# INSTANTONS AND LARGE *N*

An Introduction to Non-Perturbative Methods in Quantum Field Theory

This highly pedagogical textbook for graduate students in particle, theoretical and mathematical physics, explores advanced topics of quantum field theory. Clearly divided into two parts, the first part focuses on instantons with a detailed exposition of instantons in quantum mechanics, supersymmetric quantum mechanics, the large order behavior of perturbation theory, and Yang–Mills theories. The second part moves on to examine the large $N$ expansion in quantum field theory. The organized presentation style, in addition to detailed mathematical derivations, worked examples and applications throughout, enables students to gain practical experience with the tools necessary to start research. The author includes recent developments on the large order behavior of perturbation theory and on large $N$ instantons, and updates existing treatments of classic topics, to ensure that this is a practical and contemporary guide for students developing their understanding of the intricacies of quantum field theory.

MARCOS MARIÑO is Professor of Theoretical Physics and Mathematics at the University of Geneva where his research interests include string theory, mathematical physics and non-perturbative methods in quantum field theory.

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MARCOS MARIÑO
*University of Geneva*

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# Preface

Quantum Field Theory (QFT) is one of the pillars of modern scientific knowledge, and it is applied in many different areas of physics, from the theory of elementary particles to condensed matter. However, most QFTs cannot be solved exactly and one has to perform some type of approximation in order to extract information from them. The standard approach is a perturbative expansion in a small coupling constant, implemented diagrammatically through Feynman diagrams. This approach has been enormously successful, but it is also insufficient to address many important phenomena which are supposed to be described by QFT.

In this book I give an introduction to two methods in QFT which go beyond the standard perturbative framework: instantons and the large $N$ expansion. Both are quite general, and they have led to many useful insights on the non-perturbative aspects of QFT. The two questions that I will address in this book are the following. What are the kinds of phenomena in a QFT in which we are fundamentally misled if we use conventional perturbation theory? What kind of intuition can we get on these effects by using instantons or large $N$ physics?

It should be said from the very beginning that the main intention of this text is to give a *pedagogical* introduction to these topics. I would like to provide a useful toolbox intended for graduate students and beginners. In line with this, I provide many computational details which are usually skipped in the original literature and in many textbooks.

The treatment of the subject pays tribute to my professional bias: I am a mathematical physicist, not a phenomenologist. Therefore, in this book, after introducing an idea or a technique, I try to illustrate it with a model or example where this idea is implemented in a nice mathematical way, independently of its relevance to measurable physics. This involves typically looking at models in a low number of dimensions (either two, one or even zero). However, I hope that some of the material in this book will be useful for people with a more phenomenological orientation. After all, toy, solvable models should be useful to everybody, and they typically provide useful insights for more realistic applications.

A word about pre-requisites: my ideal reader should have a good knowledge of QFT in the path integral formulation, as well as some working knowledge of non-Abelian gauge theories and their quantization, at the level presented for example in the QFT textbook by Peskin and Schroeder. Of course, in many sections I require less background. Chapters 1 and 3 are devoted to a large extent to advanced topics in Quantum Mechanics. On the other hand, some sections are slightly more difficult. For example, Section 2.5 requires some knowledge of General Relativity. Section 4.5 is probably the most difficult one in the book, and in particular it uses the language of differentiable manifolds (although this is not essential to understand the underlying physics).

I should also comment on the choice of topics, which might strike some experts. There are some topics which I regard as basic building blocks in the theory of instantons and of the large $N$ expansion, and they should be included in any introductory text on the subject. These include instantons in Quantum Mechanics, rudiments of instantons in Yang–Mills theory, and 't Hooft's double-line notation in QCD. The choice of the remaining topics was mostly dictated by my personal taste and/or expertise, and by trying not to duplicate presentations already available in the literature. This has led to choices which might look idiosyncratic. For example, in the discussion on Yang–Mills instantons, it is useful to work out a model with an IR cutoff where semiclassical instanton calculus makes sense. One sensible choice is to introduce a Higgs field, but I have decided instead to consider Yang–Mills theory on a compact manifold. This is not treated in textbooks and it fits better my tastes. Another constraint that shaped my choices is the simple motto "no supersymmetry." Of course, supersymmetry is a wonderful laboratory for many of the ideas presented in this book, but I felt that it should not take too much of a role in an introductory text like this. Therefore, I have only indulged in one supersymmetric excursion, in Chapter 5.

Although this text is pedagogical and elementary, I have included some material which is rarely covered in advanced QFT books, and some practitioners might find it useful. Chapter 3 is devoted to the relation between instantons and the large order behavior of perturbation theory. This is a relatively old idea, but it is not as well known as I think it should be. It paves the way to a proper understanding of exponentially small effects in quantum theories. In addition, I have tried to implement in my presentation of the subject more recent ideas coming from the theory of resurgence of Jean Écalle, which at the time of this writing is being applied in different contexts. Similarly, the last chapter, devoted to instantons in large $N$ theories, puts together observations and examples which are scattered in research papers and have not found their way into any recent textbook on advanced QFT.

Since this text is a pedagogical one, I have tried to give a self-contained presentation, and I provide derivations of almost all the results. Some technical details in long calculations are relegated to the appendices. Some of the results on the heat kernel expansion on curved space, in Appendix B, are listed without a proper derivation, but this does not affect any important results in the book.

The references are listed at the end of each chapter, in the section on bibliographical notes, and they do not pretend to be exhaustive. I have tried to give due credit to seminal contributions, and sometimes to call the attention of readers to unjustly forgotten papers.

This book can be used profitably in advanced courses on Quantum Field Theory, and I have already tested it in graduate courses in Switzerland and Italy. Although I have not included exercises, there are many worked out examples which can be proposed to the students in order to develop their skills. Some parts of the book can be skipped in a one-semester course, for example Sections 1.9, 2.5 or 4.6. Alternatively, these sections could be proposed as advanced study topics.

Some students and colleagues have suggested corrections on preliminary versions of this book. I am particularly indebted to Gerald Dunne, Martin Lüscher and Ricardo Schiappa for their detailed reading of the manuscript and their valuable suggestions.

My deepest gratitude is however to Anna Serra Picamal, who has followed the writing of this book from its inception to its completion, and in addition provided me with the perfect image for its cover. For all this, and for everything else, this book is dedicated to her.