

CALENDRICAL CALCULATIONS

The Ultimate Edition

An invaluable resource for working programmers, as well as a fount of useful algorithmic tools for computer scientists, astronomers, and other calendar enthusiasts, the Ultimate Edition updates and expands the previous edition to achieve more accurate results and present new calendar variants. The book now includes algorithmic descriptions of nearly forty calendars: the Gregorian, ISO, Icelandic, Egyptian, Armenian, Julian, Coptic, Ethiopic, Akan, Islamic (arithmetic and astronomical forms), Saudi Arabian, Persian (arithmetic and astronomical), Bahá'í (arithmetic and astronomical), French Revolutionary (arithmetic and astronomical), Babylonian, Hebrew (arithmetic and astronomical), Samaritan, Mayan (long count, haab, and tzolkin), Aztec (xihuitl and tonalpohualli), Balinese Pawukon, Chinese, Japanese, Korean, Vietnamese, Hindu (old arithmetic and medieval astronomical, both solar and lunisolar), and Tibetan Phug-lugs. It also includes information on major holidays and on different methods of keeping time. The necessary astronomical functions have been rewritten to produce more accurate results and to include calculations of moonrise and moonset.

The authors frame the calendars of the world in a completely algorithmic form, allowing easy conversion among these calendars and the determination of secular and religious holidays. Lisp code for all the algorithms is available in machine-readable form.

Edward M. Reingold is Professor of Computer Science at the Illinois Institute of Technology.

Nachum Dershowitz is Professor of Computational Logic and Chair of Computer Science at Tel Aviv University.

About the Authors

Edward M. Reingold was born in Chicago, Illinois, in 1945. He has an undergraduate degree in mathematics from the Illinois Institute of Technology and a doctorate in computer science from Cornell University. Reingold was a faculty member in the Department of Computer Science at the University of Illinois at Urbana-Champaign from 1970–2000; he retired as a Professor Emeritus of Computer Science in December 2000 and moved to the Department of Computer Science at the Illinois Institute of Technology as professor and chair, an administrative post he held until 2006. His research interests are in theoretical computer science—especially the design and analysis of algorithms and data structures. A Fellow of the Association for Computing Machinery since 1996, Reingold has authored or coauthored more than 70 research papers and 10 books; his papers on backtrack search, the generation of combinations, weight-balanced binary trees, and the drawing of trees and graphs are considered classics. He has won awards for his undergraduate and graduate teaching. Reingold is intensely interested in calendars and their computer implementation; in addition to *Calendrical Calculations* and *Calendrical Tabulations*, he is the author and former maintainer of the calendar/diary part of GNU Emacs. In the accompanying photograph he is wearing a tie showing the twelve animal totems of the Chinese calendar.



Beyond his expertise in calendars, Nachum Dershowitz is a leading figure in software verification in general and the termination of programs in particular; he is an international authority on equational inference and term rewriting. Other areas in which he has made major contributions include program semantics, analysis of historical manuscripts, and combinatorial enumeration. Dershowitz has authored or coauthored more than 100 research papers and several books and has held visiting positions at prominent institutions around the globe. He has won numerous awards for his research and teaching, including the Herbrand Award for Distinguished Contributions to Automated Reasoning (2011) and Test-of-Time awards for the IEEE Symposium on Logic in Computer Science (2006), for the International Conference on Rewriting Techniques and Applications (2014), and for the International Conference on Automated Deduction (2015). Born in 1951, his graduate degrees in applied mathematics are from the Weizmann Institute in Israel. He is currently Professor of Computational Logic and Chair of Computer Science at Tel Aviv University and was elected to Academia Europaea in 2013.



Calendrical Calculations

THE ULTIMATE EDITION

EDWARD M. REINGOLD

Illinois Institute of Technology, Chicago

NACHUM DERSHOWITZ

Tel Aviv University, Israel



CAMBRIDGE
UNIVERSITY PRESS

Cambridge University Press
978-1-107-05762-3 — Calendrical Calculations
4th Edition
Frontmatter
[More Information](#)

CAMBRIDGE UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India

79 Anson Road, #06–04/06, Singapore 079906

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning, and research at the highest international levels of excellence.

www.cambridge.org

Information on this title: www.cambridge.org/9781107057623

DOI: 10.1017/9781107415058

© Cambridge University Press 2018

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First edition 1997

Second edition 2001

Third edition 2008

Fourth edition 2018

Printed in the United Kingdom by TJ International Ltd. Padstow Cornwall

A catalogue record for this publication is available from the British Library.

