1 *Aristotle on the Necessity of the Consequence*

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1. Introduction

In *Prior Analytics* A1, Aristotle explains what a deduction (a syllogism) is, and he makes clear that a syllogism involves 'necessity'[1]:

(1) A deduction is a discourse in which, certain things having been supposed, something different from the things supposed results of necessity because these things are so. (*Prior Analytics* A1, 24b18–22)[2]

Aristotle's explicit description of the conclusion of a valid syllogism as 'resulting of necessity' makes it look as though he must have in mind a modal notion. But can we say with any certainly what Aristotle understands by the 'necessity' here? Scholars have of course asked this many times, but there is no consensus and a variety of interpretations have been suggested.[3] The aim of the present paper is to focus on a source which sometimes is overlooked but which can be used to help shed light on Aristotle's understanding of the necessity in (1). This source is the examples Aristotle uses to establish where we do *not* have a syllogism. These examples are sometimes taken to be more trouble than help, and yet because they are Aristotle's own they are among the most direct textual evidence we have of his reasoning about his logic.[4]

[1] This definition appears also in *Topics* I.1, 100a25–27.
[2] Except where otherwise indicated, I have used Robin Smith's translation of the *Prior Analytics* (Smith 1989).
[3] Rini 2013 outlines various answers which scholars have suggested were available to Aristotle.
[4] van Rijen 1989 is cautious about Aristotle's examples. Ross 1957 and Łukasiewicz 1957 almost give the impression that they are embarrassed by the very idea that Aristotle might have produced counterexamples. See also Patzig 1968, who lays out a range of early views.

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While modern philosophers and logicians might have a fairly sure idea of what today we mean by logic, we have to remember that when Aristotle was setting out the material which we know as the *Prior Analytics*, he was inventing logic. And though there are passages in *An. Pr.* where he does reflect on his invention, in setting out the syllogistic he is more focused on *doing* logic, less on talking about it or reflecting on the nature of his discovery. In Chapter 3 of this volume, Robin Smith investigates passages where Aristotle is reflecting on his logic, but these are special passages. For the most part reading Aristotle's discussion of the syllogistic feels like reading an introductory logic exercise book. There is in fact so little explicit reflection on the methods that some commentators have supposed that Aristotle must have proceeded simply by trial and error. Some more explicit discussion of his overarching concept of logic would perhaps help to answer this charge, but Aristotle does not give one. Because he does not in fact tell us much about his definition of a syllogism, this makes it difficult to say precisely how he understands that feature of his system which today we call logical consequence. If we want to understand more than the most basic mechanics of the syllogistic, then we have to piece together the available evidence which we find in the *Prior Analytics*. And in doing so we have to be careful about how we use our modern tools, lest they begin to colour our picture. For example, we should not suppose that because Aristotle is after all doing *logic* then he must have a concept of logic which is relevantly like ours; rather, we need to consider whether there is textual evidence to show that his concept of logic is like ours. But this makes the 'trial and error' suggestion especially interesting. Can we find evidence to show that Aristotle had (or that he lacked) the kind of overarching appreciation of logic needed to carry his own syllogistic beyond mere trial and error? Can we find evidence that Aristotle reached any deeper understanding of the consequence relation? This chapter will show how textual evidence from *An. Pr.* provides answers to such questions and helps to show what precisely is at stake in the 'trial and error' criticism.

If we want to cash these questions out in modern terms, we might ask whether Aristotle's understanding of a syllogism extends beyond a simple substitutional view of logic. On a substitutional view of logic, an inferential schema is valid if and only if every possible substitution instance of it is truth-preserving. When we put terms into the valid schemas and generate premises involving actual truths, then the

conclusion itself is always going to be an actual truth. In order to know whether such a substitutional view is what Aristotle has in mind, we need to look not just at the (valid) syllogisms but also at the examples he offers when there is no syllogism – when a schema must be rejected. In such cases Aristotle tells us we cannot syllogize. As a rule, it is only in his instructions for constructing such examples that Aristotle offers terms – 'man', 'animal', 'moving' are among his favourites. By contrast, in cases where we can syllogize – that is, where there is a valid syllogism – Aristotle's discussion relies on the more abstract term-variables A, B and C, and he does not suggest terms. (It is worth noting that Aristotle does not have our modern labels 'valid' and 'invalid'. He expresses the distinction between what today we call a valid syllogistic schema and an invalid one by describing when 'there is a syllogism' and when 'there is no syllogism', or equivalently, when we can syllogize and when we cannot.)

