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Carlo Rovelli , Francesca Vidotto
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PART I

FOUNDATIONS

1 Spacetime as a quantum object

This book introduces the reader to a theory of quantum gravity. The theory is covariant loop quantum gravity (covariant LQG). It is a theory that has grown historically via a long, indirect path, briefly summarized at the end of this chapter. The book does not follow the historical path. Rather, it is pedagogical, taking the reader through the steps needed to learn the theory.

The theory is still tentative for two reasons. First, some questions about its consistency remain open; these will be discussed later in the book. Second, a scientific theory must pass the test of experience before becoming a reliable description of a domain of the world; no direct empirical corroboration of the theory is available yet. The book is written in the hope that some of you, our readers, will be able to fill these gaps.

This first chapter clarifies what is the problem addressed by the theory and gives a simple and sketchy derivation of the core physical content of the theory, including its general consequences.

1.1 The problem

After the detection at CERN of a particle that appears to match the expected properties of the Higgs [ATLAS Collaboration (2012); CMS Collaboration (2012)], the demarcation line separating what we know about the elementary physical world from what we do not know is now traced in a particularly clear-cut way. What we know is encapsulated into three major theories:

- *Quantum mechanics*, which is the general theoretical framework for describing dynamics
- *The* SU(3)×SU(2)×U(1) *standard model of particle physics*, which describes all matter we have so far observed directly, with its non-gravitational interactions
- *General relativity* (GR), which describes gravity, space, and time.

In spite of the decades-long continuous expectation of violations of these theories, in spite of the initial implausibility of many of their predictions (long-distance entanglement, fundamental scalar particles, expansion of the universe, black holes, . . .), and in spite of the bad press suffered by the standard model, often put down as an incoherent patchwork, so far Nature has steadily continued to say “Yes” to *all* predictions of these theories and “No” to *all* predictions of alternative theories (proton decay, signatures of extra dimensions, supersymmetric particles, new short-range forces, black holes at LHC, . . .). Anything beyond

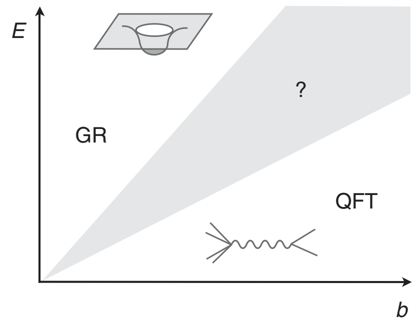

Figure 1.1 Regimes for the gravitational scattering of neutral particles in Planck units $c = \hbar = G = 1$. E is the energy in the center of the mass reference system and b the impact parameter (how close to one another the two particles come). At low energy, effective quantum field theory (QFT) is sufficient to predict the scattering amplitude. At high energy, classical general relativity (GR) is generally sufficient. In (at least parts of) the intermediate region (gray wedge) we do not have any predictive theory.

these theories is speculative. It is good to try and to dream: all good theories were attempts and dreams, before becoming credible. But lots of attempts and dreams go nowhere. The success of the above package of theories has gone far beyond anybody's expectation, and should be taken at its face value.

These theories are not the final story about the elementary world, of course. Among the open problems, three stand out:

- Dark matter
- Unification
- Quantum gravity.

These are problems of very different kind.[1] The first of these[2] is due to converging elements of empirical evidence indicating that about 85% of the galactic and cosmological matter is likely *not* to be of the kind described by the standard model. Many tentative alternative explanations are on the table, so far none convincing (Bertone 2010). The second is the old hope of reducing the number of free parameter and independent elements in our elementary description of Nature. The third, quantum gravity, is the problem we discuss here. It is not necessarily related to the first two.

The problem of quantum gravity is simply the fact that the current theories are not capable of describing the quantum behavior of the gravitational field. Because of this, we lack a predictive theory capable of describing phenomena where both gravity and quantum theory play a role. Examples are the center of a black hole, very early cosmology, the structure of Nature at very short scale, or simply the scattering amplitude of two neutral particles at small impact parameter and high energy. See Figure 1.1.

[1] To these one can add the problem of the interpretation of quantum mechanics, which is probably of still another kind.
[2] Not to be confused with the improperly called "dark energy mystery," much less of a mystery than usually advertised (Bianchi and Rovelli 2010a,b).

Observational technology has recently began to reach and probe some aspects of this regime, for instance its Lorentz invariance, and has already empirically ruled out some tentative theoretical ideas (Liberati and Maccione 2009). This is a major advance from a few years ago, when the quantum-gravitational domain appeared completely unreachable by our observation. But for the moment direct empirical information on this regime is minimal. This would be a problem if we had *many* alternative complete theories of quantum gravity to select from. But we are not in this situation: we have very few, if any. We are not at all in a situation of excessive theoretical freedom: the shortcoming in the set of fundamental laws is strident and calls for a solution, but consistency with what we know dramatically limits our freedom – which is good, since freedom is just another word for "nothing left to lose".

