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978-1-107-07490-3 - Principles of Optics for Engineers: Diffraction and Modal Analysis
By William S. C. Chang
Excerpt
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Introduction

Optics is a very old field of science. It has been taught traditionally as propagation, imaging, and diffraction of polychromatic natural light, then as interference, diffraction, and propagation of monochromatic light. Books like *Principles of Optics* by E. Wolf in 1952 gave a comprehensive and extensive in-depth discussion of properties of polychromatic and monochromatic light. Topics such as optical waveguide, fiber optics, optical signal processing, and holograms for laser light have been presented separately in more recent books. There appears to be no need for any new book in optics. However, there are several reasons to present optics differently, such as is done in this book.

Many contemporary optics books are concerned with components and instruments such as lenses, microscopes, interferometers, gratings, etc. Reflection, refraction, and diffraction of optical radiation are emphasized in these books. Other books are concerned with the propagation of laser light in devices and systems such as optical fibers, optical waveguides, and lasers, where they are analyzed more like microwave devices and systems. The mathematical techniques used in the two approaches are very different. In one case, diffraction integrals and their analysis are important. In the other case, modal analysis is important. Students usually learn optical analysis in two separate ways and then reconcile, if they can, the similarities and differences between them. Practicing engineers are also not fully aware of the interplay of these two different approaches. These difficulties can be resolved if optical analyses are presented from the beginning as solutions of Maxwell's equations and then applied to various applications using different techniques, such as diffraction or modal analysis.

The major difficulty to present optics from the solutions of Maxwell's equations is the complexity of the mathematics. Complex mathematical analyses often obscure the basic differences and similarities of the mathematical techniques and mask the understanding of basic concepts.

Optical device configurations vary from simple mirrors to complex waveguide devices. How to solve Maxwell's equations depends very much on the configuration of the components to be analyzed. The more complex the configuration, the more difficult the solution. Optics is presented in this book in the order of the complexity of the configuration in which the analysis is carried out. In this manner, the reasons for using different analytical techniques can be easily understood, and basic principles are not masked by any unnecessary mathematical complexity.

Optics in unbounded media is first presented in this book in the form of plane wave analysis. A plane wave is the simplest solution of Maxwell's equations. Propagation, refraction, diffraction, and focusing of optical radiation, even optical resonators and planar waveguides, can be analyzed and understood by plane wave analysis. It leads directly to ray optics, which is the basis of traditional optics. It provides a clear demonstration and understanding of optics without considering boundary condition or device configuration. Even sophisticated concepts such as modal expansion can also be introduced using plane waves. Plane wave analysis is the focus of the first two chapters.

Realistically, wave propagation in bulk optical components involves a finite boundary such as a lens that has a finite aperture. Plane wave analysis can no longer be used in this configuration. However, in these situations, the waves are still transverse electric and magnetic (TEM). Therefore, TEM waves are rigorously analyzed using Maxwell's equations in Chapter 3. The diffraction analysis presented in Chapter 3 is identical to traditional optical analysis. Since applications of diffraction analysis are already covered extensively in existing optics books, only a few basic applications of diffraction theory are presented here. The distinct features of our presentation here are: (1) Both the TEM assumption of the Kirchoff's integral analysis and the relation between diffraction theory and Maxwell's equations are clearly presented. (2) Modern engineering concepts such as convolution, unit impulse response, and spatial filtering are introduced.

Diffraction integrals are again used to analyze laser cavities in the first part of Chapter 4, for three reasons: (1) Laser modes are used in many applications. (2) The diffraction analysis leads directly to the concept of modes. It is instructive to recognize that they are inter-related. (3) An important consequence of laser cavity analysis is that laser modes are Gaussian. A Gaussian mode retains its functional form not only inside, but also outside of the cavity.

