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978-1-107-06392-1 - Calculus for the Ambitious
T. W. Kröner
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Calculus for the Ambitious

From the author of *The Pleasures of Counting* and *Naïve Decision Making* comes a calculus book perfect for self-study. It will open up the ideas of the calculus for any 16- to 18-year-old about to begin studies in mathematics, and will be useful for anyone who would like to see a different account of the calculus from that given in the standard texts.

In a lively and easy-to-read style, Professor Körner uses approximation and estimates in a way that will easily merge into the standard development of analysis. By using Taylor's theorem with error bounds he is able to discuss topics that are rarely covered at this introductory level. This book describes important and interesting ideas in a way that will enthuse a new generation of mathematicians.

T. W. KÖRNER is Professor of Fourier Analysis in the Department of Pure Mathematics and Mathematical Statistics at the University of Cambridge. His previous books include *The Pleasures of Counting* and *Fourier Analysis*.

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Trinity Hall, Cambridge

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A mathematics problem paper (Cambridge Scrapbook, 1859).

Bernard of Chartres used to say that we are like dwarfs on the shoulders of giants, so that we can see more than they, and things at greater distance, not by virtue of any sharpness of sight on our part, or any physical distinction, but because we are carried high and raised up by their giant size.

(*John of Salisbury* Metalogicon)

Poetry is learnt by the continual reading of the poets; painting is acquired by continual painting and designing; the art of proof, by the reading of books filled with demonstrations.

(*Galileo* Dialogue Concerning the Two Chief World Systems)

He understands ye several parts of Mathematicks . . . and which is the surest character of a true Mathematicall Genius, learned these of his owne inclination and by his owne industry without a teacher.

(*Newton* Testimonial for Edward Paget)

What one fool can do, another can.

(Ancient Simian Proverb [7])

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Introduction

Over a century ago, Silvanus P. Thompson wrote a marvellous little book [7] entitled

CALCULUS MADE EASY
Being a Very-Simplest Introduction to
Those Beautiful Methods of Reckoning
which Are Generally Called by the
Terrifying Names of the
Differential Calculus and the Integral Calculus

with the following prologue.

> Considering how many fools can calculate, it is surprising that it should be thought either a difficult or a tedious task for any other fool to learn how to master the same tricks. Some calculus-tricks are quite easy. Some are enormously difficult. The fools who write the textbooks of advanced mathematics – and they are mostly clever fools – seldom take the trouble to show you how easy the easy calculations are. On the contrary, they seem to desire to impress you with their tremendous cleverness by going about it in the most difficult way. Being myself a remarkably stupid fellow, I have had to unteach myself the difficulties, and now beg to present to my fellow fools the parts that are not hard. Master these thoroughly, and the rest will follow. What one fool can do, another can.

For a variety of reasons, the first university course in rigorous calculus is often the first course in which students meet sequences of long and subtle proofs. Sometimes the lecturer compromises and provides rigorous proofs only of the easier theorems. In my opinion, there is much to be said in favour of proving every result and much to be said in favour of proving only the hardest results, but nothing whatsoever for proving the easy results and hand-waving for the harder. The lecturer who does this resembles someone who equips themselves for tiger hunting, but only shoots rabbits.

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It is hard to learn the discipline of mathematical proof and it is hard to learn the ideas of the calculus. It seems to me, as it does to many other people, that it is possible, at least in part, to separate the two processes. Like *Calculus Made Easy*, this book is about the ideas of calculus and, although it contains a fair number of demonstrations, it contains no formal proofs.

However, Thompson wrote his book for those who use calculus as a machine for solving problems, and this book is written for those who wish, in addition, to understand how the machine works. I hope it will be found useful by able and enquiring school-children who want to see what lies ahead and by beginning mathematics students as supplementary reading. If others enjoy it or find it useful, so much the better – we can neither choose the friends of our children nor the friends of our books. Potential readers are warned that the book gets harder as it goes along and that it requires fluency in algebra.[1] It will not help you to pass exams or to discourse learnedly at the dinner table on the philosophy of the calculus. This is a book written by a professional[2] for future professionals and that is why I have called it *Calculus for the Ambitious*.

When writing this book I had in mind three sorts of users.

(1) *The desert island student.* If you are reading this book without any outside help, please remember Einstein's advice to a junior high school correspondent: 'Do not worry about your difficulties in mathematics. I can assure you mine are still greater.' If you understand everything, I shall be profoundly impressed. If you understand a great deal, I shall be delighted. If you understand something, I shall be content.

(2) *The student following another course.* I hope you find something of interest. Please remember that there are many ways of presenting the material. When my presentation clashes with your main course, either ignore the material or mentally rewrite it in accordance with that course.

(3) *The student with a helpful friend.* I hope that the spirit of Euler hovers over this book. In his autobiographical notes, he records that Johann Bernoulli refused to give him private lessons

> . . . because of his busy schedule. However, he gave me far more beneficial advice, which was to take a look at some of the more difficult mathematical books and work through them with great diligence. Should I encounter some objections or difficulties, he offered me free access to him every Saturday afternoon, and he was gracious enough to comment on the collected difficulties, which was done with such . . . advantage that, when he resolved

[1] Some of the illustrative material requires a knowledge of elementary trigonometry at the level of Exercise 1.5.3.

[2] My son claims that, when he was very young, he asked me what calculus did and I replied that it put bread, butter and jam on our table.

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> one of my objections, ten others at once disappeared, which certainly is the best method of making good progress in the mathematical sciences.

Few advisers are Bernoullis and even fewer students are Eulers, but, if you can find someone to give you occasional help in this way, there is no better way to learn mathematics.

The contents of this book do not correspond to the recommendations of any committee, have not been approved by any examination board and do not follow the syllabus of any school, university or government education department known to me. When leaving a party, Brahms is reported to have said 'If there is anyone here whom I have not offended tonight, I beg their pardon.' If any logician, historian of mathematics, numerical analyst, physicist, teacher of pedagogy or any other sort of expert picks up this book to see how I have treated their subject, I can only repeat Brahms' apology. This is an introduction and their colleagues will have plenty of opportunities to put things right later.

Since this is neither a textbook nor a reference book, I have equipped it with only a minimal bibliography and index.

Readers should join me in thanking Alison Ming for turning ill-drawn diagrams into clear figures, Tadashi Tokieda, Gareth McCaughan and three anonymous reviewers for useful comments and several Cambridge undergraduates for detecting numerous errors. In addition, I thank members of Cambridge University Press, both those known to me, like Roger Astley and David Tranah (who would have preferred the title *The Joy of dx*), and those unknown, who make publishing with the Press such a pleasant experience.

This book is dedicated to my school mathematics teachers, in particular to Mr Bone, Dr Dickinson and Mr Wynne Wilson, who went far out of their way to help a very erratic pupil. Teachers live on in the memory of their students.

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