The problem is even more serious: our successful theories are based on contradictory hypotheses. A good student following a general-relativity class in the morning and a quantum-field-theory class in the afternoon must think her teachers are chumps, or haven't been talking to one another for decades. They teach two totally different worlds. In the morning, spacetime is curved and everything is smooth and deterministic. In the afternoon, the world is formed by discrete quanta jumping over a flat spacetime, governed by global symmetries (Poincaré) that the morning teacher has carefully explained *not* to be features of our world.

Contradiction between empirically successful theories is not a curse: it is a terrific opportunity. Several of the major jumps ahead in physics have been the result of efforts to resolve precisely such contradictions. Newton discovered universal gravitation by combining Galileo's parabolas with Kepler's ellipses. Einstein discovered special relativity to solve the "irreconcilable" contradiction between mechanics and electrodynamics. Ten years later, he discovered that spacetime is curved in an effort to reconcile newtonian gravitation with special relativity. Notice that these and other major steps in science have been achieved virtually without any *new* empirical data. Copernicus, for instance, constructed the heliocentric model and was able to compute the distances of the planets from the Sun using only the data in the book of Ptolemy.[3]

This is precisely the situation with quantum gravity. The scarcity of direct empirical information about the Planck scale is not dramatic: Copernicus, Einstein, and, to a lesser extent, Newton, have understood something new about the world without new data – just comparing apparently contradictory successful theories. We are in the same privileged situation. We lack their stature, but we are not excused from trying hard.[4]

[3] This is not in contradiction with the fact that scientific knowledge is grounded on an empirical basis. First, a theory becomes reliable only after *new* empirical support. But also the discovery itself of a new theory is based on an empirical basis even when there are no *new* data: the empirical basis is the empirical content of the previous *theories*. The advance is obtained from the effort of finding the overall conceptual structure wherein these can be framed. The scientific enterprise is still finding theories explaining observations, also when *new* observations are not available. Copernicus and Einstein were scientists even when they did not make use of new data. (Even Newton, though obsessed by getting good and recent data, found universal gravitation essentially by merging Galileo's and Kepler's laws.) Their example shows that the common claim that there is no advance in physics without new data is patently false.

[4] And we stand on their shoulders.

1.2 The end of space and time

The reason for the difficulty, but also the source of the beauty and the fascination of the problem, is that GR is not just a theory of gravity. It is a modification of our understanding of the nature of space and time. Einstein's discovery is that spacetime and the gravitational field are the same physical entity.[5] Spacetime is a manifestation of a physical field. All fields we know exhibit quantum properties at some scale, therefore we believe space and time to have quantum properties as well.

We must thus modify our understanding of the nature of space and time, in order to take these quantum properties into account. The description of spacetime as a (pseudo-) Riemannian manifold cannot survive quantum gravity. We have to learn a new language for describing the world: a language which is neither that of standard field theory on flat spacetime, nor that of Riemannian geometry. We have to understand what quantum space and quantum time are. This is the difficult side of quantum gravity, but also the source of its beauty.

The way this was first understood is enlightening. It all started with a mistake by Lev Landau. Shortly after Heisenberg introduced his commutation relations

$$[q,p] = i\hbar \tag{1.1}$$

and the ensuing uncertainty relations, the problem on the table was extending quantum theory to the electromagnetic field. In a 1931 paper with Peierls (Landau and Peierls 1931), Landau suggested that once applied to the electromagnetic field, the uncertainty relation would imply that no component of the field at a given spacetime point could be measured with arbitrary precision. The intuition was that an arbitrarily sharp spatiotemporal localization would be in contradiction with the Heisenberg uncertainty relations.

Niels Bohr guessed immediately, and correctly, that Landau was wrong. To prove him wrong, he embarked on a research program with Léon Rosenfeld, which led to a classic paper (Bohr and Rosenfeld 1933) proving that in the quantum theory of the electromagnetic field the Heisenberg uncertainty relations *do not* prevent a single component of the field at a spacetime point from being measured with arbitrary precision.