The second part of Chapter 4 is focused on Gaussian beams and how different applications can be analyzed using Gaussian beams. Gaussian modes are also natural solutions of the Maxwell's equations. It constitutes a complete set. Just like any other set of modes, such as plane waves, any radiation can be represented as summation of Gaussian modes. When the diffraction integral is used in Chapter 3 to analyze waves propagating through components with finite apertures, the diffraction loss needs to be calculated by the Kirchoff's integral for each aperture. In comparison, the diffraction loss of a Gaussian beam propagating through an aperture can be calculated without any integration. Therefore, a Gaussian beam is used to represent TEM waves in many engineering applications.

Although TEM modes exist in solid-state and gas laser cavities, waves propagating in waveguides and fibers are no longer transverse electric and magnetic. Microwave-like modal analysis needs to be used to analyze optical devices that have dimensions of the order of optical wavelength.

Optical waveguides and fibers are dielectric devices. They are different from microwave devices. Microwave waveguides have closed metallic boundaries. The mathematical complexity of finding microwave waveguide modes is much simpler than that of optical waveguides.

The distinct features in the analysis of dielectric waveguides are: (1) There are analytical solutions for very few basic device configurations because of the complex boundary conditions. Analyses of practical devices need to be carried out by approximation techniques. (2) There is a continuous set of radiation modes in addition to the discrete guided-wave modes. Any abrupt discontinuity will excite radiation modes. (3) The evanescent tail of the guided-wave modes not only reduces propagation loss, but also provides access to excite the modes by coupling through evanescent fields. (4) Multiple modes are often excited in devices. The performance of the device depends on what modes have been excited.

Because of the complexity of modal analysis of optical waveguides and fibers, it is presented here in four parts.

In the first part, modes of simple waveguides and fibers are discussed in Chapter 5. Analytical solutions for planar waveguides and step–index fiber are presented. Although these are not realistic devices, they are the only solutions that can be obtained from Maxwell's equations. Modes of these simple basic devices are very useful for demonstrating various properties of the guided waves. Approximation methods are then presented to discuss modes of realistic devices. For example, the effective index method is used here to analyze channel waveguides.

Guided-wave devices operate by mutual interactions among modes. These interactions need to be analyzed in the absence of exact solutions. Therefore, several approximation methods, the perturbation technique, the coupled mode analysis, and the super mode analysis, are presented in Chapter 6. The differences and similarities of the three methods are compared and explained. Examples in applications are used to demonstrate these techniques.

In the third and fourth parts, modal analyses of passive and active guided-wave devices are presented. Passive guided-wave devices function mainly as power dividers, wavelength filters, resonators, and wavelength multiplexers. In each of these system functions, there are several different devices that could be used. Thus, devices that perform the same system function are discussed and analyzed together. Their performance is compared.

Active devices utilize electro-optical effects of the electrical signals to operate. Discussion of active guided-wave devices is complex because there are different physical mechanisms involved. How these mechanisms work is reviewed.The electrical performance, as well as the optical performance of these devices are analyzed.

In summary, when optics are presented as solutions of Maxwell's equations, the inter-relation between plane wave, diffraction, and modal analysis becomes clear. For example, the use of modal analysis is not limited to waveguides and fibers. There can be modes and modal expansion in plane wave analysis, as well as in diffraction optics. As we learn optics step by step in the order of the mathematical complexity and device configuration, we learn optical analysis from various perspectives.

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1 Optical plane waves in an unbounded medium

Engineers involved in design and the use of optical and opto-electronic systems are often required to analyze theoretically the propagation and the interaction of optical waves using different methods. Sometimes it is diffraction analysis; on other occasions, modal analysis. They are all solutions of Maxwell's equations, yet they appear to be very different. All optical analyses should be presented as solutions of Maxwell's equations so that the inter-relations between different analytical techniques are clear. In order to avoid unnecessary mathematical complexity, the simplest analysis should be presented first. In this book, optics will be presented first by plane wave analysis, followed by diffraction and modal analyses, in increasing order of complexity.