Aristotle's syllogistic unfolds in stages and it will help to follow the stages of development as laid out in *An. Pr.* My initial focus is, therefore, the non-modal (or assertoric) examples. As this paper will show, in these Aristotle can always use terms which generate instances of schemas which involve only *actual* truth. Section 2 of this chapter sets out the textual evidence for this claim. But as later sections show, in the modal syllogistic, when Aristotle gives instructions for constructing examples to establish when a conclusion does not follow, the results often go beyond actual truth and falsity, and instead require non-actual possibilities. *This suggests that at least in the modal syllogistic Aristotle has a modal view of validity which goes beyond the notion that any substitution of terms preserves actual truth*. But, first, what should we say about the non-modal? How well can the substitutional view capture what Aristotle describes there?

2. Non-modal Reasoning in *An. Pr.* (the Assertoric Syllogistic)

In order to begin to answer, we need to look at Aristotle's method for demonstrating when a premise-pair will not yield a syllogism. The first such non-modal (assertoric) example occurs in *An. Pr.* A4:

> However, if the first extreme [A] follows all of the middle [B] and the middle [B] belongs to none of the last [C], there will not be a deduction of

the extremes [A, C], for nothing necessary results in virtue of these things being so. For it is possible for the first extreme to belong to all as well as to none of the last. Consequently, neither a particular nor a universal conclusion becomes necessary; and, since nothing is necessary because of these, there will not be a deduction. Terms for belonging to every are animal, man, horse; for belonging to none, animal, man, stone. (*An. Pr.* A.4, 26a2–9)[5]

Aristotle's point is that this is a case in which from the given premise-pair – 'A belongs to all B' and 'B belongs to no C' – we have no guarantee of *any* conclusion. (Some scholars say that such a premise-pair is 'inconcludent'.) That *no* proposition can be obtained is important, and reflects the nature of syllogistic reasoning. In Aristotle's system there are just four basic forms which propositions can take. Medieval scholars used the vowels *i*, *o*, *a*, and *e* to label the different forms:

'A belongs to some C' is a particular affirmative – i.e., an *i*-proposition.
'A does not belong to some C' is a particular privative – i.e., an *o*-proposition.
'A belongs to every C' is a universal affirmative – i.e., an *a*-proposition.
'A belongs to no C' is a universal privative – i.e., an *e*-proposition.

In the syllogistic, Aristotle routinely relies on the lesson from his square of opposition, that *i* and *e* are contradictories, and that *o* and *a* are contradictories. The premises described in the preceding passage are an *a*-proposition 'A belongs to all B' and an *e*-proposition 'B belongs to no C.' By showing that there is no conclusion from these premises, Aristotle argues that there is no syllogism. He is not looking for just any conclusion but a conclusion of a specified form – in this passage the purported conclusion must itself be a proposition which links an A *predicate* to a C *subject*.

So, we need to consider whether either an affirmative conclusion 'A belongs to some C' or a privative conclusion 'A does not belong to some C' follows from the premises. This means we are looking at the following two schemas, where (*i*) and (*o*) indicate the type of the purported conclusion:

A belongs to every B		A belongs to every B	
B belongs to no C		B belongs to no C	
A belongs to some C	(*i*)	A does not belong to some C	(*o*)

[5] I have inserted the A, B and C to make clear the structure of Aristotle's text.

Although Aristotle does not explicitly state this, as proof that these premises do not yield a syllogism it clearly suffices for him to show that neither an 'affirmative particular'(*i*) nor a 'negative particular' (*o*) can be obtained. Indeed he seems to have in mind to show both that you cannot get 'A belongs to some C' as the conclusion, and that you cannot get 'A does not belong to some C.' He rules out the *i*-proposition 'A belongs to some C' as a conclusion by offering a set of terms which makes its contradictory true. These are the 'terms for belonging to none': animal, man, and stone. They give the true *e*-proposition 'Animal belongs to no stone.' Aristotle rules out a conclusion in the form of the *o*-proposition 'A does not belong to some C' by offering a set of terms which makes its contradictory 'A belongs to every C' true. These are the 'terms for belonging to every': animal, man, and horse. These give the true *a*-proposition 'Animal belongs to every horse.' When we put in the terms which Aristotle recommends, we get the following:

