# A Course on
## Cooperative Game Theory

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# Contents

# Preface

The objectives of game theory are to model and analyze interdependent decision-making circumstances. A distinction is made in the literature between cooperative and non-cooperative games in the sense that while for the former, obligatory contracts between the participants, referred to as players, is possible, such a possibility is ruled out for the latter.

Cooperative game theory has become very influential in social sciences in the recent years. This book discusses some highly important issues in cooperative game theory with examples from economics, business and sometimes from politics. The book is divided into two parts. Part 1 is composed of Chapters 1–9. Foundations of game theory and a description of the Chapters 2–13 are presented in Chapter 1. Cooperative games with transferable utility are discussed in Chapters 2–6. Chapter 2 explains some basic concepts, definitions and preliminaries. Chapter 3 analyzes set-valued solution concepts like the core, the dominance core, stable sets and different core catchers. An extensive discussion on the relations between alternative solution concepts is also made in this chapter. Two additional set-valued solution concepts, the bargaining set and the kernel that rely on a coalition structure, are presented in Chapter 4. This chapter also discusses the nucleolus, a one-point solution concept, which has interesting relations with the bargaining set and the kernel. In Chapter 5, we consider a well-known one-point solution concept, the Shapley value. A particular type of transferable-utility cooperative game with some especially attractive properties is a convex game, which has been examined in Chapter 6. Relations between the Weber set, an alternative set-valued solution concept, the core and the Shapley value for such games are also reviewed in detail in this chapter. Chapter 7 presents a systematic analysis of voting games that often arise in interactive decision-making situations. The subject of Chapter 8 is stable matching. We discuss the Gale–Shapley basic model of matching men to women or vice-versa, the concept of stable matching, matching problems in two-sided markets, matching problems when participants from one side do not have preferences and housing exchange problems. An investigation of nontransferable utility games is carried out in Chapter 9. In particular, an analytical discussion on the well-known Nash bargaining model is covered in this chapter.

Each chapter contains at least one numerical example to illustrate a concept or a result. Applied examples are provided in the chapters to indicate real-life applications of the ideas.

The book is well-suited for graduate course in cooperative game theory. However, if desired, only the necessary minimum may be chosen depending on the background of students. For instance, some sections of Chapters 2, 3, 5, 7 and 8 can be used for an undergraduate course in cooperative game theory. The mathematical prerequisites for the book are elementary calculus, real analysis, linear algebra, probability and linear programming. If any result involves advanced mathematics, we have tried to provide a self-contained explanation of the mathematics used. Each of the Chapters 2−13 contains exercises, some of which are quite simple. The difficult exercises are meant for advanced courses.

Several sections of the book were used to teach MSc (Quantitative Economics) students at the Indian Statistical Institute over the years 1998−2013. It is a pleasure to acknowledge the comments and useful suggestions that we received from our students. We are grateful to Rana Barua, Youngsub Chun, Bhaskar Dutta, Marc Fleurbaey, Anirban Kar, Francois Maniquet, Eric S. Maskin, Herve Moulin, Suresh Muthuswami, Sonali Roy, Arunava Sen and William Thomson for their suggestions. Debasmita Basu and Srikanta Kundu (Senior Research Fellows in Economics at the Indian Statistical Institute) drew the figures for Chapter 9 and helped in preparing the bibliography of Chapters 1−9.

The second part of the book (Chapters 10−13) deals with some algorithmic issues arising from investigations into cooperative game theory. This part starts with a chapter providing a brief account of linear programming and its application to the core and the nucleolus. The second chapter provides an overview of the area of algorithms and computationally hard problems and goes on to describe the issues of computational complexity in cooperative game theory. In the third chapter, computational problems related to weighted majority games are considered and dynamic programming based algorithms are carefully developed. The fourth and the final chapter of the second part discusses the Gale−Shapley algorithm in some detail along with optimality considerations and the stable matching polytope. It is hoped that along with the material in the first part, an instructor may choose to expose students to some algorithmic issues discussed in the second part. This will lead to broader insight into the subject. We would like to thank Kishan Chand Gupta and Bhargab Maharaj for reading the initial draft of the four chapters and providing useful comments.