Plane waves are the simplest form of optical waves that can be derived rigorously from Maxwell's equations. Plane wave analysis can be used to derive ray analysis, which is the basis of traditional optics. It can be applied directly to analyze many optical phenomena such as refraction, reflection, dispersion, etc. It can also be used to demonstrate sophisticated concepts such as superposition, interference, resonance, guided waves, and Fourier optics. Plane wave analyses will be the focus of discussion in Chapters 1 and 2.

However, plane wave analysis cannot be used to analyze diffraction, laser modes, optical signal processing, and propagation in small optical components such as fibers and waveguides, etc. These analyses will be the focus of discussion in subsequent chapters.

1.1 Introduction to optical plane waves

Plane wave analysis is presented here in full detail, so that the mathematical derivations and details can be fully exhibited and the physical significances of these analyses are fully explained.

1.1.1 Plane waves and Maxwell's equations

All optical waves are solutions of the Maxwell's equations (assuming there are no free carriers),

$$\nabla \times \underline{E} = \frac{-\partial \underline{B}}{\partial t}, \quad \nabla \times \underline{H} = \frac{\partial \underline{D}}{\partial t} \tag{1.1}$$

Here $\underline{E}$ is the electric field vector, $\underline{H}$ is the magnetic field vector, $\underline{D}$ is the displacement vector, and $\underline{B}$ is the magnetic induction vector. For isotropic media,

$$\underline{B} = \mu \underline{H}, \quad \underline{D} = \varepsilon \underline{E} \tag{1.2}$$

Let $\underline{i_x}$, $\underline{i_y}$, and $\underline{i_z}$, be unit vectors in the *x, y*, and *z* directions of an *x-y-z* rectangular coordinate system. Then $\underline{E}$, $\underline{H}$ and the position vector $\underline{r}$ can be written as

$$\underline{E} = E_x \underline{i_x} + E_y \underline{i_y} + E_z \underline{i_z} \qquad \underline{H} = H_x \underline{i_x} + H_y \underline{i_y} + H_z \underline{i_z} \tag{1.3a}$$

$$\underline{r} = x\underline{i_x} + y\underline{i_y} + z\underline{i_z} \tag{1.3b}$$

A special solution of Eqs. (1.1) and (1.2) is a plane wave that has no amplitude variation transverse to its direction of propagation. If we designate the *z* direction as the direction of propagation, this means that

$$\frac{\partial}{\partial x} = 0 \quad \text{and} \quad \frac{\partial}{\partial y} = 0 \tag{1.4}$$

Substituting $\partial/\partial x = 0$ and $\delta/\delta y = 0$ into the $\nabla \times \underline{E}$ and $\nabla \times \underline{H}$ equations leads to two distinct groups of equations:

$$\frac{\partial E_y}{\partial z} = \mu\, \partial H_x/\partial t, \qquad \frac{\partial H_x}{\partial z} = \varepsilon \partial E_y/\partial t; \quad or \quad \frac{\partial E_y{}^2}{\partial z^2} = \mu\varepsilon \frac{\partial^2}{\partial t^2} E_y \tag{1.5a}$$

and

$$\frac{\partial H_y}{\partial z} = -\varepsilon \partial E_x/\partial t, \qquad \frac{\partial E_x}{\partial z} = -\mu \partial H_y/\partial t; \quad or \quad \frac{\partial H_y{}^2}{\partial z^2} = \mu\varepsilon \frac{\partial^2}{\partial t^2} H_y \tag{1.5b}$$

Clearly, these are two separate independent sets of equations. E_y and H_x are related only to each other, and H_y and E_x are related only to each other. Solutions of Eq. (1.5a) are plane waves with *y* polarization of the electric field (or *x* polarization in magnetic field). Solutions of Eq. (1.5b) are plane waves with *x* polarization in the electric field $\underline{E}$ (or *y* polarization in magnetic field $\underline{H}$).

