

Convex Optimization of Power Systems

Optimization is ubiquitous in power system engineering. Drawing on powerful, modern tools from convex optimization, this rigorous exposition introduces essential techniques for formulating linear, second-order cone, and semidefinite programming approximations to the canonical optimal power flow problem, which lies at the heart of many different power system optimizations.

Convex models in each optimization class are then developed in parallel for a variety of practical applications such as unit commitment, generation and transmission planning, and nodal pricing. Presenting classical approximations and modern convex relaxations side-by-side, and a selection of problems and worked examples, this book is an invaluable resource for students and researchers from industry and academia in power systems, optimization, and control.

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Preface

The application of optimization to power systems has become so common that it deserves treatment as a distinct subject. The abundance of optimization problems in power systems can give the impression of diversity, but in truth most are merely layers on a common core: the steady-state description of power flow in a network. In this book, many of the most prominent examples of optimization in power systems are unified under this perspective.

As suggested by the title, this book focuses exclusively on convex frameworks, which by reputation are phenomenally powerful but often too restrictive for realistic, non-convex power system models. In Chapter 3, the application of classical and recent mathematical techniques yields a rich spectrum of convex power flow approximations ranging from high tractability and low accuracy to slightly reduced tractability and high accuracy. The remaining chapters explore problems in power system operation, planning, and economics, each consisting of details layered on top of the convex power flow approximations. Because all formulations can be solved using standard software packages, only models are presented, which is a departure from most books on power systems. It is a major perk of convex optimization that the user often does not need to program an algorithm to proceed.

I should comment that this book is not an up-to-date exposition of power system applications or optimization theory and that, inevitably, many important topics in both fields have been omitted. My intention has rather been to bridge modern convex optimization and power systems in a rigorous manner. While I have attempted to be mathematically self-contained, the pace assumes an advanced undergraduate level of mathematical exposure (linear algebra, calculus, and some probability) as well as familiarity with power systems and optimization. This book could be used in a course on power system optimization or as a mathematical supplement to a course in power system design, operation, or economics. It is my hope that it will also prove useful to researchers in power systems with an interest in optimization and vice versa, and to industry practitioners seeking firm foundations for their optimization applications.

Acknowledgments

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Notation

AC	Alternating current
DC	Direct current
LP	Linear programming
QP	Quadratic programming
SOC(P)	Second-order cone (programming)
SD(P)	Semidefinite (programming)
(C)QCP	(Convex) quadratically constrained programming
MI	Mixed integer
NLP	Nonlinear programming
KKT	Karush-Kuhn-Tucker (conditions)
PNE	Pure strategy Nash equilibrium
MNE	Mixed strategy Nash equilibrium
i	$\sqrt{-1}$
\mathbb{R}	The set of real numbers
\mathbb{C}	The set of complex numbers
\mathbb{Z}	The set of integers
x_i	The i^{th} entry of the vector x
x^k	The k^{th} version of the quantity x , typically corresponding to the k^{th} scenario or time period
$\text{Re } x$	The real part of x
$\text{Im } x$	The imaginary part of x
$ x $	The absolute value of x
$\ x\ $	The two-norm of x , $\sqrt{\sum_i x_i^2}$
X_{ij}	The entry at the i^{th} row and j^{th} column of the matrix X
X^T	The transpose of X
X^*	The Hermitian transpose of X . When X is scalar, the complex conjugate.
$X \succeq 0$	The matrix X is positive semidefinite.
$\text{rank } X$	The rank of X
$\text{tr } X$	The trace of X , $\sum_i X_{ii}$
$\det X$	The determinant of X
∇	The gradient operator

To condense exposition, this book employs somewhat relaxed indexing notation. Because there is little risk of ambiguity, i will often be used simultaneously as the imaginary unit and as an index, for example iq_i would be $\sqrt{-1}$ times the i^{th} entry of q . In most cases, constraint indexing will not be explicitly declared; for example,

$$g_i(x) \leq 0$$

is implicitly enforced over $i = 1, \dots, n$, which is almost always the set of nodes in the network. Similarly, the sum

$$\sum_{ij} x_{ij}$$

is over all relevant node pairs ij , which are usually those connected by lines. Indexing is denoted explicitly when it is not over a standard set, such as when summing over a subset of nodes.

This book makes extensive use of *feasible sets* as organizational tools. Given a collection of constraints $g_i(x) \leq 0$, the corresponding feasible set is

$$\{x \mid g_i(x) \leq 0\},$$

i.e., the set of points for which every constraint is satisfied.