1 Introduction

1.1 Purpose and Motivation

The purpose of this book is to present fundamentals used in the navigation and guidance of aerospace vehicles. By **fundamentals**, we mean a body of knowledge that does not change with advances in technology. **Aerospace** vehicles encompass all kinds of craft flying in the atmosphere or in space. **Navigation** is concerned with the questions: Where is the vehicle? What is its velocity? And what are its angular orientation and angular rates? **Guidance** is concerned with the question: What maneuvers should we perform to cause the vehicle to go where we want, while meeting the specifications of the mission?

For centuries, devices such as charts, magnetic compasses, drafting compasses, sextants, and clocks were sufficient to solve the navigation and guidance problems of the time. In those days, velocities over land and sea were relatively slow, navigators could count on a variety of visible references (such as landmarks, beacons, or stars), and a human pilot controlled the vehicle during the entirety of the trip. However, the advent of air and space travel has posed substantial new challenges to navigators. First, aerospace vehicles generally travel much faster and farther than their land and sea counterparts. Second, whereas navigation on land and sea can be based on visible references, an aerospace navigator may be deprived of such resources. Finally, many aerospace missions require a high level of automation, either because they are unmanned or to minimize crew fatigue.

In response to these new challenges, the aerospace community has standardized the use of avionic equipment such as radio transmitters, laser velocimeters, laser gyroscopes, and digital computers. However, despite these remarkable technological advances, the underlying mathematical principles that govern the use of data for navigation and guidance have remained the same. For instance, the fifteenth-century sea navigator would determine the location of a ship by measuring the apparent elevation of known stars using a sextant. The ship would then be located at the intersection of loci drawn on a map. In this century, the navigation of an unmanned interplanetary spacecraft is typically performed by measuring the angles between lines of sight to known heavenly bodies, such as the Sun, planets, and stars. The spacecraft is similarly found at the intersection of several loci drawn in three-dimensional space.

This book aims at an exposition of the mathematical methods used in aerospace navigation and guidance, with a strong emphasis on fundamentals rather than any particular hardware implementation. Such an exposition is possible because these methods, which use geometry, linear algebra, differential calculus, and stochastic error analysis, are hardware-independent. This book does not attempt to cover all the fundamentals; however, it is the authors' experience that the fundamentals it presents constitute a good initial body of knowledge for an engineer or researcher in the field of navigation and guidance.

Before delving earnestly into the technical material, it is worthwhile to answer the question: Why study fundamentals at all? We can give at least two compelling answers to this question. The first is purely utilitarian: by their very nature, fundamentals have a value that transcends time and technology. Hence, studying fundamentals is a better investment than studying ephemeral knowledge that may soon become obsolete due to technological advances. The second answer is foundational: fundamentals are a sound basis on which to build knowledge. As a practical consequence, fundamentals typically provide excellent guidance in the solution of engineering analysis, synthesis, and design problems.

1.2 Problem Statement

Throughout this book, we consider the following three vector quantities:

1. $x(t)$ denotes the actual state vector of the vehicle. This vector typically contains the variables that specify the position and angular orientation of the vehicle, and their time derivatives.
2. $x_r(t)$ denotes the reference state vector of the vehicle, that is, the desired value of the state vector.
3. $\hat{x}(t)$ denotes the estimated state vector, that is, the value of the state vector, as estimated by the navigator, based on the sensor measurements.

The three preceding vectors generate the following:

1. The vector $\delta x(t) = x(t) - x_r(t)$ is called the **guidance error**.
2. The vector $\tilde{x}(t) = x(t) - \hat{x}(t)$ is called the **navigation error**.

Thus, the guidance error is, roughly speaking, the difference between where the vehicle is located and where it is supposed to be. It is generally caused by disturbances acting on the vehicle, such as wind gusts, currents, and thrusting misalignment. The navigation error is, roughly speaking, the difference between where the vehicle is located and where the navigator believes it is. This error is generally due to uncertainties and errors in the readings of the sensors. It is important to realize that, even without disturbances or measurement errors, the guidance and navigation errors interact. For instance, if the vehicle is on its nominal course at some initial time t_0 (that is, $\delta x(t_0) = 0$), but the navigator believes that the vehicle is off-course (that is, $\tilde{x}(t_0) \neq 0$), this will prompt the navigator to "mis-correct" the course of the vehicle, causing $\delta x(t_0 + \tau) \neq 0$ for some small positive τ.