But Landau being Landau, even his mistakes have bite. Landau, indeed, had a younger friend, Matvei Petrovich Bronstein (Gorelik and Frenkel 1994), a brilliant young Russian theoretical physicist. Bronstein repeated the Bohr–Rosenfeld analysis using the gravitational field rather than the electromagnetic field. And here, surprise, Landau's

[5] In the mathematics of Riemannian geometry one might distinguish the metric field from the manifold and identify spacetime with the second. But in the physics of general relativity this terminology is misleading, because of the peculiar gauge invariance of the theory. If by "spacetime" we denote the manifold, then, using Einstein's words, "The requirement of general covariance takes away from space and time the last remnant of physical objectivity" (Einstein 1916). A detailed discussion of this point is given in Sections 2.2 and 2.3 of Rovelli (2004).

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Figure 1.2 The last picture of Matvei Bronstein, the scientist who understood that quantum gravity affects the nature of spacetime. Matvei was arrested on the night of August 6, 1937. He was thirty. He was executed in a Leningrad prison in February 1938.

intuition turned out to be correct (Bronstein 1936a,b). If we do not disregard general relativity, quantum theory *does* prevent the measurability of the field in an arbitrarily small region.

In August 1937, Matvei Bronstein was arrested in the context of Stalin's Great Purge; he was convicted in a brief trial and executed. His fault was to believe in communism without Stalinism (Figure 1.2).

Let us give a modern and simplified version of Bronstein's argument, because it is not just the beginning, it is also the core of quantum gravity.

Say you want to measure some field value at a location x. For this, you have to mark this location. Say you want to determine it with precision L. Say you do this by having a particle at x. Since any particle is a quantum particle, there will be uncertainties Δx and Δp associated with position and momentum of the particle. To have localization determined with precision L, you want $\Delta x < L$, and since Heisenberg uncertainty gives $\Delta x > \hbar/\Delta p$, it follows that $\Delta p > \hbar/L$. The mean value of p^2 is larger than $(\Delta p)^2$, therefore $p^2 > (\hbar/L)^2$. This is a well-known consequence of Heisenberg uncertainty: sharp location requires large momentum; which is the reason why at CERN high-momentum particles are used to investigate small scales. In turn, large momentum implies large energy E. In the relativistic limit, where rest mass is negligible, $E \sim cp$. Sharp localization requires large energy.

Now let us add GR. In GR, any form of energy E acts as a gravitational mass $M \sim E/c^2$ and distorts spacetime around itself. The distortion increases when energy is concentrated, to the point that a black hole forms when a mass M is concentrated in a sphere of radius $R \sim GM/c^2$, where G is the Newton constant. If we take L arbitrarily small, to get a sharper localization, the concentrated energy will grow to the point where R becomes larger than L. But in this case the region of size L that we wanted to mark will be hidden beyond a black hole horizon, and we lose localization. Therefore we can decrease L only up to a minimum value, which clearly is reached when the horizon radius reaches L, that is when $R = L$.

Combining the relations above, we obtain that the minimal size where we can localize a quantum particle without having it hidden by its own horizon is

$$L = \frac{MG}{c^2} = \frac{EG}{c^4} = \frac{pG}{c^3} = \frac{\hbar G}{Lc^3}. \tag{1.2}$$

Solving this for L, we find that it is not possible to localize anything with a precision better than the length

$$L_{\text{Planck}} = \sqrt{\frac{\hbar G}{c^3}} \sim 10^{-33}\ \text{cm}, \tag{1.3}$$

which is called the Planck length. Well above this length scale, we can treat spacetime as a smooth space. Below, it makes no sense to talk about distance. What happens at this scale is that the quantum fluctuations of the gravitational field, namely the metric, become wide, and spacetime can no longer be viewed as a smooth manifold: anything smaller than L_{Planck} is "hidden inside its own mini-black hole."

This simple derivation is obtained by extrapolating semiclassical physics. But the conclusion is correct, and characterizes the physics of quantum spacetime.

In Bronstein's words: "Without a deep revision of classical notions it seems hardly possible to extend the quantum theory of gravity also to [the short-distance] domain" (Bronstein 1936b). Bronstein's result forces us to take seriously the connection between gravity and geometry. It shows that the Bohr–Rosenfeld argument, according to which quantum fields can be defined in arbitrary small regions of space, fails in the presence of gravity. Therefore we cannot treat the quantum gravitational field simply as a quantum field in space. The smooth metric geometry of physical space, which is the ground needed to define a standard quantum field, is itself affected by quantum theory. What we need is a genuine quantum theory of geometry.

This implies that the conventional intuition provided by quantum field theory fails for quantum gravity. The worldview where quantum fields are defined over spacetime is the common world-picture in quantum field theory, but it needs to be abandoned for quantum gravity. We need a genuinely new way of doing physics, where space and time come *after*, and not *before*, the quantum states. Space and time are semiclassical approximations to quantum configurations. The quantum states are not quantum states *on* spacetime. They are quantum states *of* spacetime. This is what loop quantum gravity provides (Figure 1.3).

