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Excerpt
More Information

Part I

Basic Theory and Observations

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Introduction

1.1 What is dynamo theory?

Dynamo theory is concerned with the manner in which magnetic fields are generated and maintained in planets, stars and galaxies. The Earth, Sun and Milky Way provide examples of most immediate interest, for which a huge quantity of observational detail is now available; and yet the fundamental theory applies equally to any sufficiently large mass of electrically conducting fluid, either liquid metal or ionised gas ('plasma' when fully ionised), under the combined effects of global rotation and convective motion, this usually having a turbulent character. This turbulence may be either 'strong turbulence' of a type familiar in aerodynamics and meteorology, or 'weak turbulence' – a field of weakly interacting random waves internal to the fluid. Either way, it is the combination of rotation and convection that turns out to be particularly conducive to the spontaneous growth of magnetic fields in fluid systems of sufficient spatial extent.

It is this latter requirement that has made laboratory realisation of the self-exciting dynamo process such a great challenge for experimentalists. The triple requirements of sufficient conductivity, scale and turbulent intensity have placed huge demands on the design of experiments, and it is only over the last decade that the necessary conditions have been achieved and that self-exciting dynamo action has been convincingly demonstrated. These experimental achievements have run in parallel with great computational achievements in modelling the dynamo process both in planetary liquid cores and in stellar convection zones. Theoretical progress, so essential for a full understanding of the dynamo process, has been much stimulated by the great advances on observational, experimental and numerical fronts.

In this introductory chapter, we first set out some of the historical background, with reference to subsequent chapters where specific issues are treated in detail.

1.2 Historical background

1.2.1 The geodynamo

A very complete history of magnetism over the past millennium may be found in Stern (2002). We content ourselves here with some of the highlights of a fascinating story.[1]