The problem of combined navigation and guidance can now, in broad terms, be stated as that of *causing the vehicle to follow its desired path, despite disturbances and navigation uncertainties*. In other words, we want to go where we are supposed to go, despite the facts that we don't know exactly where we are and that our motion is undergoing unpredictable disturbances.

We present the following argument to show how such an achievement is, in principle, possible and to justify why we consider the problems of navigation and guidance together. Assume that we guide the vehicle based on navigation, that is, we take guidance decisions assuming that our estimate of $x(t)$ is indeed $x(t)$ itself. This results in an **estimated guidance error** $\hat{x}(t) - x_r(t)$, that is, the difference between where we believe we are and where we are supposed to be. Note that the estimated guidance error can also be viewed as the guidance error we would incur if the navigation error happened to be zero. Assume that we have a bound on the estimated guidance error, that is, we have found a positive number ϵ_g such that $||\hat{x}(t) - x_r(t)|| \leq \epsilon_g$. Also assume that we have a bound on the navigation error, that is, we have found a positive number ϵ_n such that $||x(t) - \hat{x}(t)|| \leq \epsilon_n$. Then, a simple application of the triangle inequality yields

$$\begin{aligned} ||\delta x(t)|| &= ||x(t) - x_r(t)|| \\ &= ||(x(t) - \hat{x}(t)) + (\hat{x}(t) - x_r(t))|| \\ &\leq ||x(t) - \hat{x}(t)|| + ||\hat{x}(t) - x_r(t)|| \end{aligned} \tag{1.1}$$

and therefore

$$||\delta x(t)|| \leq \epsilon_n + \epsilon_g, \tag{1.2}$$

which implies that we can guarantee a bound on the true guidance error. In other words, small navigation error and small estimated guidance error imply good guidance.

Therefore, in practice, we proceed as follows. First, we develop navigation strategies and quantify their errors. Then, we obtain guidance laws and analyze their performance in the absence of navigation errors. Finally, we assess the degradation of performance of the guidance laws due to the navigation errors. Note that this procedure assumes that a guidance law that is designed based on perfect knowledge of $x(t)$ would work similarly well if, in its implementation, $x(t)$ were replaced by $\hat{x}(t)$. This **separation property** is formally justified in Chapter 7.

1.3 Scope of the Book

This book applies systems and control theory to the motion of aerospace vehicles. In broad terms, systems theory deals with signals and cause-effect relationships between them, whereas control theory seeks to achieve performance of systems in the presence of uncertainty. Let us clarify what we mean.

1.3.1 Systems Theory

For now, let us define a signal as a vector function of time – in Chapter 2, we give a more precise definition of the term *signal*, but here vector functions of time suffice. Then, a system is simply a portion of the universe where we postulate that some signals cause other signals. We call the cause-signals inputs and the effect-signals outputs, and we represent their cause-effect relationship in a block diagram such

Figure 1.1. Block diagram representation of a system.

as in Figure 1.1, where the inputs enter the system and the outputs exit the system. Formally, we write

$$y = S(u). \tag{1.3}$$

For instance, in a spring system, such as in Figure 1.2, we typically postulate that the force is the input and the deformation is the output.

The following taxonomy is used to characterize systems.

1. **Linear versus nonlinear.** In a linear system, the superposition principle holds, whereby a linear combination of inputs produces as output the same linear combination of the respective outputs. In other words, for all inputs u_1, u_2 and real numbers α_1, α_2, we have

$$S(\alpha_1 u_1 + \alpha_2 u_2) = \alpha_1 S(u_1) + \alpha_2 S(u_2). \tag{1.4}$$

   Otherwise, the system is called nonlinear. For instance, a spring system is often modeled as linear.
2. **Static versus dynamic.** In a static, or **memoryless**, system, the output at any time, $y(t)$, depends only on the input at the same time, $u(t)$, but does not depend on the past time history of input $u(\tau)$, $\tau < t$. Otherwise, the system is called dynamic. Typically, the mathematical description of a static system is in terms of algebraic equations, whereas that of a dynamic system is in terms of differential equations. For instance, the spring system of Figure 1.2 is static, whereas a mass-spring system, such as in Figure 1.3, is dynamic.
3. **Time invariant versus time varying.** In a time invariant system, an arbitrary time shift of the input produces exactly the same time shift of the output. In other words, let the system Δ_τ represent the time shift of magnitude τ, so that $(\Delta_\tau(u))(t) = u(t - \tau)$. Then, for a time invariant system, for all signals u and time shift magnitudes τ, we have