Library of Congress Cataloging-in-Publication Data

Names: Reingold, Edward M., 1945– author. | Dershowitz, Nachum, author.

Title: Calendrical calculations : the ultimate edition / Edward M. Reingold,

Illinois Institute of Technology, Chicago, Nachum Dershowitz, Tel Aviv University, Israel.

Description: Fourth edition. | Cambridge : Cambridge University Press, 2017.

Identifiers: LCCN 2017024295 | ISBN 9781107057623 (hardback)

| ISBN 9781107683167 (paperback)

Subjects: LCSH: Calendar—Mathematics.

Classification: LCC CE12 .R45 2017 | DDC 529/.3—dc23

LC record available at <https://lccn.loc.gov/2017024295>

ISBN 978-1-107-05762-3 Hardback

ISBN 978-1-107-68316-7 Paperback

Additional resources for this publication available at www.cambridge.org/calendricalcalculations

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.

לקהלות הקדש
שמסרו נפשם על קדשת השם
יהי זכרם ברוך

Cambridge University Press
978-1-107-05762-3 — Calendrical Calculations
4th Edition
Frontmatter
[More Information](#)

Contents

<i>List of Frontispieces</i>	<i>page</i> xii
<i>List of Figures</i>	xiii
<i>List of Tables</i>	xiv
<i>List of Calendar Functions</i>	xvi
<i>Abbreviations</i>	xxiv
<i>Mathematical Notations</i>	xxvi
<i>Preface</i>	xxxi
The Ultimate Edition	xxxv
Calendrical Tabulations	xxxvii
The Cambridge University Press Web Site	xxxvii
The Authors' Web Page	xxxvii
Acknowledgments	xxxviii
References	xxxix
<i>Credits</i>	xli
<i>License and Limited Warranty and Remedy</i>	xlii
1 Calendar Basics	1
1.1 Calendar Units and Taxonomy	4
1.2 Fixed Day Numbers	10
1.3 Negative Years	15
1.4 Epochs	15
1.5 Julian Day Numbers	16
1.6 Unix Time Representation	19
1.7 Mathematical Notation	20
1.8 Search	23
1.9 Dates and Lists	25
1.10 Mixed-Radix Notations	27
1.11 A Simple Calendar	29
1.12 Cycles of Days	33
1.13 Simultaneous Cycles	35
1.14 Cycles of Years	39
1.15 Approximating the Year Number	46
1.16 Warnings about the Calculations	47
References	49

I Arithmetical Calendars

2	The Gregorian Calendar	55
	2.1 Structure	55
	2.2 Implementation	59
	2.3 Alternative Formulas	63
	2.4 The Zeller Congruence	67
	2.5 Holidays	69
	References	71
3	The Julian Calendar	75
	3.1 Structure and Implementation	75
	3.2 Roman Nomenclature	77
	3.3 Roman Years	81
	3.4 Olympiads	82
	3.5 Seasons	83
	3.6 Holidays	84
	References	85
4	The Coptic and Ethiopic Calendars	89
	4.1 The Coptic Calendar	89
	4.2 The Ethiopic Calendar	91
	4.3 Holidays	92
	References	93
5	The ISO Calendar	95
	Reference	97
6	The Icelandic Calendar	99
	References	102
7	The Islamic Calendar	105
	7.1 Structure and Implementation	105
	7.2 Holidays	108
	References	109
8	The Hebrew Calendar	113
	8.1 Structure and History	114
	8.2 Implementation	119
	8.3 Inverting the Molad	125
	8.4 Holidays and Fast Days	128
	8.5 The Drift of the Hebrew Calendar	133
	8.6 Personal Days	134
	8.7 Possible Days of the Week	137
	References	139
9	The Ecclesiastical Calendars	143
	9.1 Orthodox Easter	145

9.2	Gregorian Easter	147
9.3	Astronomical Easter	150
9.4	Movable Christian Holidays	150
	References	152
10	The Old Hindu Calendars	155
10.1	Structure and History	155
10.2	The Solar Calendar	158
10.3	The Lunisolar Calendar	160
	References	166
11	The Mayan Calendars	169
11.1	The Long Count	170
11.2	The Haab and Tzolkin Calendars	171
11.3	The Aztec Calendars	177
	References	181
12	The Balinese Pawukon Calendar	185
12.1	Structure and Implementation	185
12.2	Conjunction Days	190
	References	192
13	Generic Cyclical Calendars	195
13.1	Single Cycle Calendars	195
13.2	Double Cycle Calendars	198
13.3	Summary	200