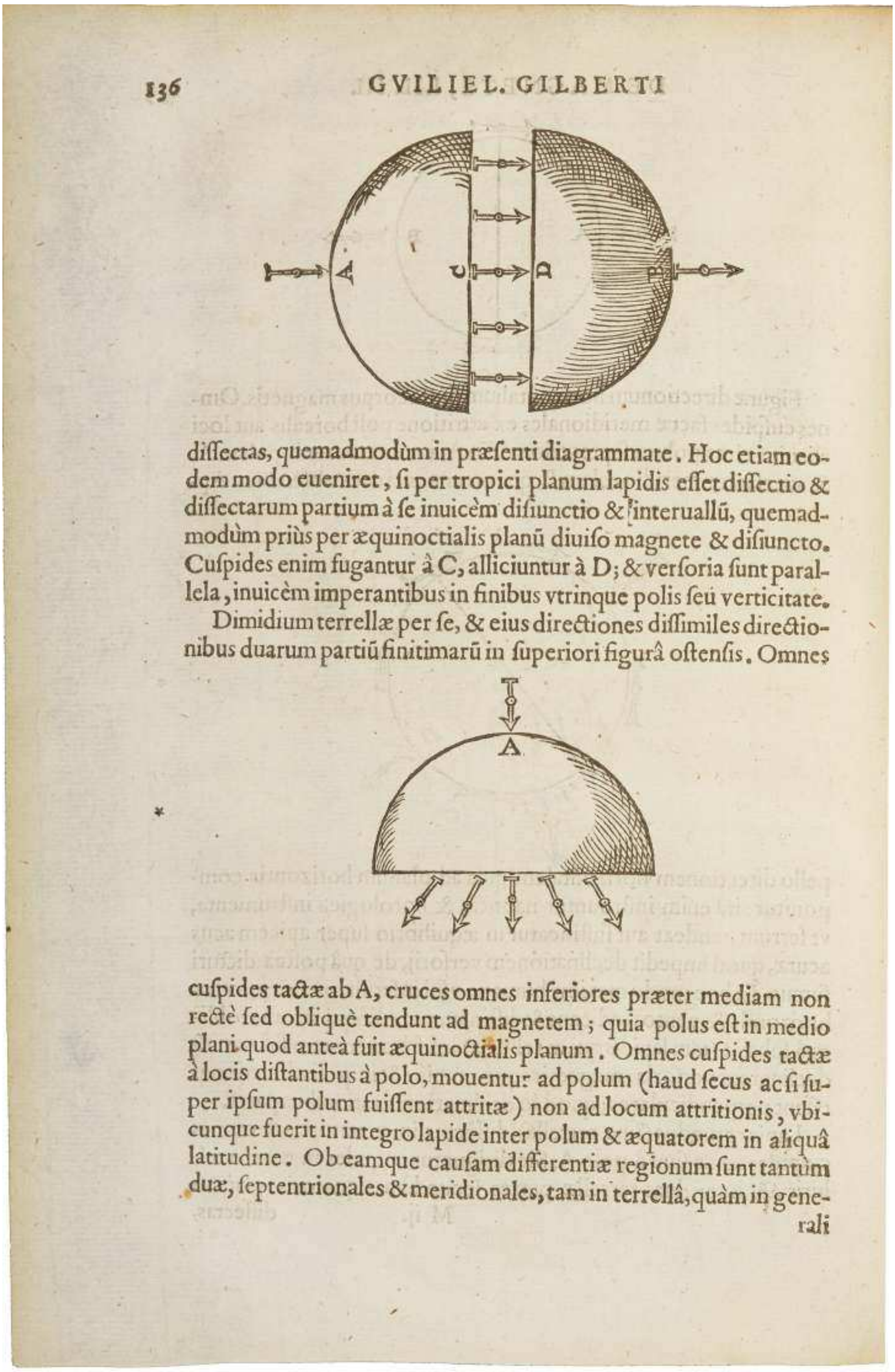
Every child who has played with a magnetic compass knows that the compass needle points North; but he learns as she grows older that magnetic North is not quite the same as 'true' North defined by the Pole star; or to put it differently, that the magnetic dipole axis is slightly inclined to the axis of rotation of the Earth. This worrying mismatch was already known to the Chinese of the Sung dynasty. In his great work *Science and Civilisation in China*, Joseph Needham (1962) quotes the *Mêng Chhi Pi Than* of Shen Kua (c.1088), which he translates thus: "Magicians rub the point of a needle with the lodestone; then it is able to point to the south. But it always inclines slightly to the east, and does not point directly at the south". So the 'declination' of the field was known, at least to the Chinese, 930 years ago. It was rediscovered and charted out by the early navigators of the fifteenth and sixteenth centuries and in particular by Christopher Columbus whose great voyage of discovery in 1492 opened new windows in the Western World. We recognise this declination now as a manifestation of a crucial departure from axisymmetry which is essential for the Earth's internal dynamo to operate.

In his seminal work *De Magnete*, William Gilbert (1600) recognised that 'magnet Earth' could be modelled by a spherical lodestone – his 'terrella' – over whose surface he was able to measure the magnetic field and plot its direction. Figure 1.1 shows a page from the second (1613) edition of this book, describing the sort of measurement that Gilbert was able to make. He spoke scathingly of earlier fanciful speculations concerning magnetism and was a pioneer of the 'scientific approach' based on careful observation and experiment.

The distinction between local magnetic north and 'true' north is often indicated on large-scale maps. The small print usually warns that the angle between the two directions changes irregularly by up to 1° in 6 years. This is the 'secular variation' of the magnetic field which was known to navigators of the seventeenth century and was no doubt a considerable nuisance to them. Edmund Halley considered the possible causes of this secular variation (Halley 1692) and concluded that

> the external parts of the globe may well be reckoned as the shell, and the internal as a nucleus or inner globe included within ours, with a fluid medium between …only this outer Sphere having its turbinating motion some small matter either swifter or slower than the inner Ball.