(a) The *y*-polarized plane wave

For a cw optical plane wave with a single angular frequency ω that has a time variation, $e^{j\omega t}$, and for lossless media (i.e. the medium has a real value of ε), there is a well-known solution of Eq. (1.5a) in the complex notation. It is

$$E_y = E_y^f e^{-j\beta z} e^{j\omega t}, \quad H_x = H_x^f e^{-j\beta z} e^{j\omega t}, \quad H_x^f = -\sqrt{\frac{\varepsilon}{\mu}} E_y^f, \tag{1.6a}$$

where $\beta = \omega\sqrt{\mu\varepsilon}$. The real time domain expression for the complex E_y shown in (1.6a) is $\left|E_y^f\right| \cos(\beta z - \omega t + \varphi)$ where φ is the phase of $\left|E_y^f\right|$ at $z = 0$ and $t = 0$. The angular frequency ω is related to the optical frequency f by $\omega = 2\pi f$. This wave is known as a *y*-polarized forward propagating wave in the $+z$ direction. The phase of

E_y, i.e. $\beta z - \omega t = \beta(z - v_p t)$, is a constant when $z = v_p t$. Thus v_p is known as the phase velocity of the plane wave.

If the medium in which the plane wave propagates is free space, then $\varepsilon = \varepsilon_o$ and the free space phase velocity is $c_o = 1/\sqrt{\mu\varepsilon_o} \equiv 3 \times 10^8 \text{ m s}^{-1}$. In free space, the optical wave length for a frequency f is λ_o, where $f\lambda_o = c_o$. If the medium is a lossless dielectric material with a permittivity ε, then its index of refraction is $n = \sqrt{\varepsilon/\varepsilon_o}$, $\beta = n\beta_o = n\omega\sqrt{\mu\varepsilon_o}$. If ε is a function of wavelength, the medium is said to be dispersive.

There is also a second solution for the same polarization of the electric field,

$$E_y = E_y^b e^{j\beta z} e^{j\omega t}, \quad H_x = H_x^b e^{j\beta z} e^{j\omega t}, \quad H_x^b = \sqrt{\frac{\varepsilon}{\mu}} E_y^b \tag{1.6b}$$

This solution is a backward propagating wave because the phase of E_y, i.e. $\beta z + \omega t = \beta(z + v_p t)$, at any time t is a constant when $z = -v_p t$ and $v_p = \omega/\beta$.

If the permittivity has a loss component, $\varepsilon = \varepsilon_r - j\varepsilon_\sigma$, then

$$\beta = \omega\sqrt{\mu(\varepsilon_r - j\varepsilon_\sigma)} = \beta_r - j\beta_\sigma \tag{1.7}$$

The phase velocity of light is now $v_p = c = \omega/\beta_r$. The amplitude of the plane wave decays as $e^{-\beta_\sigma z'}$ for forward waves and $e^{+j\beta_\sigma z'}$ for backward waves. In comparison with the phase velocity of free space, the ratio of the phase velocities, c_o/c, is the effective refractive index of the plane wave, $n = c_o\beta_r/\omega = c_o/c$. The wavelength in the medium is $\lambda = \lambda_o/n$. In addition to β, or phase velocity, the loss of optical waves in the medium is an important consideration in applications.

(b) The x-polarized plane wave

A similar solution exists for the x-polarized electric field and H_y. For the forward wave,

$$H_y = H_y^f e^{-j\beta z} e^{j\omega t}, \quad E_x = E_x^f e^{-j\beta z} e^{j\omega t}, \quad E_x^f = \sqrt{\frac{\mu}{\varepsilon}} H_y^f \tag{1.8a}$$

For the backward wave,

$$H_y = H_y^b e^{+j\beta z} e^{j\omega t}, \quad E_x = E_x^b e^{j\beta z} e^{j\omega t}, \quad E_x^b = -\sqrt{\frac{\mu}{\varepsilon}} H_y^b \tag{1.8b}$$

