and quantum anomalies	253
8	Finite density: the Reissner–Nordström black hole and strange metals	259
8.1	The Reissner–Nordström strange metal	261
8.2	The AdS ₂ near-horizon geometry and the emergent local quantum criticality	267
8.3	The zero sound and conductivity of the RN metal in the Galilean continuum	271
8.4	The scaling atlas of emergent holographic quantum critical phases	281

<i>Contents</i>		vii
9	Holographic photoemission and the RN metal: the fermions as probes	291
9.1	The holographic encoding of fermions	294
9.2	The discovery of the holographic Fermi surfaces	304
9.3	Computing fermion spectral functions: Schrödinger potentials and the matching method	312
9.4	The physics of the holographic fermions: confinement, semi-holography and black-hole stability	323
10	Holographic superconductivity	328
10.1	The black hole in AdS with scalar hair	330
10.2	The phenomenology of holographic superconductivity	340
10.3	Observing the origin of T_c : the pair susceptibility of the strange metal	355
10.4	The phase diagram of holographic superconductivity	362
10.5	The zero-temperature states of holographic superconductors	368
11	Holographic Fermi liquids: the stable Fermi liquid and the electron star as holographic dual	377
11.1	The cohesive Landau Fermi liquid from hard-wall holography	381
11.2	The electron star as the dual of holographic fermions	390
11.3	The landscape of holographic Fermi liquids: radial re-quantisation and instabilities	403
12	Breaking translational invariance	417
12.1	Transport and un-particle physics: the memory-matrix formalism	420
12.2	Periodic potentials in holographic superconductors and the optical conductivity	432
12.3	Lattice potentials and the fermion spectral functions of the Reissner–Nordström metal	437
12.4	Unidirectional potentials becoming strong: Bianchi VII geometry and the quantum smectic	442
12.5	The dual of translational-symmetry breaking as gravity with a mass	452
12.6	Holographic crystallisation: the spontaneous breaking of translational symmetry	463
13	AdS/CMT from the top down	470
13.1	Top-down AdS/CMT models from supergravity	472
13.2	Probe-brane holography from Dp/Dq -brane intersections	483

Cambridge University Press
978-1-107-08008-9 - Holographic Duality in Condensed Matter Physics
Jan Zaanen, Ya-Wen Sun, Yan Liu and Koenraad Schalm
Frontmatter
[More information](#)

viii	<i>Contents</i>	
14	Outlook: holography and quantum matter	504
14.1	The UHOs: the unidentified holographic objects	505
14.2	Is holographic matter extreme quantum matter?	511
14.3	The final message for condensed matter	540
	<i>References</i>	543
	<i>Index</i>	570

Preface

Not so long ago, two large and quite old fields in physics, string theory and condensed matter physics, were more or less at the opposite ends of the physics building. During the 40 or so years of its history, string theory has developed into a high art of “mathematical machine building”, propelled forwards by the internal powers of mathematics as inspired by physics. Yet, it has suffered greatly for the shortcoming that its theoretical answers are always beyond the reach of experimental machinery. Modern condensed matter physics is in the opposite corner. It has been propelled forwards by continuously improving experiments, which have delivered one serendipitous discovery after another during the last few decades. However, its interpretational framework rests by and large on equations developed 40 years or so ago. There has been an increasing sense that it is these that fall short in trying to explain the strongly interacting quantum many-body systems as realised by electrons in high- T_c superconductors and other unconventional materials.

All this changed dramatically in 2007 when physicists started to feed condensed matter questions to the most powerful mathematical machine of string theory: the holographic duality in the title of the book, also known as the “anti-de Sitter/conformal field theory” (AdS/CFT) correspondence. This book introduces the explosion of answers that has followed since then.