[1] Parts of this section are an edited version of the introduction to a Union Lecture (Moffatt 1992) delivered at the IUGG General Assembly (Vienna 1991).



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diſſectas, quemadmodùm in præſenti diagrammate. Hoc etiam eodem modo eueniret, ſi per tropici planum lapidis eſſet diſſectio & diſſectarum partium à ſe inuicèm diſiunctio & interuallũ, quemadmodùm priùs per æquinoctialis planũ diuiſo magnete & diſiuncto. Cuſpides enim fugantur à C, alliciuntur à D; & verſoria ſunt parallela, inuicèm imperantibus in finibus vtrinque polis ſeu verticitate.

Dimidium terrellæ per ſe, & eius directiones diſſimiles directionibus duarum partiũ finitimarũ in ſuperiori figurâ oſtenſis. Omnes

A

cuſpides tactæ ab A, cruces omnes inferiores præter mediam non rectè ſed obliquè tendunt ad magnetem; quia polus eſt in medio plani quod anteà fuit æquinoctialis planum. Omnes cuſpides tactæ à locis diſtantibus à polo, mouentur ad polum (haud ſecus ac ſi ſuper ipſum polum fuiſſent attritæ) non ad locum attritionis, vbicunque fuerit in integro lapide inter polum & æquatorem in aliquâ latitudine. Ob eamque cauſam differentiæ regionum ſunt tantùm duæ, ſeptentrionales & meridionales, tam in terrellâ, quàm in generali

Figure 1.1 A page from the 1613 edition of Gilbert's *De Magnete* showing the result of an experiment conducted with his 'terrella', modelling the Earth's magnetic field. [Courtesy of the Wren Library, Trinity College, Cambridge.]

This was a prophetic vision as far as the inner structure of the Earth is concerned, but also remarkable in its perception of the need for *differential* rotation, now recognised as a further key element in the dynamo process.

A discovery of great importance was made by Oersted (1820), namely that current in a wire produces a magnetic field whose field lines embrace the wire. This led Ampère (1822) to propose that an east/west current must flow within the Earth. He wrote[2]

> L'idée la plus simple, et celle qui se présenterait immédiatement à celui qui voudrait expliquer cette direction constante de l'aiguille, ne serait-elle pas d'admettre dans la terre un courant électrique, dans une direction telle que le nord se trouvât à gauche d'un homme qui, couché sur sa surface pour avoir la face tournée du côté de l'aiguille, recevrait ce courant dans la direction de ses pieds à sa tête, et d'en conclure qu'il a lieu, de l'est à l'ouest, dans une direction perpendiculaire au méridien magnétique ?

A modern understanding of the origin of these currents is based both on Ampère's law, essentially that electric current is the source of magnetic field, and on Faraday's law of induction (Faraday 1832). By painstaking experiments, Faraday discovered that if a conductor moves across a magnetic field, and if a path is available for the completion of a current circuit, then in general current will flow in that circuit. For this achievement, Faraday was awarded the Copley Medal of the Royal Society of London. The citation records that

> he gives indisputable evidence of electric action due to terrestrial magnetism alone. An important addition is thus made to the facts which have long been accumulating for the solution of that most interesting problem, the magnetism of the Earth.

It was in fact more than an important addition; it was the key ingredient of the dynamo process, although this was not recognised till much later.

