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Fluid statics and dynamics

1.1 Introduction

Our experience suggests that most students from geosciences, natural sciences, and other backgrounds are not required to take a fluid mechanics class, and most engineering students who took a fluid mechanics class earlier may still need to reinforce their physical understanding of the fluid mechanics concepts. Thereby, this chapter is intended to provide readers with some basics of fluid mechanics (particularly, in terms of physics) that are essential to understand and appreciate flow through geologic media.

Fluid Mechanics is the study of the forces on fluids. These fluids can be either a gas or a liquid. Fluid Mechanics includes both **fluid statics** (the study of fluids at rest) and **fluid dynamics** (the study of fluids in motion). Notice that the fluid mechanics serves as the fundamental principles in a number of disciplines in science and engineering. For instance, atmospheric science is built upon fluid mechanics, as is cardiology – the study of the blood flow through our veins and arteries. The study of the infiltration of water and its subsequent movement in unsaturated subsurface media (vadose zone hydrology) also relies on fluid mechanics, as do studies of the movement of groundwater in geologic media (groundwater hydrology). Other disciplines include surface hydrology (the study of movement of water on land surfaces, in canals and rivers, and in oceans, lakes, estuaries, etc.), and petroleum engineering that studies the movement of oil and gas in geologic reservoirs. Examples also include hydraulics which studies fluid motions in pipes or man-made conduits (conduit flow systems).

This chapter first introduces readers to the basic continuum assumption (i.e., volume averaging concept). Then, definitions of fluid properties and their units follow. Forces of static fluids and the relationship between forces and energies are discussed subsequently. The chapter then examines fluid dynamics, in which the fixed and moving coordinate systems are introduced, the time derivatives associated with these two coordinate systems are brought forth, and fluid acceleration is investigated. Using force balance and acceleration concepts, we derive the Bernoulli equation for flow through pipe; we elucidate its physical meaning in term of conservation of energy, which leads to the concept of total, pressure, velocity, and potential heads. Head loss due to viscous forces ignored by the Bernoulli equation then explains the limitation of the equation for flow through real-world systems. Subsequently, the influence of the relative strength of the viscous force to the inertia force of the fluid (i.e., Reynolds Number) is discussed on flow behavior (i.e., laminar or turbulent flow). At the end, this chapter sets the stage for the discussion of flow through geologic media in next few chapters, which is generally considered as laminar flow.

1.1.1 Fluids and Solids

Discussion of the states of a matter is a way to explain the definition of fluids and solids. The states of a matter are also known as phases of the matter or states of an aggregation, which are: solid, liquid, and gas. Under a solid state, the molecules of matters are limited to only vibration about a fixed position. This restriction gives the material both a definite volume and a definite shape. Any material under this condition is called solid phase. As some energy is added to a solid, its molecules begin to vibrate more rapidly until they break out of their fixed positions. The solid then becomes a liquid. The molecules of a liquid are free to move throughout the liquid, but are held from escaping the liquid surface by the intermolecular forces. This gives a liquid a definite volume, but no definite shape. As more energy is added to a liquid, a small number of molecules gain enough energy to break away completely, and escape into the adjacent space. Finally, a threshold temperature is reached at which molecules throughout the liquid are becoming energetic enough to escape. Then, vapor bubbles form and rise to the surface, and the liquid becomes a gas. The molecules of a gas are free to move in every possible way; a gas has neither a definite shape nor a definite volume but expands to fill any container in which it is placed. Both liquids and gases are considered to be fluids.

Strictly speaking, there is no quantitative criterion to clearly distinguish solids from fluids. Nevertheless, a fluid is defined as a substance whose particles can move and change their relative position easily in contrast to a solid. Specifically, a fluid is a substance that deforms continuously without substantial resistance, when a shear stress (explained later) is applied to it, no matter how small the magnitude of the applied shear stress is. A solid can be defined as a material of which the shape or the relative position of the constituent elements changes only by an insignificantly small amount, when forces acting on it change, either in magnitude or direction. Thus, the distinction between solids and fluids is rather relative.

While fluids can mean liquids or/and gas, in the context of understanding the basic processes of subsurface hydrology, we are typically interested in the liquid phase of water when we deal with problems related to water resources. Notice that in certain cases we may have to deal with liquid and/or gas phases of nonaqueous fluids in the case of groundwater pollution problems.