'terms for belonging to every'		'terms for belonging to none'	
Animal belongs to every man		Animal belongs to every man	
Man belongs to no horse		Man belongs to no stone	
Animal belongs to every horse	(*a*)	Animal belongs to no stone	(*e*)

From the given premises, no conclusion of the required form 'becomes necessary' because the different sets of terms give different (true) results. One set of terms gives a true *a*-proposition, which shows that you cannot obtain an *o*-conclusion (and so not an *e*-conclusion). The other set gives a true *e*-proposition, so you cannot obtain an *i*-conclusion (and so not an *a*-conclusion). So you cannot obtain *any* conclusion of the required form. That is to say, there is no conclusion relating an A predicate to a C subject. So Aristotle says 'there is no syllogism'. The rejected conclusions – both the *i* and the *o* – are particulars, and showing that no particular AC proposition logically follows *also* establishes that no universal AC proposition follows, but Aristotle does not comment on this.

Aristotle uses this method right through the assertoric syllogistic of *An. Pr.* A4–6, in order to establish which non-modal, *assertoric* premise-pairs do not yield a syllogism. Chapter A4 deals with schemas in what Aristotle calls the 'first figure'. (The preceding schema is in the first figure.) Chapter A5 deals with schemas in the 'second figure',

and chapter A6 deals with the 'third figure'. Taking A, B and C as our terms, we can represent the figures schematically:

First Figure	Second Figure	Third Figure
Pred-Subj	Pred-Subj	Pred-Subj
A-B	A-B	A-C
B-C	A-C	B-C
A-C	B-C	A-B

Aristotle calls the term which occurs twice in the premises the 'middle term'. The other terms he calls the 'extremes'. So in the first figure, the B term is the middle and drops out of the conclusion. In the second figure, the A term is the middle, and in the third figure, C is the middle.

Triples of terms together with the definitions of the three figures give us Aristotle's 'recipe' for constructing examples to show which schemas do not syllogize. Some awkwardness arises right away. As a rule Aristotle does not himself actually put the terms into these examples for us; instead, he sketches what he reasons we need to have in order to be able to do so ourselves. He offers proofs of validity of the valid syllogisms, but leaves the final business of showing when we cannot syllogize as homework for us, his readers. So, the evidence of these examples is not straight, direct evidence. We have to do our homework, and that involves some pencil work and therefore some small amount of interpretation on our own part. So if these examples do provide evidence of how Aristotle was thinking about the nature of his logic, then in order to access that evidence that we have to unpackage it. The tables which follow give a complete list of the term-triples which Aristotle offers for showing which assertoric, non-modal schemas do not let us syllogize and should be interpreted in the way I have described. These schemas are listed by the *An. Pr.* chapter in which they appear – that is, A4, A5, or A6, in bold – and by the line numbers of the relevant explanations. The term-triples are listed in the top line of each entry. The term-triples are numbered using square brackets, for greater ease of reference. And where Aristotle indicates that we should construct an example using the term-triples, the example is included. The terms give two true premises, but in each case no conclusion follows from the premises, and Aristotle's point is that in none of these can we syllogize.