$$S\left(\Delta_\tau(u)\right) = \Delta_\tau\left(S(u)\right). \tag{1.5}$$

   Otherwise, the system is called time varying. For instance, a spring system where the spring stiffness is constant is a time invariant system.
4. **Causal versus anticipatory.** In a dynamic, strictly causal (resp. causal) system, $y(t)$ is determined by $u(\tau)$, $\tau < t$ (resp. $\tau \leq t$), but does not depend on $u(\tau)$, $\tau \geq t$ (resp. $\tau > t$). Otherwise, the system is called **noncausal** or anticipatory. For instance, the mass-spring system of Figure 1.3 is strictly causal.
5. **Lumped parameter versus distributed parameter.** In a lumped parameter system, the mathematical description is in terms of a finite number of ordinary differential equations (ODEs), whereas in a distributed parameter system, it is

Figure 1.2. Spring system.

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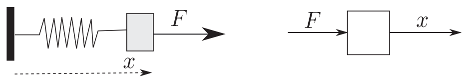

Figure 1.3. Mass-spring system.

in terms of an infinite number of ODEs or a partial differential equation. For instance, the mass-spring system of Figure 1.3 is lumped parameter.

6. **Deterministic versus stochastic.** In a stochastic system, some signals are random processes, that is, they are unpredictable but exhibit statistical regularity – these terms are defined precisely in Chapter 3. For instance, an aircraft flying through stochastic turbulence is a stochastic system.

In view of the preceding taxonomy, the systems considered in this book are mostly linear, dynamic, time varying, strictly causal, lumped parameter, and stochastic. This is the class of systems that is most relevant to the study of navigation and guidance.

1.3.2 Control Theory

A generic control system is represented in Figure 1.4, where:

1. The system P, called the **plant**, is given and is to be controlled
2. The system C, called the **controller**, is unknown and is used to control the plant
3. The input u_1, called **exogenous**, is not under the authority of the designer
4. The input u_2, called **endogenous**, is under the authority of the designer
5. The output y_1, called the **performance output**, is used to quantify the performance of the controller
6. The output y_2, called the **measured output**, is available to the designer for feedback to achieve performance

The goal of control, then, is to find a system C that causes the signal y_1 to be small despite uncertainties in the signal u_1.

REMARK 1.1 *Note that in this book, nonlinearities in dynamics do not play a significant role, which is justified in Chapter 2. For this reason, this book is separate from the literature on nonlinear dynamic systems (see, e.g., [44], [69], [41], [64]). Also note that in this book, the uncertainties are in the signals rather than the dynamics of plants. This is because, owing to sufficient instrumentation, dynamic models of aerospace systems are relatively accurate, even though the reading of sensors may be corrupted by noise. In that respect, this book is separate from the literature on adaptive control systems (see, e.g., [55], [3], [48]) and model predictive control systems (see, e.g., [16], [20], [79]).*

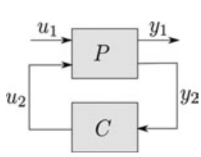

Figure 1.4. Generic control system.

1.3.3 Aerospace Applications

In general, the application of systems and control theory to the motion of aerospace vehicles may cover four areas:

1. **Path planning.** The determination of a nominal flight path for a vehicle to accomplish specified objectives subject to specified constraints. This corresponds to choosing $x_r(t)$ in (1.1).
2. **Navigation.** The estimation of the state of the vehicle, from outputs of specified imperfect sensors, and using principles of geometry and astronomy. This corresponds to computing $\hat{x}(t)$ in (1.1).
3. **Guidance.** A strategy for following the nominal flight path, given off-nominal conditions, and despite disturbances and navigation uncertainties.
4. **Attitude control.** A strategy for maintaining an angular orientation consistent with the guidance law and the constraints, despite disturbances.

Note that these four areas may overlap. For instance, guidance and attitude control interact in airplane flight where, typically, a change of trajectory requires a change of attitude (for example, in a bank-to-turn aircraft). Another instance is that of guidance and path planning, which are coupled in homing guidance: when a disturbance perturbs the pursuer's trajectory, rather than returning to the original nominal path, the pursuer plans a new path to achieve intercept.