II Astronomical Calendars

14	Time and Astronomy	203
14.1	Position	204
14.2	Time	206
14.3	The Day	212
14.4	The Year	219
14.5	Astronomical Solar Calendars	226
14.6	The Month	227
14.7	Rising and Setting of the Sun and Moon	240
14.8	Times of Day	245
14.9	Lunar Crescent Visibility	249
	References	253
15	The Persian Calendar	257
15.1	Structure	257
15.2	The Astronomical Calendar	259
15.3	The Arithmetical Calendar	261
15.4	Holidays	265
	References	266

16	The Bahá'í Calendar	269
	16.1 Structure	269
	16.2 The Arithmetical Calendar	271
	16.3 The Astronomical Calendar	273
	16.4 Holidays and Observances	277
	References	278
17	The French Revolutionary Calendar	281
	17.1 The Original Form	283
	17.2 The Modified Arithmetical Form	284
	References	286
18	Astronomical Lunar Calendars	289
	18.1 The Babylonian Calendar	289
	18.2 Astronomical Easter	292
	18.3 The Observational Islamic Calendar	293
	18.4 The Classical Hebrew Calendar	297
	18.5 The Samaritan Calendar	300
	References	302
19	The Chinese Calendar	305
	19.1 Solar Terms	306
	19.2 Months	309
	19.3 Conversions to and from Fixed Dates	316
	19.4 Sexagesimal Cycle of Names	318
	19.5 Common Misconceptions	321
	19.6 Holidays	322
	19.7 Chinese Age	324
	19.8 Chinese Marriage Auguries	325
	19.9 The Japanese Calendar	326
	19.10 The Korean Calendar	327
	19.11 The Vietnamese Calendar	329
	References	330
20	The Modern Hindu Calendars	335
	20.1 Hindu Astronomy	341
	20.2 Calendars	347
	20.3 Sunrise	351
	20.4 Alternatives	354
	20.5 Astronomical Versions	358
	20.6 Holidays	362
	References	371
21	The Tibetan Calendar	375
	21.1 Calendar	375
	21.2 Holidays	379
	References	382

Coda 385

III Appendices

A Function, Parameter, and Constant Types 389

 A.1 Types 389

 A.2 Function Types 394

 A.3 Constant Types and Values 409

B Cross References for Functions and Constants 415

C Sample Data 445

 References 467

D Lisp Implementation 469

 D.1 Basics 469

 D.1.1 Lisp Preliminaries 469

 D.1.2 Basic Code 474

 D.1.3 The Egyptian and Armenian Calendars 476

 D.1.4 Cycles of Days 477

 D.1.5 Akan Calendar 478

 D.2 The Gregorian Calendar 479

 D.3 The Julian Calendar 484

 D.4 The Coptic and Ethiopic Calendars 489

 D.5 The ISO Calendar 490

 D.6 The Icelandic Calendar 491

 D.7 The Islamic Calendar 493

 D.8 The Hebrew Calendar 494

 D.9 The Ecclesiastical Calendars 502

 D.10 The Old Hindu Calendars 503

 D.11 The Mayan Calendars 505

 D.12 The Balinese Pawukon Calendar 510

 D.13 General Cyclical Calendars 513

 D.14 Time and Astronomy 513

 D.15 The Persian Calendar 535

 D.16 The Bahá'í Calendar 538

 D.17 The French Revolutionary Calendar 541

 D.18 Astronomical Lunar Calendars 543

 D.19 The Chinese Calendar 549

 D.20 The Modern Hindu Calendars 557

 D.21 The Tibetan Calendar 570

 References 572

Index 575

Envoi 617

About the Cover 618

List of Frontispieces

Page from a 1998 Iranian synagogue calendar	<i>page xxx</i>
Two pages of Scaliger's <i>De Emendatione Temporum</i>	xliv
Swedish almanac for February, 1712	54
Give us our eleven days	74
Ethiopic computus	88
Banker's calendar from 1884	94
Icelandic oak calendar wheels	98
Illustration of Mohammed instituting the lunar calendar	104
Gezer calendar	112
Finger calculation for the date of Easter	142
Stone astrolabe from India	154
Mayan New Year ceremonies	168
Balinese <i>plintangan</i>	184
Painting of Joseph Scaliger	194
Kepler's mystical harmony of the spheres	202
Arabian lunar stations	256
Shrine of the Bāb on Mount Carmel	268
<i>Vendémiaire</i> by Laurent Guyot	280
Lagash calendar month names	288
The 12 traditional Chinese calendrical animals	304
Stone slab from Andhra Pradesh with signs of the zodiac	334
Tibetan calendar carving	374
Chinese New Year greeting card	384
Page from a 1911 Turkish calendar	388
Sixteenth-century Hebrew astrolabe	414
Japanese calendar volvelles	444
Rasmus Sørnes astronomical clock number 3	468
First page of the index to Scaliger's <i>De Emendatione Temporum</i>	574
Blue and white glazed jar from the reign of Kāng Xī	616