Rejected assertoric schemas

A4: (First Figure)			
26a2–9	animal-man-horse;	animal-man-stone	[1]
a	Animal belongs to all men	Animal belongs to all men	
e	Man belongs to no horse	Man belongs to no stone	
	Animal belongs to all horses	Animal belongs to no stone	
26a11–12	science-line-medicine;	science-line-unit	[2]
e	Science belongs to no line	Science belongs to no line	
e	Line belongs to no medicine	Line belongs to no unit	
	Science belongs to all medicine	Science belongs to no unit	
26a34–36	good-condition-wisdom;[a]	good condition-ignorance	[3]
i	Good belongs to some condition	Good belongs to some condition	
a	Condition belongs to all wisdom	Condition belongs to all ignorance	
	Good belongs to all wisdom	Good belongs to no ignorance	
26a38	white-horse-swan;	white-horse-raven	[4]
i	White belongs to some horse	White belongs to some horse	
e	Horse belongs to no swan	Horse belongs to no raven	
	White belongs to all swans	White belongs to no raven	
26b6–10	animal-man-white; [swan and snow in place of white]		[5]
a	Animal belongs to every man	Animal belongs to every man	
o	Man does not belong to some swan	Man does not belong to some snow	
	Animal belongs to every swan	Animal belongs to no snow	
26b10–14	inanimate-man-white [swan and snow in place of white]		[6]
e	Inanimate belongs to no man	Inanimate belongs to no man	
o	Man does not belong to some snow	Man does not belong to some swan	
	Inanimate belongs to some snow	Inanimate does not belong to some swan	
26b24–25	animal-white-horse	animal-white-stone	[7]
i	Animal belongs to some white	Animal belongs to some white	
i	White belongs to some horse	White belongs to some stone	
	Animal belongs to every horse	Animal belongs to no stone	
o	Animal does not belong to some white	Animal does not belong to some white	
i	White does not belong to some horse	White does not belong to some stone	
	Animal belongs to every horse	Animal belongs to no stone	
i	Animal belongs to some white	Animal belongs to some white	
o	White does not belong to some horse	White does not belong to some stone	
	Animal belongs to every horse	Animal belongs to no stone	
o	Animal does not belong to some white	Animal does not belong to some white	
o	White does not belong to some horse	White does not belong to some stone	
	Animal belongs to every horse	Animal belongs to no stone	

[a] Aristotle's terms in [3] and [4] will show that *oa* and *oe* are also inconcludent.

In [5] and [6] Aristotle first offers white as one of the recommended terms but then in elaborating on the proof, he suggests that we "let swan and snow also be selected from among those white things of which man is not predicated." Where white names an accident, swan

and snow name substances and so generate examples in which the truth value of the purported conclusions is easy to appreciate. As the terms in [5] and [6] make clear, we never need move beyond simple actual-world truths.

The terms offered in [7] are given to show that a family of schemas can all be quickly rejected. These all involve combinations of various particular premises. Aristotle uses the same terms to establish that in each such case there is no deduction. Here is the passage:

Nor will there be a deduction in any way if both the intervals are particular, whether positively or privatively, or if one is stated positively and the other privatively, or if one is indeterminate and the other determinate, or both are indeterminate. Common terms for all are animal, white, horse; animal, white, stone. (*An. Pr.* A4, 26b22–25)

A5: (Second Figure)

27a18–20	substance-animal-man;	substance-animal-number[a]	[8]
a	Substance belongs to every animal	Substance belongs to every animal	
a	Substance belongs to every man	Substance belongs to every number	
	Animal belongs to every man	Animal belongs to no number	
27a20–23	line-animal-man;	line-animal-stone	[9]
e	Line belongs to no animal	Line belongs to no animal	
e	Line belongs to no man	Line belongs to no stone	
	Animal belongs to every man	Animal belongs to no stone	
27b4–6	animal-substance-raven;	animal-white-raven (27b5-6)	[10]
o	Animal does not belong to some substance	Animal does not belong to some white	
a	Animal belongs to every raven	Animal belongs to every raven	
	Substance belongs to every raven	White belongs to no raven	
27b6–8	animal-substance-unit;	animal-substance-science	[11]
i	Animal belongs to some substance	Animal belongs to some substance	
e	Animal belongs to no unit	Animal belongs to no science	
	Substance belongs to all unit	Substance belongs to no science	
27b16–23	[no terms for belonging];[b]	black-snow-animal	[12]
		e Black belongs to no snow	
		o Black does not belong to some animal	
		Snow belongs to no animal	
27b26–28	[no terms for belonging];[c]	white-swan-stone	[13]
		a White belongs to every swan	
		i White belongs to some stone	
		Swan belongs to no stone	

27b28–32	white-animal-raven;	white-stone-raven	[14]
i	White belongs to some animal	White belongs to some stone	
e	White belongs to no raven	White belongs to no raven	
	Animal belongs to every raven	Stone belongs to no raven	
27b33	white-animal-swan;	white-animal-snow	[15]
i	White belongs to some animal	White belongs to some animal	
a	White belongs to all swans	White belongs to all snow	
	Animal belongs to all swans	Animal belongs to no snow	
27b36–38	white-animal-man;	white-animal-inanimate	[16]

[a] Aristotle calls substance the middle.