At about the same time, in two great papers Carl Friedrich Gauss (1832, 1838) established the spherical harmonic decomposition of the Earth's magnetic field and the technique by which secular variation of the field could be quantified. The traditional unit of field intensity in geomagnetism, and equally in astrophysics, is of course the Gauss (G), and it is arguably regrettable that the Système International of units now favours the tesla ($1\mathrm{T} = 10^4$ G). Gauss' spherical harmonic decomposition allows us to extrapolate the Earth's field (assumed potential) down to the core–mantle boundary (CMB), to map the contours of constant radial field at the CMB, and to do so at different epochs using all available data (Bloxham et al. 1989, see Chapter 4). In these maps, the dipole ingredient of the field is still quite evident at

[2] This translates, somewhat freely, as follows: "The simplest idea that must occur immediately to anyone attempting to explain the constant direction of the compass needle is this: there must exist an electric current in the Earth, such that, if a man were to lie on the surface of the Earth with the north to his left and his face turned in the direction of the needle, he would sense this current in the direction from his feet to his head; and should we not therefore conclude that this current flows from east to west perpendicular to the magnetic meridian?"

the CMB, but there is also a strong presence of quadrupole, octupole and higher-order ingredients, as is to be expected from the nature of downward extrapolation towards the region where the 'source' currents are confined. The slow evolution of the pattern (i.e. its secular variation) is also evident.

The high point and climax of electromagnetic theory in the nineteenth century came with the publication of James Clerk Maxwell's *Treatise on Electricity and Magnetism* (Maxwell 1873). Maxwell built on Faraday's discoveries and completed the system of equations that bear his name. It is interesting to note however that in a late chapter of the treatise, devoted to *Terrestrial Magnetism*, Maxwell comes nowhere near to any explanation of the real nature of the phenomenon. He confines himself to a description of Gauss' techniques for the determination of the Earth's field and its time variation, and his demonstration that the dominant source for the field is of internal rather than external origin; but as to the root cause of the phenomenon, he writes in sonorous tones:

> The field of investigation into which we are introduced by the study of terrestrial magnetism is as profound as it is extensive. . . . What cause [is it], whether exterior to the Earth or in its inner depths, [that] produces such enormous changes in the Earth's magnetism, that its magnetic poles move slowly from one part of the globe to another? . . . These immense changes in so large a body force us to conclude that we are not yet acquainted with one of the most powerful agents in nature, the scene of whose activity lies in those inner depths of the Earth, to the knowledge of which we have so few means of access.

It was the science of seismology that was to provide the vital means of access, establishing the existence first of a liquid outer core (Jeffreys 1926, who concluded that "the central core is probably fluid, but its viscosity is uncertain"), and secondly of an inner solid core (Lehmann 1936, Bullen 1946); both inner and outer cores are believed to be important for the operation of the geodynamo.[3]

One of the earliest discussions of possible causes of terrestrial magnetism was given by Arthur Schuster (1911) in his Presidential Address to the Physical Society of London. Schuster discussed the arguments for and against a system of electric currents in the Earth's interior and concluded that "the difficulties which stand in the way of basing terrestrial magnetism on electric currents inside the Earth are insurmountable" – strong words, which have since been invalidated with the passage of time and the birth and advance of magnetohydrodynamics. Nevertheless, even as late as 1940 in their great treatise on *Geomagnetism*, Chapman & Bartels (1940) came to the same defeatist conclusion as Schuster. They discussed Larmor's (1919) suggestion concerning the possibility of self-exciting dynamo action (see below) but stated that "Cowling, however, has shown that such self-excitation

[3] An illuminating discussion of the developments leading to these discoveries is given by Brush (1980).

is not possible. Consequently, Schuster's view still holds, that the difficulties . . . are insuperable". Cowling (1934) had not in fact shown that such self-excitation is not possible: he had merely shown that it was not possible for axisymmetric systems (for Cowling's anti-dynamo theorem, see §§6.4 and 6.5), and yet the tilt of the magnetic dipole which had been known for centuries shows that we are dealing with an emphatically non-axisymmetric system. Nevertheless the fact that Chapman & Bartel could be so easily persuaded that Cowling's theorem closed the matter is an indication of the powerful influence that this theorem then had – this no doubt because it was one of the few exact results of the subject. The year 1940 marked a high point in the collection and systematisation of geomagnetic data, but it also marks the nadir as regards real understanding of the origins of terrestrial magnetism,