List of Figures

1.1	Tel ‘Aroer calendar plaque	<i>page</i> 8
1.2	Meaning of a “day” in various calendars	14
2.1	A corrective term in the Gregorian calendar calculation	64
8.1	Molad of Nisan versus the actual moment of the new moon	120
8.2	First day of Passover versus the Spring equinox	134
9.1	Garrigues nomogram for the date of Easter	144
9.2	Distribution of Gregorian Easter dates	149
10.1	Old Hindu lunisolar calendar	161
11.1	Haab month signs	172
11.2	Tzolkin name signs	174
14.1	Differences in local time relative to Washington, D.C.	207
14.2	Standard time zones of the world as of January, 2017	209
14.3	Difference between DT and UT for –500 to 1600	213
14.4	Difference between DT and UT for 1620 to 2012	214
14.5	Equation of time	216
14.6	Equation of time wrapped onto a cylinder	217
14.7	Length of the year	222
14.8	Length of the synodic month	228
19.1	Possible numberings of the months on the Chinese calendar	312
19.2	Hypothetical Chinese year	314
19.3	Distribution of Chinese New Year dates	323
20.1	Modern Hindu lunisolar calendar	338
20.2	Hindu calculation of longitude	343
20.3	The traditional Hindu equation of time in 1000 C.E.	358
20.4	Tithi time differences, 1000–1001 C.E., in hours	363

List of Tables

Abbreviations	<i>page</i> xxiv
Mathematical notations	xxvi
1.1 Mean year and month lengths on various calendars	11
1.2 Epochs for various calendars	17
1.3 Functions $\delta(d)$ for use in formula (1.69)	35
1.4 Constants describing the leap-year structure of various calendars	40
3.1 Roman nomenclature	79
9.1 Comparative dates of Passover and Easter, 9–40 c.e.	151
10.1 Samvatsaras	157
10.2 Hindu solar (<i>saura</i>) months	159
12.1 Pawukon day names	186
12.2 The 210-day Balinese Pawukon calendar	191
13.1 Constants for generic arithmetic calendars	200
14.1 Arguments for solar-longitude	224
14.2 Solar longitudes and dates of equinoxes and solstices	225
14.3 Arguments for nth-new-moon	231
14.4 Arguments for nth-new-moon	231
14.5 Arguments for lunar-longitude	233
14.6 Arguments for lunar-latitude	237
14.7 Arguments for lunar-distance	239
14.8 Significance of various solar depression angles	244
15.1 Astronomical versus arithmetic Persian calendars, 1000–1800 A.P.	264
19.1 Solar terms of the Chinese year	307
20.1 Suggested correspondence of lunar stations and asterisms	340

20.2	Hindu calendar solar and lunar events, 1982	341
20.3	Hindu sine table	344
20.4	Śaka offsets for various eras	347
20.5	The cycle of <i>karaṇas</i>	370
20.6	The cycle of yogas	370
	Sample data—diurnal, Gregorian, Julian, Egyptian, Armenian	447
	Sample data—Akan, Coptic, Ethiopic, ISO, Icelandic, Islamic, Hebrew	448
	Sample data—Persian, Bahá'í, French Revolutionary, Easter	449
	Sample data—Mayan, Aztec, Balinese Pawukon, Babylonian, Samaritan	450
	Sample data—Chinese, Hindu Solar, Hindu Lunisolar, Tibetan	451
	Sample data—times of day, solar longitude, seasons	452
	Sample data—lunar times	453
	Sample holiday dates, 2000–2007	454
	Sample holiday dates, 2008–2015	455
	Sample holiday dates, 2016–2023	456
	Sample holiday dates, 2024–2031	457
	Sample holiday dates, 2032–2039	458
	Sample holiday dates, 2040–2047	459
	Sample holiday dates, 2048–2055	460
	Sample holiday dates, 2056–2063	461
	Sample holiday dates, 2064–2071	462
	Sample holiday dates, 2072–2079	463
	Sample holiday dates, 2080–2087	464
	Sample holiday dates, 2088–2095	465
	Sample holiday dates, 2096–2103	466