The first (Jan) and last (Koenraad) of this book’s authors are from such opposite corners. As soon as the seminal work of Herzog, Kovtun, Sachdev and Son in 2007 showed that these two subjects have dealings with each other, Jan and Koenraad recognised the potential and met up, almost literally half-way up the stairs. As has been characteristic for the development at large, it took us remarkably little effort to get on speaking terms, despite our superficially very different backgrounds. Shrouded by differences in language, string theory and condensed matter had already been on a collision course for a while, meeting each other on the common ground of quantum criticality/conformal field theory. In the years that

Cambridge University Press

978-1-107-08008-9 - Holographic Duality in Condensed Matter Physics

Jan Zaanen, Ya-Wen Sun, Yan Liu and Koenraad Schalm

Frontmatter

[More information](#)

followed this dialogue only intensified and the upbeat tone of this book is a testimony of the great time we had together. The largest part of that time was shared with the two middle authors (Ya-Wen and Yan), who came to Leiden as postdocs in 2010, freshly graduated from the Chinese Academy of Science in Beijing. The seeds for this book were planted when Jan received the invitation to become the 2012 Solvay Professor in Brussels, with the request to organise an AdS/condensed-matter-theory lecture course. What you see before you grew from the notes of this course, joined together with lecture notes by Koenraad at the 2012 Cargèse and 2013 Crete schools.

This is an incredibly fast-moving field, and many pages had to be added to describe the developments that happened since the summer of 2012. In January 2014 we stopped playing catch up, and we decided to get it out as quickly as possible, given the high demand for such a text at this moment. The first nine months of 2014 turned into a writing frenzy for all of us, and the result is lying in front of you. We are well aware that in certain regards the book therefore has its limitations and that the text will already be obsolete as soon as it appears. Examples of significant developments that occurred since our cut-off are a holographically inspired theory of incoherent metals¹ and a holographic solution of the anomalous temperature scaling of the Hall angle as observed in high- T_c superconductors.² Nor do we claim this to be a comprehensive review that does justice to all of the papers which have been published on the subject. What we have done is to provide an introduction to serve a non-expert readership that wishes to be informed about the main developments. Our aim has been to catch the mainstream, those developments where one discerns a consensus in the expert community that these are the most significant accomplishments. As authors we found it quite obvious how to make this selection and we sincerely believe that our choices will be approved by the AdS/CMT experts. We felt that we just had the role of humble narrators working on the chronicles of a monumental physics odyssey. We wish to take you on board and we hope you will enjoy it as much as we do!

We are in the first instance indebted to numerous holographists who contributed to our understanding of the correspondence. We are particularly grateful to Andrea Amoretti, Steffen Klug, Richard Davison, Andrey Bagrov, Petter Sæterskog and Balázs Meszéná for their thorough proofreading of the manuscripts and their many helpful suggestions, and to Mihael Petač for his help on the figures. Both the Leiden and Madrid physics departments gave us all the room to concentrate on the writing of this book. We acknowledge financial support of various funding organisations, in particular the Solvay Foundation and the Dutch Foundation of Fundamental

¹ S. Hartnoll, *Nature Phys.* **11**, 54 (2015), arXiv:1405.3651.

² M. Blake, A. Donos, *Phys. Rev. Lett.* **114**, 021601 (2015), arXiv:1406.1659.

Cambridge University Press
978-1-107-08008-9 - Holographic Duality in Condensed Matter Physics
Jan Zaanen, Ya-Wen Sun, Yan Liu and Koenraad Schalm
Frontmatter
[More information](#)

Preface

xi

Research on Matter (FOM) in the initial stages of this project, as well as the Spanish MINECO's "Centros de Excelencia Severo Ochoa" Programme under grant SEV-2012-0249, the Netherlands Organisation for Scientific Research/Ministry of Science and Education (NWO/OCW), and a grant from the Templeton Foundation: the opinions expressed in this publication are those of the authors and do not necessarily reflect the views of the John Templeton Foundation. Jan Zaanen and Koenraad Schalm acknowledge the hospitality of various institutions during the writing process: the Aspen Center of Physics, supported by the National Science Foundation under Grant No. PHYS-1066293, the Kavli Institute for Theoretical Physics, supported by the National Science Foundation under Grant No. NSF PHY11-25915, and the physics department of Harvard University in particular.

Jan Zaanen, Ya-Wen Sun, Yan Liu and Koenraad Schalm
Leiden and Madrid