In summary, both equations (1.5a) and (1.5b) are second-order differential equations. Mathematically, each of them has two independent solutions, which are the forward and the backward propagating waves. However, Eqs. (1.5a) and (1.5b) are also two separate set of equations. The solution for Eq. (1.5a) describes a plane wave polarized in the y direction. The solution of Eq. (1.5b) describes a plane wave polarized in the x direction. Both waves have the same direction of propagation. $\underline{\beta}$ is usually designated as a propagation vector along the direction of propagation z that has magnitude β,

$$\underline{\beta} = \beta \underline{i_z}, \quad \underline{z} = z\underline{i_z}, \quad \beta z = \underline{\beta} \bullet \underline{z} \tag{1.9}$$

The forward wave has $+\underline{\beta}$, the backward wave has $-\underline{\beta}$.

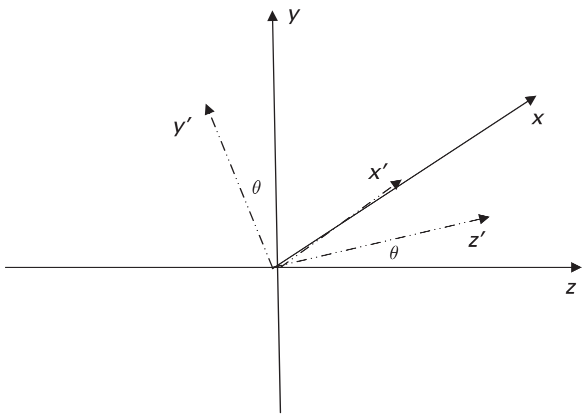

Figure 1.1 Illustration of x-y-z and x'-y'-z' coordinates.

It is important to note that, along any direction of propagation, there are always plane waves with two orthogonal polarizations. In each polarization, there are always two solutions, the forward wave and the backward wave. The propagation constant β *and phase velocity will depend on the medium and the frequency.*

1.1.2 Plane waves in an arbitrary direction

Frequently, plane waves in other directions of propagation need to be expressed mathematically for analysis. As an example, let there be another x'-y'-z' rectangular coordinate which is related to the x-y-z coordinate by

$$\underline{i_{x'}} = \underline{i_x}, \quad \underline{i_{y'}} = \cos\theta \underline{i_y} - \cos\left(\frac{\pi}{2} - \theta\right)\underline{i_z}, \quad \underline{i_{z'}} = \cos\left(\frac{\pi}{2} - \theta\right)\underline{i_y} + \cos\theta \underline{i_z} \tag{1.10}$$

The x-y-z and the x'-y'-z' coordinates are illustrated in Figure 1.1. The x'-y'-z' coordinate is just the x-y-z coordinate rotated by angle θ about the x axis. The x and x' axes are the same.

Let there be a plane wave propagating along the z' direction. The solutions for the y' and x' polarized plane waves have already been given in Eqs. (1.6) and (1.8). However, these solutions could also be expressed in the x, y, and z coordinates, where

$$\beta z' = \underline{\beta} \bullet \underline{z'} = \beta\cos\theta z + \beta\cos\left(\frac{\pi}{2} - \theta\right)y \tag{1.11}$$

$$\underline{\beta} = \beta \underline{i_{z'}} = \beta\cos\theta \underline{i_z} + \beta\cos\left(\frac{\pi}{2} - \theta\right)\underline{i_y} \tag{1.12}$$

$$\mathrm{e}^{\pm j\beta z'} = \mathrm{e}^{\pm j\underline{\beta} \bullet \underline{z'}} = \mathrm{e}^{\pm j\underline{\beta} \bullet \underline{r}} \tag{1.13}$$