The post-war years saw a profound transformation in the situation, to the point at which a dynamo theory of the origin of the Earth's magnetic field is now universally accepted among geophysicists. The progress in dynamo theory has been dramatic, and the theory applies with equal force to planets other than the Earth. Statements in textbooks since the 1980s are as vigorously positive as Schuster's (1912) statement was negative. Thus, for example, Jacobs (1994) writes, "There has been much speculation on the origin of the Earth's magnetic field. . . . The only possible means seems to be some form of electromagnetic induction, electric currents flowing in the Earth's core"; and Cook (2009) writes, "There is no theory other than a dynamo theory that shows any signs of accounting for the magnetic fields of the planets". It is a dynamo theory based on the principles of magnetohydrodynamics, and ultimately on a suitable exploitation of Faraday's law of induction, that has led to this remarkable revolution in our understanding of Nature.

1.2.2 The solar dynamo

Galileo's celebrated discovery of sunspots dates back to the MDCXIII publication of his *Istoria e Dimostrazioni*. Figure 1.2 shows Galileo's representation of the sunspots that he had by then observed. This apparent 'imperfection' in God's creation caused consternation in the powerful Catholic Church of that epoch; but paradoxically, it is this very imperfection and the manner in which it has evolved over the last four centuries that has provided a prime source of information concerning the physics of the surface layers of the Sun. This will be discussed in detail in Chapter 5; for the moment, we need only note Maunder's discovery of the 11-year sunspot cycle (Maunder 1904), and Hale's discovery of the relatively strong magnetic field in sunspots (Hale 1908) and the polarity laws that govern their behaviour.

Since the 1990s, the science of 'helioseismology' (analysis of the spectrum of solar oscillations, Christensen-Dalsgaard et al. 1996) has provided a wealth of

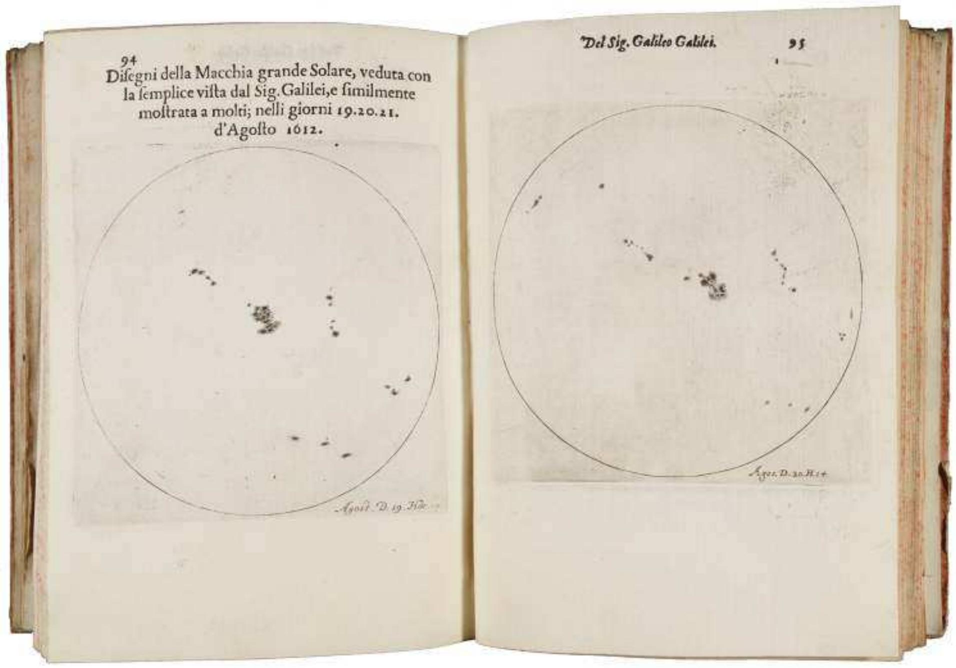

Figure 1.2 Galileo's volume *Istoria e Dimostrazioni*, showing here the record of his observations of sunspots on successive days in August 1612 (Galileo 1613). [Courtesy of the Wren Library, Trinity College, Cambridge.]

