

Nonlinear Optical Systems

Guiding graduate students and researchers through the complex world of laser physics and nonlinear optics, this book provides an in-depth exploration of the dynamics of lasers and other relevant optical systems, under the umbrella of a unitary spatio-temporal vision.

Adopting a balanced approach, the book covers traditional as well as special topics in laser physics, quantum electronics and nonlinear optics, treating them from the viewpoint of nonlinear dynamical systems. These include laser emission, frequency generation, solitons, optically bistable systems, pulsations and chaos and optical pattern formation. It also provides a coherent and up-to-date treatment of the hierarchy of nonlinear optical models and of the rich variety of phenomena they describe, helping readers to understand the limits of validity of each model and the connections among the phenomena. It is ideal for graduate students and researchers in nonlinear optics, quantum electronics, laser physics and photonics.

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Preface

The aim of our book is to provide a unified and compact vision of the tree of nonlinear optical models and of the wealth of phenomena that can be described by them. In doing that, we adopt the viewpoint of the general field of nonlinear dynamical systems, even if we keep the treatment at a certain level of simplicity, performing an in-depth analysis but avoiding all of the technicalities which are not strictly necessary.

The discussion encompasses static aspects, temporal phenomena and spatial effects, including both those arising in the longitudinal direction in which the light beam propagates and those which occur in the transverse directions. The selected material gathers and organizes a wealth of knowledge scattered in a vast literature from the sixties of the past century to our days.

The volume is subdivided into three parts of decreasing extent. The first seventeen chapters derive from the fundamental laws which govern electromagnetic radiation and matter, a variety of models that describe the radiation–matter interaction both in free propagation and in optical cavities, and discuss mainly the stationary solutions of such models. Most space is devoted to two-level systems, but attention is paid also to parametric systems and to the effects of atomic coherence in multilevel systems. Part II (Chapters 18–25) illustrates the dynamical aspects of lasers and other amplifying or absorbing systems and, in particular, the onset of instabilities that lead to phenomena of spontaneous pulsations and chaos. Part III (Chapters 26–30) deals with the phenomena which arise in the transverse section of light beams, such as Gaussian modes, spontaneous spatial pattern formation and cavity solitons.

The book combines topics that are usually considered in courses on laser physics/quantum electronics and nonlinear optics. The natural attention to the standard laser is extended to other kinds of laser, such as lasers with saturable absorber or injected signal, to other light sources as the optical parametric oscillator and to passive systems that exhibit optical bistability. The description of classic laser phenomena such as, for example, relaxation oscillations, giant pulses and mode locking, is naturally extended to the spontaneous undamped pulsations which emerge from temporal instabilities, providing a modern vision of multimodal phenomena in lasers. The same holds for the extension of the classic analysis of Gaussian beams to the spatial instabilities which produce patterns in the beam cross section. These features allow one to connect in a natural way to the vast field of the phenomena which arise in nonlinear dynamical systems in general.

The volume is addressed to students and teachers of graduate courses and to researchers in the areas of nonlinear optics, laser physics/quantum electronics/photonics and dynamics of nonlinear optical systems.

The treatment is consistently limited to the semiclassical theory but, whenever possible, we avail ourselves of the description of the radiation field in terms of photons, as happens, for example, in the description of quadratic and cubic nonlinearities.

All models treated in this book are derived, of course, from the fundamental set of Maxwell equations and the Schrödinger equation. We have taken special care in constructing a solid, coherent and logically compact building of models, with clear interconnections among them and a very linear and economical set of derivations. This point is especially important, because in the literature one meets models of very different kinds, for example models that include explicitly the field propagation, models expressed in terms of modal amplitudes, or models that include the transverse diffraction effects, and it is necessary to have a clear global picture of how all such models branch from the tree of the Maxwell–Schrödinger equations. Particular attention has been devoted to avoiding all phenomenological steps that it is possible to avoid, to describing the physical aspects of all steps with precision and to making quite clear the limits of validity of each model.

The vision provided aims at being comprehensive, but the length of the volume is kept reasonable thanks to a strict selection of the topics discussed and to the fact that for related topics we refer the reader to excellent textbooks, reviews or research articles in the literature. Unavoidably, the selection of the topics presented here is affected by our personal preferences and by the limits of our knowledge.

In this connection, it is necessary to spell out some aspects that in this book are treated only marginally or not treated at all. First, the polarization of light is included only in its simplest configuration of linear polarization, and polarization effects are not discussed. The same is true for fluctuations, not only of quantum but also of classical origin. Noise is considered only when necessary, and its effects are not discussed.

Insofar as the atoms are concerned, they are described only in terms of their internal degrees of freedom, whereas their external degrees, i.e. their motion, are taken into account only to include the collisional broadening of the atomic line or to describe inhomogeneously broadened lasers. This implies that here we do not consider the case of cold atomic systems, i.e. we assume that the atoms are sufficiently hot that their momenta are very large compared with the momentum transfer from the ponderomotive force, hence the atomic motion is unaffected and the atomic density does not change appreciably in time.

We would like to mention also not only that the material presented in this book is better organized than in the literature, but also that a few of the results are even new, being included to improve the balance and the impact of the discussion. Examples can be found in Sections 14.3, 20.3, 21.2, 21.3 and 23.1.

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