1.1.2 Dimensions and Units

Variables used in fluid mechanics are expressed in terms of basic dimensions (e.g., mass, length, and time), which in turn are quantified by three basic units: English (US), Metric (rest of the world) and SI–International system units (scientific community). Before introducing fluid properties, we will review the dimensions and units that are of interest in the study of subsurface hydrology.

A fundamental variable in fluid mechanics is force; its SI unit is Newton (N), after Sir Isaac Newton (1643–1727). From fundamental physics, we are all familiar with Newton’s second law: the relation between the mass m of an object, its acceleration $\mathbf{a}$, and the applied force $\mathbf{F}$ is $\mathbf{F} = m\mathbf{a}$. Acceleration (the rate of change in velocity), $\mathbf{a}$**,** and force, $\mathbf{F}$**,** are vectors (as indicated by their bold font symbols); the direction of the force vector is the same as the direction of the acceleration vector. Scalars are quantities that

Table 1.1. *Units of measurement of fundamental dimensions in three systems*

Fundamental Dimension	International System (SI) Units	Metric or Centimeter-gram-second (cgs) Unit	English (Imperial) Unit
Mass	kilogram (kg)	gram (g)	slug
Force	newton (N)	dyne (dyn)	pound (lb)
Length	meter (m)	centimeter (cm)	foot (ft)
Time	second (s)	second (s)	second (s)
Temperature	kelvin (K)	degree Celsius (°C)	degree Fahrenheit (°F)

Table 1.2. *Systéme International (SI) units and prefixes*

Unit	Abbreviation	Prefix	Symbol	Multiplication factor
Joule (N·m)	J	Tera	T	10^{12}
Kelvin	K	Giga	G	10^{9}
Kilogram	kg	Mega	M	10^{6}
Meter	m	Kilo	k	10^{3}
Newton	N	Milli	m	10^{-3}
Pascal (N m^{-2})	Pa	Micro	μ	10^{-6}
Radian	rad	Nano	n	10^{-9}
Second	s	Pico	p	10^{-12}
Watt (J s^{-1})	W	Femto	f	10^{-15}

require only magnitude, whereas vectors are quantities that require not only a magnitude, but also a direction to specify them completely.

Recall that mass is a measure of how much material an object contains, but weight is a measure of the gravitational force exerted on that material in a gravitational field. Thus, mass and weight are proportional to each other, with the acceleration due to gravity being the proportionality constant. The dimension of **F** follows directly from Newton's second law, as shown in the following:

$$\text{Force}: \ \mathbf{F} = m\boldsymbol{a} \tag{1.1.1}$$

Dimensions of force are denoted by $[MLT^{-2}]$ where M, L, and T denote dimensions of mass, length, and time, respectively. The Newton (N) or $kg{\cdot}ms^{-2}$ is often used as its units. For instance, a force of 1 Newton indicates the force applied on a substance of 1 Kg mass to accelerate it by 1 m/s^2.

Three systems of units listed in Table 1.1 have been used widely in various branches of science and engineering. The SI units (Table 1.2) are the most popular. Some useful prefixes in the metric system are also given in Table 1.2.

1.1.3 Fluid as a Continuum

Like any scientific study of the real world systems, fluid mechanics makes some basic assumptions about the materials being studied. For example, fluids are composed of molecules that collide with one another. The properties of a fluid in fluid mechanics refer to properties of a point in the fluid. However, if we picked a point at random in a fluid, that point might be on the inside of an atomic particle or alternatively might be in the space between atomic particles or molecules due to the discrete nature of the fluid matter. The fluid "properties" associated with a point as being defined in fluid mechanics therefore would depend upon the location of the measurement point. This can create a discontinuous distribution of properties in space for a substance. This discontinuity violates the assumption of smoothness and continuity embedded in calculus and differential equations. To avoid this problem, we have to adopt the ***continuum assumption*** in the analysis of fluid mechanics. In other words, the fluid is idealized macroscopically as being continuous throughout its entirety. The molecules are pictured as being "smeared" or "averaged" to eliminate spaces between atomic particles and molecules. Although mathematically we define a physical or chemical attribute of the fluid at a point, we should remember that the "point" actually represents a small volume in the real space over which the average fluid properties are defined. The spatial volume is large enough to contain many molecules (e.g., there are 6.022×10^{23} molecules in a mole of gas, also called Avogadro's Number).