information concerning the flow field within the solar interior. In particular, through helioseismology, the differential rotation throughout most of the solar convection zone has been determined, and the presence of the 'tachocline', a layer of rapid shear at the base of the convection zone postulated by Spiegel & Zahn (1992), has been confirmed (Charbonneau et al. 1999). This in turn has led to renewed debate concerning the 'mean-field electrodynamics' applicable to the Sun through the '$\alpha\omega$-mechanism', matters that will be discussed in detail in later chapters.

The birth of solar dynamo theory proper is generally attributed to Joseph Larmor, Lucasian Professor at the University of Cambridge, who, exactly 100 years before publication of this book, posed the question "How could a rotating body such as the Sun become a magnet?" (Larmor 1919); and the question was certainly a natural one since the origin of the magnetic field of the Sun was at that time a total mystery.

And not only the Sun! We now know that a magnetic field is a normal accompaniment of any cosmic body that is both fluid (wholly or in part) and rotating. There appears to be a universal validity about this statement which applies quite irrespective of the length-scales considered. For example, on the planetary length-scale, Jupiter shares with the Earth the property of strong rotation (its rotation period being approximately 10 hours) and it is believed to have a fluid interior composed of an alloy of liquid metallic hydrogen and helium (Hide 1974); it exhibits a surface magnetic field of order 10 G in magnitude (as compared with the Earth's field of order 1 G). On the stellar length-scale, magnetic fields as weak as 1 G are hard to detect in general; there are however numerous examples of stars which rotate with periods ranging from several days to several months, and with detectable surface magnetic fields in the range 10^2 to 3×10^4 G (Preston 1967); and on the galactic length-scale, our own galaxy rotates about the normal to the plane of its disc with a period of order 3×10^8 years and exhibits a galactic-scale magnetic field roughly confined to the plane of the disc whose typical magnitude is of order 3 or 4×10^{-6} G.

The detailed character of these naturally occurring magnetic fields and the manner in which they evolve in time will be described in subsequent chapters; for the moment it is enough to note that it is the mere existence of these fields (irrespective of their detailed properties) which provides the initial motivation for the various investigations which will be described in this book.

Larmor put forward three alternative and very tentative suggestions concerning the origin of the Sun's magnetic field, only one of which has in any sense stood the test of time. This suggestion, which is fundamental to hydromagnetic dynamo theory, was that, just as for the Earth, motion of the electrically conducting fluid within the rotating body, might by its inductive action in flowing across the magnetic field generate just those currents $\mathbf{J}(\mathbf{x})$ required to provide the self-same field $\mathbf{B}(\mathbf{x})$.

1.3 The homopolar disc dynamo

This type of 'bootstrap' effect is most simply illustrated with reference to a system consisting entirely of solid (rather than fluid) conductors. This is the 'homopolar' disc dynamo (Bullard 1955) illustrated in Figure 1.3. A solid copper disc rotates about its axis with angular velocity Ω, and a current path between its rim and its axle is provided by the wire twisted as shown in a loop round the axle. This system can be unstable to the growth of magnetic perturbations. For suppose that a current $I(t)$ flows in the loop; this generates a magnetic flux Φ across the disc, and, provided the conductivity of the disc is not too high,[4] this flux is given by $\Phi = M_0 I$ where M_0 is the mutual inductance between the loop and the rim of the disc. Rotation of the disc leads to an electromotive force $\mathcal{E} = \Omega\Phi/2\pi$ which drives the current I, and the equation for $I(t)$ is then

[4] This proviso is necessary as is evident from the consideration that a superconducting disc would not allow any flux to cross its rim; a highly conducting disc in a time-dependent magnetic field tends to behave in the