List of Calendar Functions

1.1	rd	<i>page</i> 12	1.57	thursday	33
1.3	jd-epoch	18	1.58	friday	33
1.4	moment-from-jd	18	1.59	saturday	33
1.5	jd-from-moment	18	1.60	day-of-week-from- fixed	33
1.6	mjd-epoch	19	1.62	kday-on-or-before	34
1.7	fixed-from-mjd	19	1.65	kday-on-or-after	34
1.8	mjd-from-fixed	19	1.66	kday-nearest	34
1.9	unix-epoch	19	1.67	kday-before	34
1.10	moment-from-unix	19	1.68	kday-after	34
1.11	unix-from-moment	19	1.76	akan-day-name	38
1.12	fixed-from-moment	20	1.77	akan-name-difference	38
1.13	fixed-from-jd	20	1.78	akan-day-name-epoch	38
1.14	jd-from-fixed	20	1.79	akan-name-from-fixed	38
1.16	sign	20	1.80	akan-day-name-on- or-before	38
1.18	time-from-moment	21			
1.37	list-of-fixed-from- moments	26			
1.40	positions-in-range	27	2.3	gregorian-epoch	58
1.43	time-from-clock	28	2.4	january	59
1.44	clock-from-moment	28	2.5	february	59
1.45	angle-from-degrees	29	2.6	march	59
1.46	egyptian-epoch	30	2.7	april	59
1.47	fixed-from-egyptian	30	2.8	may	59
1.48	alt-fixed-from- egyptian	31	2.9	june	59
1.49	egyptian-from-fixed	31	2.10	july	59
1.50	armenian-epoch	31	2.11	august	59
1.51	fixed-from-armenian	31	2.12	september	59
1.52	armenian-from-fixed	31	2.13	october	59
1.53	sunday	33	2.14	november	59
1.54	monday	33	2.15	december	59
1.55	tuesday	33	2.16	gregorian-leap-year?	59
1.56	wednesday	33	2.17	fixed-from-gregorian	60
			2.18	gregorian-new-year	60

2.19	gregorian-year-end	60	3.14	auc-year-from-julian	81
2.20	gregorian-year-range	60	3.15	olympiad-start	82
2.21	gregorian-year- from-fixed	61	3.16	julian-year-from- olympiad	82
2.23	gregorian-from-fixed	62	3.17	olympiad-from- julian-year	82
2.24	gregorian-date- difference	62	3.18	spring	83
2.25	day-number	62	3.19	summer	83
2.26	days-remaining	63	3.20	autumn	83
2.27	last-day-of- gregorian-month	63	3.21	winter	83
2.28	alt-fixed-from- gregorian	66	3.22	cycle-in-gregorian	83
2.29	alt-gregorian-from- fixed	66	3.23	julian-season-in- gregorian	84
2.30	alt-gregorian-year- from-fixed	67	3.24	julian-in-gregorian	85
2.32	independence-day	69	3.25	eastern-orthodox- christmas	85
2.33	nth-kday	69	4.1	coptic-epoch	90
2.34	first-kday	69	4.2	coptic-leap-year?	90
2.35	last-kday	69	4.3	fixed-from-coptic	90
2.36	labor-day	69	4.4	coptic-from-fixed	91
2.37	memorial-day	70	4.5	ethiopic-epoch	92
2.38	election-day	70	4.6	fixed-from-ethiopic	92
2.39	daylight-saving-start	70	4.7	ethiopic-from-fixed	92
2.40	daylight-saving-end	70	4.8	coptic-in-gregorian	93
2.41	christmas	70	4.9	coptic-christmas	93
2.42	advent	71	5.1	fixed-from-iso	96
2.43	epiphany	71	5.2	iso-from-fixed	96
2.44	unlucky-fridays-in- range	71	5.3	iso-long-year?	97
2.45	unlucky-fridays	71	6.1	icelandic-epoch	100
3.1	julian-leap-year?	75	6.2	icelandic-summer	100
3.2	julian-epoch	76	6.3	icelandic-winter	100
3.3	fixed-from-julian	76	6.4	fixed-from-icelandic	101
3.4	julian-from-fixed	77	6.5	icelandic-from-fixed	101
3.5	kalends	77	6.6	icelandic-leap-year?	101
3.6	nones	77	6.7	icelandic-month	102
3.7	ides	77	7.1	islamic-epoch	106
3.8	ides-of-month	77	7.2	islamic-leap-year?	107
3.9	nones-of-month	78	7.3	fixed-from-islamic	107
3.10	fixed-from-roman	80	7.4	islamic-from-fixed	108
3.11	roman-from-fixed	81	7.5	islamic-in-gregorian	109
3.12	year-rome-founded	81	7.6	mawlid	109
3.13	julian-year-from-auc	81			