Adopting the continuum assumption, fluid mechanics considers fluids to be continuous. That is, fluid properties such as density and viscosity are taken to be well-defined at infinitely small points, and are continuous from one point to another. Thus, the fact that the fluid is made up of discrete molecules is ignored. The same continuum concept also applies to the definition of forces, temperature, pressure, energy, and other variables of interest in fluid mechanics. For example, in considering the action of a set of forces on a fluid, one can either account for the behavior of each and every molecule of fluid in a given flow field (can you imagine considering behavior of 6.022×10^{23} molecules together in a single mole of a substance) or simplify the problem by considering the averaged effect on the molecules in a given volume. The continuum assumption permits fluid mechanics to use the latter approach. The continuum hypothesis is an approximation, however. Therefore, it may lead to results which are not of desired accuracy or behavior. That being said, under the right circumstances or observation scales, the continuum hypothesis produces extremely accurate results.

1.1.4 Fluid Properties

With the introduction of the continuum assumption, definitions of fluid properties (i.e., specific weight, mass density, specific gravity, viscosity) are given as follows.

Specific weight, γ. Specific weight is the gravitational force (Mass of the fluid, M, times gravitational acceleration, g) per unit volume (V) of fluid, or simply the weight (W) per unit volume:

Table 1.3. *Densities of some common fluids*

Fluid	Density (g/cm^3) at 20°C and 1 atm, unless noted otherwise
water	0.998
water (0°C)	1.000
water (4°C)	1.000
water (100°C)	0.958
gasoline	0.726
ethyl alcohol	0.791
air	1.204
sea water	1.030
tetrachloroethene	1.622
glycerin	1.260
mercury	13.546

$$\frac{Mg}{V} = \frac{W}{V} \tag{1.1.2}$$

Specific weight is generally represented by the symbol γ (gamma). Water at 20°C has a specific weight of 9.79×10^3 N/m^3. In contrast, the specific weight of air at the same temperature and at atmospheric pressure is 11.9 N/m^3.

Mass density, ρ. The mass per unit volume is *mass density* or simply *density*; it has the units of kilograms per cubic meter and is given the symbol ρ (rho). Because specific weight is weight per unit volume, mass density will be given by the specific weight at the earth’s surface divided by g. The mass density of water at 10°C is 1000 kg/m^3. For air at 20°C, the mass density is 1.20 kg/m^3 (see Table 1.3 for densities of some common fluids).

Specific gravity, SG. The ratio of the specific weight of a given fluid to the specific weight of water at a standard reference temperature is defined as *specific gravity*, SG. The standard reference temperature for water is often taken as 4°C, where the specific weight of water at atmospheric pressure is 9810 N/m^3. With this reference, the specific gravity (SG) of mercury with a specific weight of 133 kN/m^3 at 20°C is

$$SG_{Hg} = \frac{133\ \text{kN/m}^3}{9.81\ \text{kN/m}^3} = 13.6 \tag{1.1.3}$$

Because SG is a ratio of specific weights, it is dimensionless and independent of the system of units used.

Dynamic Viscosity (μ) and Kinematic Viscosity (υ). Viscosity is a fluid property, which describes the fluid’s resistance to shearing or tangential motion under an applied shear stress (see Section 1.2.1). Of course, different fluids deform at different rates. For example, oil and honey deform slower than water, and therefore we intuitively recognize that both are more viscous than water. But which one is more

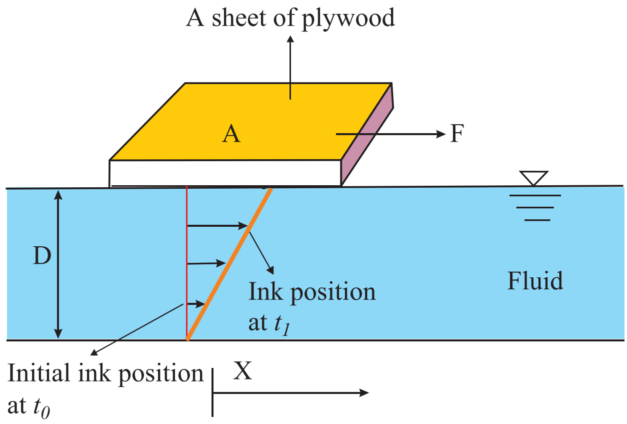

Figure 1.1. An illustrative experimental set-up demonstrating the relation between shear stress and velocity gradient.

viscous (oil or honey)? To differentiate viscosities of different fluids, quantitative descriptors for the viscosity of different fluids are needed.