[b] As Ross explains, no terms are given here since Aristotle has already shown that second figure *ee* is inconcludent, so you would only need a proof that *eo* is inconcludent if the *o* was not an *e*. But if the *o* is not an *e* then there must also be an *i* (since if the *e* is false – that it is not so that none – then it is so that some). But if there is an *i* as well as an *o* we get Festino, and then Aristotle puts it this way: "But it is not possible to find terms of which the extremes are related positively and universally, if M belongs to some O, and does not belong to some O." [This is Jenkinson's translation, in Aristotle 1985.]

[c] Here we require the second *i* not to be an *a*, so we have to have an *o* as well as an *i*. But second figure *ao* is *not* inconcludent since we have Baroco.

The terms in [16] occur in a brief passage in which – as in [7] earlier – we find a sweeping description of several separate premise combinations, none of which produces a valid syllogism. In [16] Aristotle explains that all such second-figure combinations can be shown to be invalid by the same sets of terms – and the terms here in [16] are strikingly similar to the terms in [7]. The full passage in which he offers the term-triples for the second figure is as follows:

> But neither does a deduction come about if the middle term belongs or does not belong to some of each extreme, or belongs to one and does not belong to the other, or not to all of either, or indeterminately. Common terms for all these are white, animal, man; white, animal, inanimate. (*An. Pr.* A5, 27b36–39)

With white as the middle term, each of these various premise combinations is straightforward and does not involve any modal suppositions.

i	White belongs to some animal	White belongs to some animal
i	White belongs to some man	White belongs to something inanimate
	Animal belongs to every man	Animal belongs to nothing inanimate
i	White belongs to some animal	White belongs to some animal
o	White does not belong to some man	White does not belong to something inanimate
	Animal belongs to every man	Animal belongs to nothing inanimate
o	White does not belong to some animal	White does not belong to some animal
i	White belongs to some man	White belongs to something inanimate
	Animal belongs to every man	Animal belongs to nothing inanimate

o	White does not belong to some animal	White does not belong to some animal
o	White does not belong to some man	White does not belong to something inanimate
	Animal belongs to every man	Animal belongs to nothing inanimate

A6: (Third Figure)

28a30–33	animal-horse-man;	animal-inanimate-man	[17]
a	Animal belongs to every man	Animal belongs to every man	
e	Horse belongs to no man	Inanimate belongs to no man	
	Animal belongs to every horse	Animal belongs to nothing inanimate	
28a33	animal-horse-inanimate;	man-horse-inanimate	[18]
e	Animal belongs to nothing inanimate	Man belongs to nothing inanimate	
e	Horse belongs to nothing inanimate	Horse belongs to nothing inanimate	
	Animal belongs to every horse	Man belongs to no horse	
28b22–24	animate-man-animal;	["We cannot get terms"]	[19]
		(because of Datisi)	
a	Animate belongs to every animal		
o	Man does not belong to some animal		
	Animate belongs to every man		
28b36-38	animal-man-wild;	animal-science-wild	[20]
i	Animal belongs to something wild	Animal belongs to something wild	
e	Man belongs to nothing wild	Science belongs to nothing wild	
	Animal belongs to every man	Animal belongs to no science	
29a2	animal-science-wild;	animal-man-wild	[21]
o	Animal does not belong to some wild	Animal does not belong to some wild	
e	Science belongs to nothing wild	Man belongs to nothing wild	
	Animal belongs to no science	Animal belongs to all men	
29a3	["We cannot get terms"]	raven-snow-white;	[22]
	(because of Ferison)	*e* White belongs to no raven	
		o Snow does not belong to some white	
		Raven belongs to no snow	
29a9–10	animal-man-white;	animal-inanimate-white	[23]

In [23], we again have a passage in which Aristotle gives a sweeping description of a number of separate premise combinations, none of which produces a valid syllogism and all of which Aristotle says can be shown to be invalid by the same terms. The full passage in which these term-triples appear is as follows:

Nor will there be a deduction in any way if each term belongs or does not belong to some of the middle, or if one belongs and the other does not belong, or if one belongs to some and the other not to every, or if they belong