xviii *List of Calendar Functions*

8.1	nisan	114	8.41	adda-season-in-gregorian	133
8.2	iyyar	114	8.42	hebrew-in-gregorian	133
8.3	sivan	114	8.43	hanukkah	134
8.4	tammuz	114	8.44	hebrew-birthday	135
8.5	av	115	8.45	hebrew-birthday-in-gregorian	135
8.6	elul	115	8.46	yahrzeit	136
8.7	tishri	115	8.47	yahrzeit-in-gregorian	137
8.8	marheshvan	115	8.49	shift-days	138
8.9	kislev	115	8.50	possible-hebrew-days	139
8.10	tevet	115	9.1	orthodox-easter	146
8.11	shevat	115	9.2	alt-orthodox-easter	147
8.12	adar	115	9.3	easter	148
8.13	adarii	115	9.4	pentecost	152
8.14	hebrew-leap-year?	115	10.1	hindu-epoch	156
8.15	last-month-of-hebrew-year	115	10.2	hindu-day-count	156
8.16	hebrew-sabbatical-year?	115	10.3	arya-solar-year	157
8.17	hebrew-epoch	119	10.4	arya-jovian-period	157
8.19	molad	120	10.5	jovian-year	158
8.20	hebrew-calendar-elapsed-days	121	10.6	arya-solar-month	158
8.21	hebrew-year-length-correction	122	10.7	fixed-from-old-hindu-solar	158
8.22	hebrew-new-year	122	10.8	old-hindu-solar-from-fixed	159
8.23	last-day-of-hebrew-month	122	10.9	arya-lunar-month	160
8.24	long-marheshvan?	122	10.10	arya-lunar-day	162
8.25	short-kislev?	122	10.11	old-hindu-lunar-leap-year?	163
8.26	days-in-hebrew-year	123	10.13	old-hindu-lunar-from-fixed	165
8.27	fixed-from-hebrew	123	10.14	fixed-from-old-hindu-lunar	166
8.28	hebrew-from-fixed	123	11.1	mayan-epoch	171
8.29	fixed-from-molad	126	11.2	fixed-from-mayan-long-count	171
8.30	yom-kippur	128	11.3	mayan-long-count-from-fixed	171
8.31	passover	129	11.4	mayan-haab-ordinal	173
8.32	omer	129	11.5	mayan-haab-epoch	173
8.33	purim	129	11.6	mayan-haab-from-fixed	173
8.34	ta-anit-esther	130			
8.35	tishah-be-av	130			
8.36	yom-ha-zikkaron	131			
8.37	sh-ela	131			
8.38	birkath-ha-hama	132			
8.39	samuel-season-in-gregorian	132			
8.40	alt-birkath-ha-hama	132			

11.7	mayan-haab-on-or-before	173	12.8	bali-week-from-fixed	187
11.8	mayan-tzolkin-epoch	175	12.9	bali-dasawara-from-fixed	188
11.9	mayan-tzolkin-from-fixed	175	12.10	bali-dwiwara-from-fixed	188
11.10	mayan-tzolkin-ordinal	175	12.11	bali-luang-from-fixed	188
11.11	mayan-tzolkin-on-or-before	176	12.12	bali-sangawara-from-fixed	188
11.12	mayan-year-bearer-from-fixed	176	12.13	bali-asatawara-from-fixed	189
11.13	mayan-calendar-round-on-or-before	177	12.14	bali-caturwara-from-fixed	189
11.14	aztec-correlation	177	12.15	bali-on-or-before	189
11.15	aztec-xihuitl-ordinal	178	12.16	kajeng-keliwon	190
11.16	aztec-xihuitl-correlation	178	12.17	tumpek	190
11.17	aztec-xihuitl-from-fixed	178	14.1	urbana	204
11.18	aztec-xihuitl-on-or-before	179	14.2	greenwich	204
11.19	aztec-tonalpohualli-ordinal	179	14.3	mecca	204
11.20	aztec-tonalpohualli-correlation	179	14.4	jerusalem	204
11.21	aztec-tonalpohualli-from-fixed	180	14.5	acre	205
11.22	aztec-tonalpohualli-on-or-before	180	14.6	direction	205
11.23	aztec-xiuhmopilli-from-fixed	180	14.7	arctan	205
11.24	aztec-xihuitl-tonalpohualli-on-or-before	181	14.8	zone-from-longitude	208
12.1	bali-pawukon-from-fixed	187	14.9	universal-from-local	208
12.2	bali-epoch	187	14.10	local-from-universal	208
12.3	bali-day-from-fixed	187	14.11	standard-from-universal	208
12.4	bali-triwara-from-fixed	187	14.12	universal-from-standard	208
12.5	bali-sadwara-from-fixed	187	14.13	standard-from-local	208
12.6	bali-saptawara-from-fixed	187	14.14	local-from-standard	210
12.7	bali-pancawara-from-fixed	187	14.15	ephemeris-correction	212
			14.16	dynamical-from-universal	212
			14.17	universal-from-dynamical	212
			14.18	julian-centuries	212
			14.19	j2000	212
			14.20	equation-of-time	217
			14.21	apparent-from-local	218
			14.22	local-from-apparent	218
			14.23	apparent-from-universal	218
			14.24	universal-from-apparent	218