Consider a sheet of plywood floating on a static water body in a tank (Figure 1.1). First, we use an ink pen to draw a vertical line from the bottom of the plywood to the bottom of the water body, and we then apply a force tangentially to the plywood. We expect to observe the phenomenon that the ink position will be displaced as shown in the figure. Based on the displacement of the ink, we can define the velocity of the movement of water, v, at any vertical location (z), that is,

$$v(z) = \frac{\Delta x(z)}{\Delta t} \tag{1.1.4}$$

where Δx(z) is the horizontal displacement at elevation z over a time interval Δt.

From the experiment, we will find that the maximum velocity exists at the top surface of the water body and the minimum velocity at the bottom. This difference is attributed to high friction between the fluid and the tank walls and the low friction between the fluid and the air. Now, we define the velocity gradient at a point in the fluid as

$$\alpha(z) = \frac{v(z)}{z} \tag{1.1.5}$$

where z is the vertical distance from the bottom of the tank to the elevation where the velocity is defined. If the displacement of the ink increases linearly with z, the velocity gradient then will be constant.

Suppose we repeat this experiment several times with different forces, F, and plot the shear stress (F/A) versus $\alpha(z)$ (or the velocity gradient), we will observe a relationship showing that the value of α increases with the stress (F/A) as illustrated in Figure 1.2. That is,

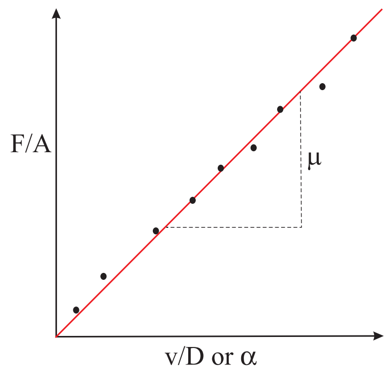

Figure 1.2. An illustration of the relationship between shear stress and the vertical velocity gradient of a Newtonian fluid. D is the depth of the fluid (see Figure 1.1).

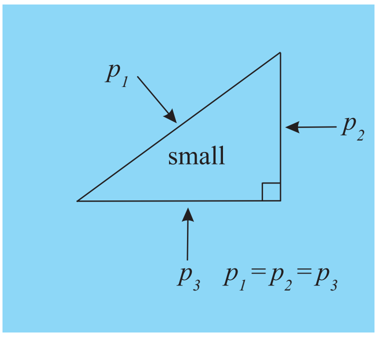

Figure 1.3. Pressure acts equally in all directions in fluids.

$$\frac{F}{A} \propto \alpha(z) \tag{1.1.6}$$

This relationship has been used to classify various fluids into categories such as Newtonian or non-Newtonian fluids. Specifically, a **Newtonian fluid** (e.g., water, air, and so on.) is a fluid whose shear stress is linearly proportional to the velocity gradient in the direction perpendicular to the plane of shear (Figure 1.3). The constant of proportionality is known as viscosity. On the other hand, a **non-Newtonian fluid** is a fluid in which the relationship between the shear stress and the velocity gradient is nonlinear (such as plastics, blood, body wash and hand soap). That is, the viscosity changes with the applied shear force.

In subsurface hydrology, we generally deal with water, which is considered to be a Newtonian fluid. Therefore, F/A is a linear function of $v(z)/z$ for water, and we can explicitly write the relationship as

$$\frac{F}{A} = \mu \frac{v(z)}{z} \tag{1.1.7}$$

Table 1.4. *Dynamic and kinematic viscosity of water in imperial units*

Temperature *t* (°F)	**Dynamic Viscosity *μ* (lb s/ft^2) × 10^{-5}**	**Kinematic Viscosity *ν* (ft^2/s) × 10^{-5}**
32	3.732	1.924
40	3.228	1.664
50	2.730	1.407
60	2.344	1.210
70	2.034	1.052
80	1.791	0.926
90	1.580	0.823
100	1.423	0.738
120	1.164	0.607
140	0.974	0.511
160	0.832	0.439
180	0.721	0.383
200	0.634	0.339
212	0.589	0.317