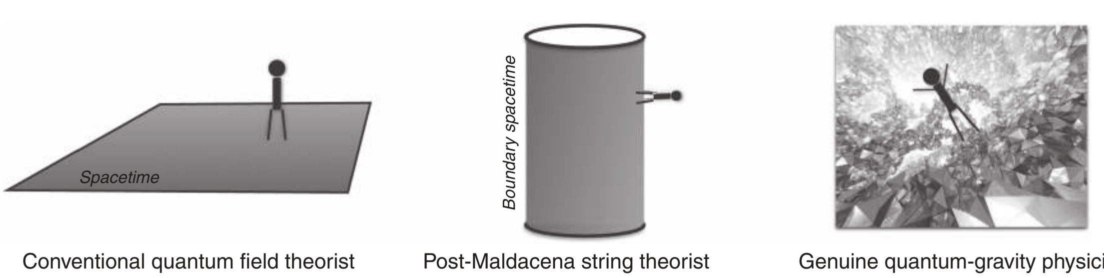

Figure 1.3 Pre-general-relativistic physics is conceived on spacetime. The recent developments of string theory, with bulk physics described in terms of a boundary theory, are a step toward the same direction. Genuine full quantum gravity requires no spacetime at all.

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1.3 Geometry quantized

The best guide we have toward quantum gravity is provided by our current quantum theory and our current gravity theory. We cannot be sure whether the basic physics on which each of these theories is grounded still applies at the Planck scale, but the history of physics teaches that vast extrapolation of good theories often works very well. The Maxwell equations, discovered with experiments in a small lab, turn out to be extremely good from nuclear to galactic scale, some 35 orders of magnitudes away, more than our distance from the Planck scale. General relativity, found at the solar system scale, appears to work remarkably well at cosmological scales, some 20 orders of magnitudes larger, and so on. In science, the best hypothesis, until something new appears empirically, is that what we know extends.

The problem, therefore, is not to *guess* what happens at the Planck scale. The problem is: is there a consistent theory that merges general relativity and quantum theory? This is the form of thinking that has been extraordinarily productive in the past. The physics of guessing, the physics of "why not try this?" is a waste of time. No great idea came from the blue sky in the past: good ideas come either from experiments or from taking seriously the empirically successful theories. Let us therefore take seriously geometry and the quantum and see, in the simplest possible terms, what a "quantum geometry" implies.

General relativity teaches us that geometry is a manifestation of the gravitational field. Geometry deals with quantities such as area, volume, length, angles, . . . These are quantities determined by the gravitational field. Quantum theory teaches us that fields have quantum properties. The problem of quantum gravity is therefore to understand what are the quantum proprieties of geometrical quantities such as area, volume, et cetera.

The quantum nature of a physical quantity is manifested in three forms:

1. In the possible discretization (or "quantization") of the quantity itself
2. In the short-scale "fuzziness" implied by the uncertainty relations
3. In the probabilistic nature of its evolution (given by the transition amplitudes).

We focus here on the first two of these (probabilistic evolution in a gravitational context is discussed in the next chapter), and consider a simple example of how they can come about, namely, how space can become discrete and fuzzy. This example is elementary and is going to leave some points out, but it is illustrative and it leads to the most characteristic aspect of loop quantum gravity: the existence of "quanta of space."

Let us start by reviewing basic quantum theory in three very elementary examples; then we describe an elementary geometrical object; and finally we see how the combination of these two languages leads directly to the quanta of space.

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Harmonic oscillator

Consider a mass m attached to a spring with elastic constant k. We describe its motion in terms of the position q, the velocity v and the momentum $p = mv$. The energy $E = \frac{1}{2}mv^2 + \frac{1}{2}kq^2$ is a positive real number and is conserved. The ”quantization postulate” from which the quantum theory follows is the existence of a Hilbert space $\mathcal{H}$ where (p,q) are non-commuting (essentially) self-adjoint operators satisfying (Born and Jordan 1925)

$$[q,p] = i\hbar. \tag{1.4}$$

This is the ”new law of nature” (Heisenberg 1925) from which discretization can be computed. These commutation relations imply that the energy operator $E(p,q) = \frac{p^2}{2m} + \frac{k}{2}q^2$ has discrete spectrum with eigenvalues ($E\psi^{(n)} = E_n\psi^{(n)}$)

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right), \tag{1.5}$$

where $\omega = \sqrt{k/m}$. That is, energy is “quantized”: it comes in discrete quanta. Since a free field is a collection of oscillators, one per mode, a quantum field is a collection of discrete quanta (Einstein 1905a). The quanta of the electromagnetic field are the photons. The quanta of Dirac fields are the particles that make up ordinary matter. We are interested in the elementary quanta of gravity.