For the y' polarized plane wave propagating in the $+z'$ direction,

$$\underline{E_{y'}} = E^f_{y'}\underline{i_{y'}}\mathrm{e}^{-j\underline{\beta}\bullet\underline{z'}}\mathrm{e}^{j\omega t} = \underline{E^f_{y'}}\,\mathrm{e}^{-j\underline{\beta}\bullet\underline{r}}\mathrm{e}^{j\omega t}$$

$$= \left(E^f_{y'}\cos\theta\underline{i_y} - E^f_{y'}\sin\theta\underline{i_z}\right)\mathrm{e}^{-j\underline{\beta}\bullet\underline{r}}\mathrm{e}^{j\omega t} \tag{1.14}$$

$$\underline{H_x} = \underline{H_{x'}} = -\sqrt{\frac{\varepsilon}{\mu}}E^f_{y'}\mathrm{e}^{-j\underline{\beta}\bullet\underline{r}}\mathrm{e}^{j\omega t}\underline{i_x} \tag{1.15}$$

For the y' polarized backward plane wave propagating in the $-z'$ direction,

$$\underline{E_{y'}} = E^b_{y'}\mathrm{e}^{+j\underline{\beta}\bullet\underline{r}}\mathrm{e}^{j\omega t}\underline{i_{y'}}, \quad \underline{H^b_{x'}} = \sqrt{\frac{\varepsilon}{\mu}}E^b_{y'}\mathrm{e}^{+j\underline{\beta}\bullet\underline{r}}\mathrm{e}^{j\omega t}\underline{i_{x'}} \tag{1.16}$$

For the x' polarized plane wave propagating in the $+z'$ direction,

$$\underline{E_{x'}} = \underline{E_x} = E^f_{x'}\mathrm{e}^{-j\underline{\beta}\bullet\underline{r}}\mathrm{e}^{j\omega t}\underline{i_{x'}} \tag{1.17}$$

$$\underline{H_{y'}} = \sqrt{\frac{\varepsilon}{\mu}}E^f_{x'}\mathrm{e}^{-j\underline{\beta}\bullet\underline{r}}\mathrm{e}^{j\omega t}\underline{i_{y'}} = \sqrt{\frac{\varepsilon}{\mu}}E^f_{x'}\left(\cos\theta\underline{i_y} - \sin\theta\underline{i_z}\right)\mathrm{e}^{-j\underline{\beta}\bullet\underline{r}}\mathrm{e}^{-j\omega t} \tag{1.18}$$

For the x' polarized backward wave plane wave propagating in the $-z'$ direction,

$$\underline{E_{x'}} = \underline{E_x} = E^b_{x'}\mathrm{e}^{+j\underline{\beta}\bullet\underline{r}}\mathrm{e}^{j\omega t}i_{x'} \tag{1.19}$$

$$\underline{H_{y'}} = -\sqrt{\frac{\varepsilon}{\mu}}E^b_{x'}\mathrm{e}^{+j\underline{\beta}\bullet\underline{r}}\mathrm{e}^{j\omega t}\underline{i_{y'}} \tag{1.20}$$

The preceding example can be generalized for any orientation of the x', y', and z' coordinates with respect to the x, y, and z coordinates. Any plane wave propagating in the z' direction can have two mutually perpendicular polarizations, $\underline{i_a}$ and $\underline{i_b}$. $\underline{i_{z'}}$, $\underline{i_a}$ and $\underline{i_b}$ are mutually perpendicular to each other, i.e. $\underline{i_a} \bullet \underline{i_b} = \underline{i_a} \bullet \underline{\beta} = \underline{i_b} \bullet \underline{\beta} = 0$.

$$\text{Let } \underline{i_a} = \underline{i_{x'}} \quad \text{and} \quad \underline{i_b} = \underline{i_{y'}} \tag{1.21}$$