The subject of this book is navigation and guidance, as defined earlier.

1.4 Examples

In this section, we outline how navigation and guidance are used to help accomplish a number of prototypical aerospace missions. Technical details are treated in subsequent chapters; the purpose here is simply to give the reader an appreciation of the importance of navigation and guidance throughout the spectrum of aerospace missions.

1.4.1 Transoceanic Jetliner Flight

In a modern jetliner, navigation is typically accomplished using the Global Positioning System (GPS) and other electronic aids. Path planning and guidance are typically accomplished by specifying a sequence of waypoints over which the aircraft is to fly. Path planning, navigation, guidance, and control are typically integrated through the Flight Management System (FMS), with an ergonomic display for use by the pilots, such as in Figure 1.5.

In such missions, navigation errors may, for instance, take the jetliner over forbidden territory, with tragic results. Guidance errors may, for instance, cause a landing aircraft to overshoot or undershoot the runway, also with tragic results.

1.4.2 Intelligence, Surveillance, and Reconnaissance with Unmanned Aerial Vehicle

A large proportion of military unmanned aerial vehicle (UAV) sorties are tasked with collecting information through intelligence, surveillance, and reconnaissance

Figure 1.5. A control display unit (CDU) used to control the first flight management system (FMS), UNS-1. Image courtesy of FAA Instrument Flying Handbook, FAA-H-8083-15B.

(ISR) missions [59]. In these missions, navigation is typically accomplished by combining GPS estimates, inertial navigation system (INS) estimates, and radar telemetry. Path planning and guidance are typically accomplished by a human remote-operator specifying a sequence of waypoints over which the UAV is to fly. Path planning, navigation, guidance, and control are typically integrated in a command center, with ergonomic displays for remote-operators, such as in Figure 1.6.

In such missions, navigation errors may compromise the UAV, especially when combined with a loss of radio link. Of special concern in adversarial situations is the possibility of GPS spoofing that, through a maliciously generated navigation error, may deliver an autonomous UAV to the enemy [22]. Guidance errors may also occur and jeopardize the UAV.

1.4.3 Homing Guidance of Heat-Seeking Missile

Homing missiles are capable of intercepting a target despite its maneuvers, as follows. Some missiles carry a heat seeker, such as in Figure 1.7, a device that can determine the line of sight from the missile to a heat source, for example, the exhaust of an enemy aircraft engine. An effective way to guide a homing missile is then to steer it so that its turn rate is proportional to the turn rate of the line of sight. Hence, in this situation, navigation consists of estimating the turn rate of the line of sight, and guidance consists of steering the missile so that its turn rate is proportional to that of the line of sight. This process can be automated through an autopilot such as in Figure 1.8.

Figure 1.6. General Atomics MQ-1 Predator control station. Image courtesy of U.S. Air Force, Master Sgt. Steve Horton.

Figure 1.7. U.S. Marine Corps Lance Cpl. Leander Pickens arms an AIM-9 Sidewinder missile on a FA-18C Hornet. Image courtesy of U.S. Navy ID 980220-N-0507F-003.