The magic circle: discreteness is kinematics

Consider a particle moving on a circle, subject to a potential $V(\alpha)$. Let its position be an angular variable $\alpha \in S^1 \sim [0,2\pi]$ and its hamiltonian $H = \frac{p^2}{2c} + V(\alpha)$, where $p = c\,d\alpha/dt$ is the momentum and c is a constant (with dimensions ML^2). The quantum behavior of the particle is described by the Hilbert space $L_2[S^1]$ of the square integrable functions $\psi(\alpha)$ on the circle and the momentum operator is $p = -i\hbar\, d/d\alpha$. This operator has a discrete spectrum, with eigenvalues

$$p_n = n\hbar, \tag{1.6}$$

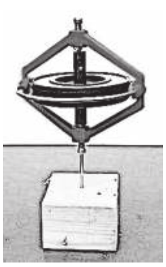
independently from the potential. We call “kinematic” the properties of a system that depend only on its basic variables, such as its coordinates and momenta, and “dynamic” the properties that depend on the hamiltonian, or, in general, on the evolution. Then it is clear that, in general, discreteness is a *kinematic* property.[6]

The discreteness of p is a direct consequence of the fact that α is in a compact domain. (The same happens for a particle in a box.) Notice that $[\alpha,p] \neq i\hbar$ because the derivative of the function α on the circle diverges at $\alpha = 0 \sim 2\pi$: indeed, α is a discontinuous function on S^1. Quantization must take into account the global topology of phase space. One of the

[6] Not so for the discreteness of the *energy*, as in the previous example, which of course depends on the form of the hamiltonian.

many ways to do so is to avoid using a discontinuous function like α and use instead a continuous function like $s = \sin(\alpha)$ or/and $c = \cos(\alpha)$. The three observables s, c, p have closed Poisson brackets $\{s, c\} = 0, \{p, s\} = c, \{p, c\} = -s$ correctly represented by the commutators of the operator $-i\hbar d/d\alpha$, and the multiplication operators $s = \sin(\alpha)$ and $c = \cos(\alpha)$. The last two operators can be combined into the complex operator $h = e^{i\alpha}$. In this sense, the correct elementary operator of this system is not α, but rather $h = e^{i\alpha}$. (We shall see that for the same reason the correct operator in quantum gravity is not the gravitational connection but rather its exponentiation along "loops." This is the first hint of the"loops" of LQG.)

Angular momentum



Let $\vec{L} = (L^1, L^2, L^3)$ be the angular momentum of a system that can rotate, with components $\{L^i\}$, with $i = 1, 2, 3$. The total angular momentum is $L = |\vec{L}| = \sqrt{L^i L^i}$ (summation on repeated indices always understood unless stated). Classical mechanics teaches us that $\vec{L}$ is the generator (in the sense of Poisson brackets) of infinitesimal rotations. Postulating that the corresponding quantum operator is also the generator of rotations in the Hilbert space, we have the quantization law (Born *et al.* 1926)

$$[L^i, L^j] = i\hbar\, \varepsilon^{ij}{}_k L^k, \tag{1.7}$$

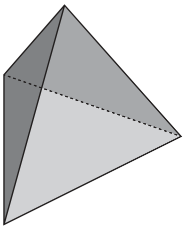
where $\varepsilon^{ij}{}_k$ is the totally antisymmetric (Levi-Civita) symbol. SU(2) representation theory (reviewed in the Complements to this chapter) then immediately gives the eigenvalues of L, if the operators $\vec{L}$ satisfy the above commutation relations. These are

$$L_j = \hbar\sqrt{j(j+1)}, \qquad j = 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \ldots \tag{1.8}$$

That is, total angular momentum is quantized. Notice that the quantization of angular momentum is a purely kinematical prediction of quantum theory: it remains the same irrespective of the form of the hamiltonian, and in particular irrespective of whether or not angular momentum is conserved. Notice also that, as for the magic circle, discreteness is a consequence of compact directions in phase space: here the space of the orientations of the body.

This is all we need from quantum theory. Let us move on to geometry.

Geometry



Pick a simple geometrical object, an elementary portion of space. Say we pick a small tetrahedron τ, not necessarily regular.

The geometry of a tetrahedron is characterized by the length of its sides, the area of its faces, its volume, the dihedral angles at its edges, the angles at the vertices of its faces, and so on. These are all local functions of the gravitational field, because geometry is the same thing as the gravitational field. These geometrical quantities are related to one