14.25	midnight	218	14.67	topocentric-lunar- altitude	239
14.26	midday	218	14.68	approx-moment-of- depression	240
14.27	sidereal-from-moment	219	14.69	sine-offset	241
14.28	obliquity	220	14.70	moment-of-depression	241
14.29	declination	220	14.71	morning	241
14.30	right-ascension	220	14.72	dawn	241
14.31	mean-tropical-year	221	14.73	evening	242
14.32	mean-sidereal-year	221	14.74	dusk	242
14.33	solar-longitude	223	14.75	refraction	242
14.34	nutation	223	14.76	sunrise	242
14.35	aberration	223	14.77	sunset	243
14.36	solar-longitude-after	224	14.78	urbana-sunset	243
14.37	season-in-gregorian	225	14.79	cfs-alert	243
14.38	urbana-winter	225	14.80	jewish-sabbath-ends	243
14.39	precession	225	14.81	jewish-dusk	243
14.40	sidereal-solar- longitude	225	14.82	observed-lunar- altitude	243
14.41	solar-altitude	226	14.83	moonrise	245
14.42	estimate-prior- solar-longitude	227	14.84	moonset	245
14.44	mean-synodic-month	227	14.85	padua	246
14.45	nth-new-moon	230	14.86	local-zero-hour	247
14.46	new-moon-before	231	14.87	local-from-italian	247
14.47	new-moon-at-or-after	232	14.88	italian-from-local	247
14.48	lunar-longitude	232	14.89	daytime-temporal- hour	247
14.49	mean-lunar-longitude	234	14.90	nighttime- temporal-hour	248
14.50	lunar-elongation	234	14.91	standard-from-sundial	248
14.51	solar-anomaly	234	14.92	jewish-morning-end	248
14.52	lunar-anomaly	234	14.93	asr	249
14.53	moon-node	234	14.94	alt-asr	249
14.54	lunar-node	234	14.95	arc-of-light	250
14.55	sidereal-lunar- longitude	234	14.96	simple-best-view	250
14.56	lunar-phase	235	14.97	shaukat-criterion	250
14.57	lunar-phase-at-or- before	235	14.98	arc-of-vision	251
14.58	lunar-phase-at-or- after	235	14.99	bruin-best-view	251
14.59	new	236	14.100	yallop-criterion	251
14.60	first-quarter	236	14.101	lunar-semi-diameter	252
14.61	full	236	14.102	lunar-diameter	252
14.62	last-quarter	236	14.103	visible-crescent	252
14.63	lunar-latitude	236	14.104	phasis-on-or-before	252
14.64	lunar-altitude	238	14.105	phasis-on-or-after	253
14.65	lunar-distance	238			
14.66	lunar-parallax	239	15.1	persian-epoch	258