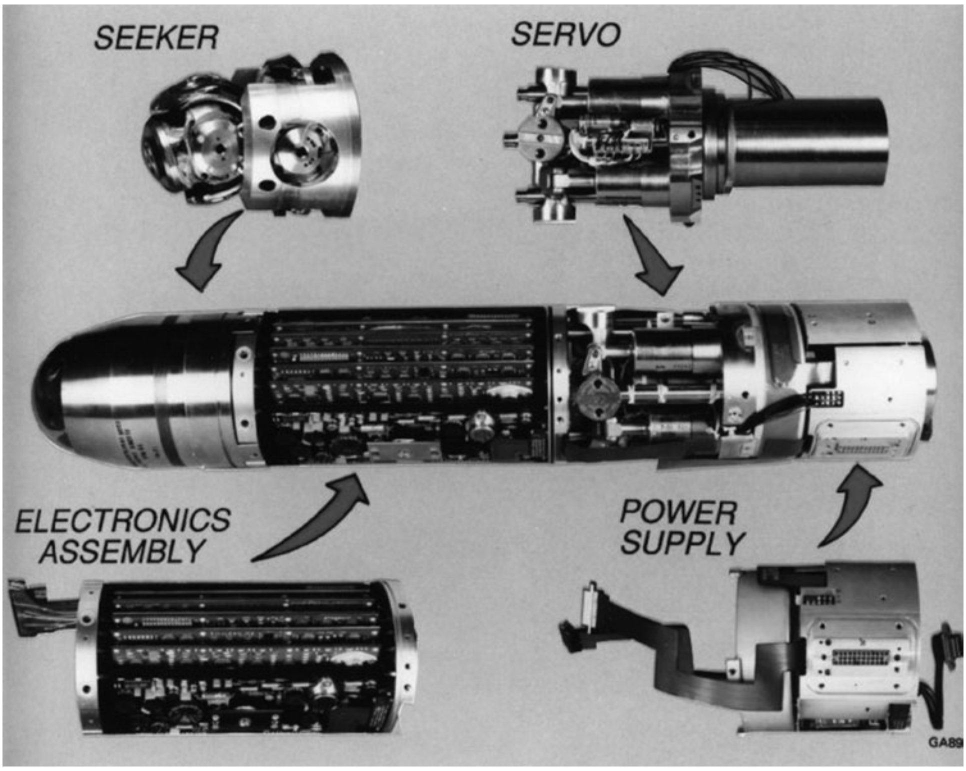

Figure 1.8. AIM-9R Sidewinder subsystems. The autopilot is in the Electronics Assembly. Image courtesy of the U.S. Navy.

For homing missiles, navigation and guidance errors may cause the missile to miss the target, with possible collateral damage. The problem is aggravated when the missile autopilot has a sluggish dynamic response.

1.4.4 Spacecraft Orbital Maneuvers

For spacecraft orbiting Earth, navigation is typically accomplished using radar telemetry from Earth-based tracking stations, such as in Figure 1.9. The outcome of navigation is the orbit determination, that is, the computation of the six orbital elements of the spacecraft. Note that the particular mission of each spacecraft typically puts stringent requirements on its orbit, for example, geosynchronous or Sun synchronous [18]. Guidance is typically achieved through changes of orbit via impulsive burns, as in Figure 1.10. Navigation and guidance are typically integrated and monitored in mission control centers, such as in Figure 1.11.

In space missions, both navigation and guidance errors may leave the spacecraft on an orbit that is useless for the mission at hand.

1.4.5 Interplanetary Travel

For interplanetary travel, navigation is typically accomplished using measurements of angles between lines of sight to bright heavenly objects and radar telemetry from

Figure 1.9. Tracking and Data Relay Satellite System (TDRSS) – Guam remote site. Image courtesy of NASA.

Earth-based tracking stations, as in Figure 1.12. Here also the outcome of navigation is the orbit determination, guidance consists of orbit changes via impulsive burns, and these activities are integrated in mission control centers.

The "final frontier" of interplanetary travel is fraught with significant technical challenges. Here also, both navigation and guidance errors may doom the spacecraft and the mission.

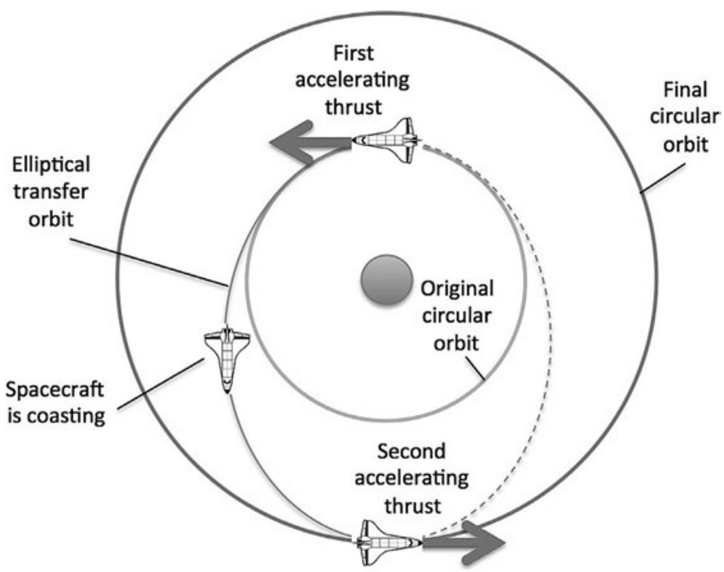

Figure 1.10. Orbital maneuver – Hohmann transfer orbit.