Then the general solutions for the case of $\underline{i_a}$ polarization are:

$$\underline{E^f_a} = E^f_a\mathrm{e}^{-j\underline{\beta}\bullet\underline{r'}}\mathrm{e}^{j\omega t}\underline{i_{x'}} \quad \underline{H^f_a} = \sqrt{\frac{\varepsilon}{\mu}}E^f_a\mathrm{e}^{-j\underline{\beta}\bullet\underline{r'}}\mathrm{e}^{j\omega t}\underline{i_{y'}} \tag{1.22}$$

$$\underline{E^b_a} = E^b_a\mathrm{e}^{+j\underline{\beta}\bullet\underline{r'}}\mathrm{e}^{j\omega t}\underline{i_{x'}} \quad \underline{H^b_a} = -\sqrt{\frac{\varepsilon}{\mu}}E^b_a\mathrm{e}^{+j\underline{\beta}\bullet\underline{r'}}\mathrm{e}^{j\omega t}\underline{i_{y'}} \tag{1.23}$$

$$\underline{\beta} = \beta_{x'}\underline{i_{x'}} + \beta_{y'}\underline{i_{y'}} + \beta_{z'}\underline{i_{z'}} \qquad \beta^2 = \beta_{x'}{}^2 + \beta_{y'}{}^2 + \beta_{z'}{}^2 \tag{1.24}$$

Here, $\underline{\beta}$ makes angles $\theta_{x'}$, $\theta_{y'}$, and $\theta_{z'}$ with respect to the x', y', and z' axes, with $\beta_{x'}/\beta = \cos\theta_{x'}$, $\beta_{y'}/\beta = \cos\theta_{y'}$, and $\beta_{z'}/\beta = \cos\theta_{z'}$. The general solutions for the case of $\underline{i_b}$ polarization are:

$$\underline{E_b^f} = E_b^f e^{-j\underline{\beta}\bullet\underline{r'}} e^{j\omega t}\underline{i_{y'}} \quad \underline{H_a^f} = -\sqrt{\frac{\varepsilon}{\mu}} E_b^f e^{-j\underline{\beta}\bullet\underline{r'}} e^{j\omega t}\underline{i_{x'}} \tag{1.25}$$

$$\underline{E_b^b} = E_a^b e^{+j\underline{\beta}\bullet\underline{r'}} e^{j\omega t}\underline{i_{y'}} \quad \underline{H_a^b} = \sqrt{\frac{\varepsilon}{\mu}} E_a^b e^{+j\underline{\beta}\bullet\underline{r'}} e^{j\omega t}\underline{i_{x'}} \tag{1.26}$$

It is important to recognize that when there is a wave solution containing various terms, any term that has the form shown in Eqs. (1.17) to (1.26) represents a plane wave propagating in the direction of $\underline{\beta}$.

1.1.3 Evanescent plane waves

Eqs. (1.22) to (1.26) described propagating plane waves that have real $\beta_{x'}$, $\beta_{y'}$, and $\beta_{z'}$ values. The maximum real $\beta_{x'}$ and $\beta_{y'}$ values of propagating plane waves are limited to $\sqrt{\beta_{x'}{}^2 + \beta_{y'}{}^2} < \omega\sqrt{\mu\varepsilon}$, i.e. $0 < \theta_{x'}$, $\theta_{y'}$, and $\theta_{z'} < \pi/2$. Nevertheless, Maxwell's equation is still satisfied even if $\sqrt{\beta_x^2 + \beta_{y'}^2}$ is larger than β. In that case Eq. (1.24) can only be satisfied if $\beta_{z'}$ is imaginary. When $\beta_{z'}$ is imaginary, the z' variation is a real decaying or growing exponential function, $e^{\pm\sqrt{\beta_{x'}{}^2+\beta_{y'}{}^2-\beta^2}z'}$. In any passive medium, the plane wave cannot grow without energy input. Thus the solution must decay exponentially in the z' direction. Any solution with imaginary $\beta_{z'}$ is called an evanescent wave. Such solutions do not propagate in the z direction. They do not have a phase velocity. Evanescent waves are excited usually in the vicinity of a boundary with an incident wave applied across the boundary. It is only a near field, meaning that it is negligible at locations far away from the boundary.[1] It is interesting to note that when $\beta_{x'} = \beta$, $\beta_{y'} = \beta_{z'} = 0$, it is no longer a plane wave propagating in the z' direction. It is a plane wave propagating in the $+x'$ direction.