where μ (mu) is the constant of proportionality (i.e., the slope of the linear relationship between *F/A* and α) and is called the **dynamic viscosity** of the fluid. If we repeat the experiment using a different fluid, the slope of the line may change, reflecting its different resistance to the applied shear stress. We can replace the slope *v(z)/z* by a differential *dv/dz*, where z is the elevation. Also replacing *F/A* by the symbol τ (tau) for shear stress, we obtain

$$\tau = \mu \frac{dv}{dz} \tag{1.1.8}$$

Equation (1.1.8) is called **Newton's Law of viscosity**.

Now let us examine the dimension of each variable of the preceding equation.

Note M, T, and L stand for the dimension of mass, time, and length, respectively.

τ is the shear stress $= \frac{F}{A} = \frac{m\,a}{L^2} = \frac{[\mathrm{M}]\left[v\mathrm{T}^{-1}\right]}{\left[\mathrm{L}^2\right]} = [\mathrm{M}]\left[\frac{\mathrm{L}}{\mathrm{T}^2}\right]\cdot\left[\frac{1}{\mathrm{L}^2}\right] = \left[\frac{\mathrm{M}}{\mathrm{LT}^2}\right]$

μ = dynamic viscosity $\left[\frac{\mathrm{M}}{\mathrm{LT}}\right]$

$\frac{dv}{dz}$ = velocity gradient $\left[\frac{\mathrm{LT}^{-1}}{\mathrm{L}}\right] = \left[\frac{1}{\mathrm{T}}\right]$

The units for the corresponding variables are

Force: $F = ma$ 1 Newton (N) = 1 $\frac{\mathrm{kg\ m}}{\mathrm{s}^2}$

Pressure or stress: $p = F/A$ 1 Pascal (Pa) = 1 $\frac{\mathrm{N}}{\mathrm{m}^2} = 1\ \frac{\mathrm{kg}}{\mathrm{ms}^2}$

Viscosity: $\mu = \frac{\tau}{dv/dz}$ 1 Centipoise (cP) = $10^{-2} P = 10^{-3}\,\frac{\mathrm{N}}{\mathrm{m}^2}\,s = 10^{-3}$Pa-s

Note that viscosity varies as a function of temperature, and it generally decreases as temperature increases, or vice versa. Instead of dynamic viscosity, the kinematic viscosity is also used occasionally (Table 1.4).

Table 1.5. *Dynamic and kinematic viscosity of water in SI units*

Temperature T (°C)	**Dynamic Viscosity μ (Pa s, N s/m^2) $\times$ 10^{-3}**	**Kinematic Viscosity ν (m^2/s) $\times$ 10^{-6}**
0	1.787	1.787
5	1.519	1.519
10	1.307	1.307
20	1.002	1.004
30	0.798	0.801
40	0.653	0.658
50	0.547	0.553
60	0.467	0.475
70	0.404	0.413
80	0.355	0.365
90	0.315	0.326
100	0.282	0.29

Table 1.6. *Densities and specific weight of water in imperial units*

		Specific Weight γ	
Temperature T (°F)	**Density ρ (slugs/ft^3)**	**(lb/ft^3)**	**(lb/US gallon)**
32	1.940	62.42	8.3436
40	1.940	62.43	8.3451
50	1.940	62.41	8.3430
60	1.938	62.37	8.3378
70	1.936	62.30	8.3290
80	1.934	62.22	8.3176
90	1.931	62.11	8.3077
100	1.927	62	8.2877
120	1.918	61.71	8.2498
140	1.908	61.38	8.2048
160	1.896	61	8.1537
180	1.883	60.58	8.0969
200	1.869	60.12	8.0351
212	1.860	59.83	7.9957

Kinematic Viscosity (υ) = the ratio of dynamic viscosity (μ) to mass density (ρ).

$$\upsilon = \frac{\mu}{\rho} = \left[\frac{L^2}{T}\right] \tag{1.1.9}$$

Tables 1.5, 1.6, and 1.7 list water properties at different temperatures and their units.

1.2 Fluid Statics

Fluid statics is concerned with the balance of forces on a fluid parcel which stabilizes it and brings it to rest. In the following, some fundamental concepts related to fluid statics will be discussed.