

## STOCHASTIC ANALYSIS OF SCALING TIME SERIES

From Turbulence Theory to Applications

Multiscale systems, involving complex interacting processes that occur over a range of temporal and spatial scales, are present in a broad range of disciplines, from financial trading to atmospheric dynamics. Turbulent flows are a classical example of such a complex system. Several methodologies exist to retrieve multi-scale information from a given time series obtained from a complex dynamical system; however, each method has its own advantages and limitations.

This book presents the mathematical theory behind the stochastic analysis of scaling time series, including a general historical introduction to the problem of intermittency in turbulence, as well as how to implement this analysis for a range of different applications. Covering a variety of statistical methods, such as Fourier analysis, structure-function analysis, wavelet transforms, and Hilbert-Huang transforms, it provides readers with a more thorough understanding of each technique and when they should be applied. New techniques to analyze stochastic processes, including empirical mode decomposition, autocorrelation function of increments, and detrended analysis, are also explored.

The final part of the book contains a selection of case studies, on the topics of turbulence and ocean sciences, to demonstrate how these statistical methods can be applied in practice. With MATLAB codes available online, this book is of value to students and researchers in Earth sciences, physics, geophysics, and applied mathematics.

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From Turbulence Theory to Applications

FRANÇOIS G. SCHMITT & YONGXIANG HUANG



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## Preface

Fluid geoscience, which includes the study of the atmosphere, the oceans, hydrology, and climate, is a special topic in science: it is the application, in some sense, of the Navier-Stokes and transport equations, as well as other fluid mechanics theoretical concepts, into the “dirty real-world”. In the theoretical realm and in the laboratory, where all conditions are controlled, a desired phenomenon can be isolated and studied. In the field, however, conditions are not controlled, leading to a multitude of mixed phenomena, and measurements are the result of a multiscale complex system. In such a system, stochastic fluctuations are often superposed onto deterministic forcing associated with astronomic cycles. Scaling concepts are very important when approaching and studying geophysical fields, but the stochastic scaling properties are perturbed by forcing of, say, the day-night cycle, the annual cycle, or the tidal cycle in oceanography. In such cases, scaling methodologies may fail. In this book, we shall discuss such issues, and provide new tools that are more adapted to retrieving stochastic scaling information from different turbulence and real-world situations.

Fluid geoscience is also a special topic, in the sense that it is extremely difficult to find adequate and solid theories. The underlying fundamental equations (e.g., Navier-Stokes) are nonlinear, and turbulence belongs to the last field of classical physics, which is still unsolved. The resolution of Navier-Stokes equations belongs to the Clay Institute’s one-million-dollar problems. This explains why the field of fluid geosciences has only progressed, in some sense, so slowly. Nowadays, many models used are based on crude large-scale approximations of dynamics, based on averages of Navier-Stokes equations. These models are inaccurate and no theory really tackles the nonlinearities and intermittenencies of the natural (fluid) turbulent-like world. New theories and new models are needed. The solution may reside in generalizations and continuations of Kolmogorov’s seminal 1941 work, when a scaling law was proposed to describe velocity fluctuations at different scales. Multifractal models are continuations of these works from the 1940s, and their aim

has been to find statistical laws (such as thermodynamics in statistical physics for an equilibrium system) able to describe and model, at different scales, the fluctuations of fields of interest.

Such a general objective explains why we have devoted the first chapters of this book to historical developments on scaling and multiscaling issues in the field of turbulence and passive scalar turbulent transport. We have then followed this with an overview of several scaling methods, including our own results and proposals. After these methodological presentations, we have given the remaining chapters to emphasising applications in the fields of turbulence, and the real worlds of the ocean and the atmosphere.

Many more applications are possible, and it is with this aim, and to help other colleagues willing to develop their own approach and compare their results with other methodologies, that we have installed for free download on a dedicated website most of the Matlab codes used in this book.

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Finally, we would like to dedicate this book to our families.