1.1.4 Intensity and power

In optics, only time-averaged power can be detected directly by means of detectors or by recording media such as film. The time-averaged power per unit area is known commonly as the intensity. In comparison with rf and microwaves, intensity analysis plays a much more important role in optics. From text books on electromagnetic theory, it is well known that the total time-averaged power in the direction of propagation is [1]

$$P_{av} = \frac{1}{2}\mathrm{Re}\int_S \underline{E} \times \underline{H}^* \bullet \underline{i_{z'}} \mathrm{d}s = \int_S \underline{I} \bullet \underline{i_{z'}} \mathrm{d}s, \quad \underline{I} = \frac{1}{2} Re[\underline{E} \times \underline{H}] \tag{1.27}$$

[1] It is important to note that although the mathematical solution of a plane wave exists for β_x or β_y values larger than $\omega\sqrt{\mu\varepsilon}$, such a solution is important only if those plane waves are excited in specific applications such as total internal reflection. Otherwise, the solutions have no practical significance.

The integration is carried out over the entire surface of the plane wave, S. The * designates the complex conjugate of the variable. Re designates the real part of the complex quantity. Therefore the time-averaged power per unit area in the direction of propagation z' in either polarization is

$$I_{z'} = \frac{1}{2}\mathrm{Re}\, E_a H_a^* = \frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}} E_a E_a^* \qquad \text{or} \qquad I_{z'} = \frac{1}{2}\mathrm{Re}\, E_b H_b^* = \frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}} E_b E_b^* \tag{1.28}$$

Note that although the total I *is the sum of the* I*s in each polarization, the total* I *carries no information about polarization breakdown. Although the complex amplitude of the plane wave has a phase, its intensity* I *has no phase information. For plane waves in a lossless medium, i.e.* $\varepsilon_\sigma = 0$*, its intensity* I *is a constant. In media with loss, the decay of the time-averaged power is* $e^{-2\beta_\sigma z'}$ *for a forward wave and* $e^{+2\beta_\sigma z'}$ *for a backward wave.*

In microwaves, $\underline{I}$ is known as the Poynting vector. In x-y-z coordinates, the intensity along the z direction is ½ Re $E_x H_y^*$, the intensity along the y direction is ½ Re $E_z H_x^*$, and the intensity along the x direction is ½Re $E_y H_z^*$.

1.1.5 Superposition and plane wave modes

Plane waves in different direction of propagation (or plane wave modes) can be superimposed simultaneously. This is known as the superposition theory in linear media. Many interesting optical phenomena can be understood by superposition of plane waves. Three examples are presented here to illustrate the effects of superposition. They are important concepts in many applications.

(a) Plane waves with circular polarization

Let us consider superposition of two plane waves of equal magnitude, polarized in x and y, with a $\pi/2$ phase difference.

$$\underline{E} = E_o\left(\underline{i_x} + j\underline{i_y}\right) \tag{1.29}$$

The real time domain form of this wave is

$$\underline{E} = E_o\left[\cos(\beta z - \omega t + \varphi)\underline{i_x} + \sin(\beta z - \omega t + \varphi)\underline{i_y}\right] \tag{1.30}$$

So that, at any time t, the polarization rotates at different z positions. This type of wave is known as a circular polarized optical wave because the polarization of $\underline{E}$ rotates as it propagates. When these two waves have unequal amplitudes they give rise to an elliptical polarized plane wave.

(b) Interference of coherent plane waves

Let us consider two plane waves of equal amplitude at the same ω and y polarization. They propagate at different directions of propagation $\underline{\beta}$ in the x–z plane. Their $\underline{\beta}$s lie in the x–z plane and make angles, θ and ζ, with respect to the z axis. Mathematically, the waves are