15.2	tehran	259	18.1	moonlag	290
15.3	midday-in-tehran	259	18.2	babylon	290
15.4	persian-new-year- on-or-before	259	18.3	babylonian-criterion	290
15.5	fixed-from-persian	260	18.4	babylonian-new- month-on-or-before	291
15.6	persian-from-fixed	260	18.5	babylonian-epoch	291
15.7	arithmetic-persian- leap-year?	262	18.6	babylonian-leap-year?	291
15.8	fixed-from- arithmetic-persian	262	18.7	fixed-from-babylonian	291
15.9	arithmetic-persian- year-from-fixed	263	18.8	babylonian-from-fixed	292
15.10	arithmetic-persian- from-fixed	264	18.9	astronomical-easter	293
15.11	nowruz	265	18.10	islamic-location	293
16.1	ayyam-i-ha	271	18.11	fixed-from- observational-islamic	293
16.2	bahai-epoch	271	18.12	observational- islamic-from-fixed	294
16.3	fixed-from-bahai	272	18.13	month-length	294
16.4	bahai-from-fixed	273	18.14	early-month?	295
16.5	bahai-location	274	18.15	alt-fixed-from- observational-islamic	295
16.6	bahai-sunset	274	18.16	alt-observational- islamic-from-fixed	296
16.7	astro-bahai-new- year-on-or-before	274	18.17	saudi-criterion	296
16.8	fixed-from-astro-bahai	275	18.18	saudi-new-month- on-or-before	296
16.9	astro-bahai-from-fixed	276	18.19	fixed-from-saudi- islamic	296
16.10	bahai-new-year	277	18.20	saudi-islamic-from- fixed	297
16.11	naw-ruz	277	18.21	hebrew-location	297
16.12	feast-of-ridvan	277	18.22	observational- hebrew-first-of-nisan	298
16.13	birth-of-the-bab	278	18.23	observational- hebrew-from-fixed	298
17.1	paris	283	18.24	fixed-from- observational-hebrew	298
17.2	midnight-in-paris	283	18.25	classical-passover-eve	298
17.3	french-new-year- on-or-before	283	18.26	alt-observational- hebrew-from-fixed	299
17.4	french-epoch	283	18.27	alt-fixed-from- observational-hebrew	300
17.5	fixed-from-french	284	18.28	samaritan-location	300
17.6	french-from-fixed	284	18.29	samaritan-noon	300
17.7	french-leap-year?	284	18.30	samaritan-new- moon-after	300
17.8	arithmetic-french- leap-year?	285	18.31	samaritan-new- moon-at-or-before	300
17.9	fixed-from- arithmetic-french	285			
17.10	arithmetic-french- from-fixed	286			

xxii *List of Calendar Functions*

18.32	samaritan-epoch	301	19.25	chinese-day-name- on-or-before	320
18.33	samaritan-new- year-on-or-before	301	19.26	chinese-new-year	322
18.34	fixed-from-samaritan	301	19.27	dragon-festival	324
18.35	samaritan-from-fixed	302	19.28	qing-ming	324
			19.29	chinese-age	325
19.1	current-major- solar-term	306	19.30	double-bright	325
19.2	chinese-location	306	19.31	bright	325
19.3	chinese-solar- longitude-on-or-after	308	19.32	blind	325
19.4	major-solar-term- on-or-after	308	19.33	widow	325
19.5	current-minor- solar-term	308	19.34	chinese-year- marriage-augury	326
19.6	minor-solar-term- on-or-after	309	19.35	japanese-location	326
19.7	midnight-in-china	309	19.36	korean-location	328
19.8	chinese-winter- solstice-on-or-before	309	19.37	korean-year	328
19.9	chinese-new-moon- on-or-after	310	19.38	vietnamese-location	329
19.10	chinese-new-moon- before	310	20.1	hindu-sidereal-year	336
19.11	chinese-no-major- solar-term?	313	20.2	hindu-sidereal-month	336
19.12	chinese-prior-leap- month?	313	20.3	hindu-synodic-month	336
19.13	chinese-new-year- in-sui	316	20.4	hindu-sine-table	342
19.14	chinese-new-year- on-or-before	316	20.5	hindu-sine	343
19.15	chinese-epoch	316	20.6	hindu-arcsin	344
19.16	chinese-from-fixed	317	20.7	hindu-mean-position	344
19.17	fixed-from-chinese	318	20.8	hindu-creation	344
19.18	chinese- sexagesimal-name	319	20.9	hindu-anomalistic- year	345
19.19	chinese-name- difference	319	20.10	hindu-anomalistic- month	345
19.20	chinese-year-name	320	20.11	hindu-true-position	345
19.21	chinese-month- name-epoch	320	20.12	hindu-solar-longitude	345
19.22	chinese-month-name	320	20.13	hindu-zodiac	346
19.23	chinese-day-name- epoch	320	20.14	hindu-lunar-longitude	346
19.24	chinese-day-name	320	20.15	hindu-lunar-phase	346
			20.16	hindu-lunar-day- from-moment	346
			20.17	hindu-new-moon- before	346
			20.18	hindu-calendar-year	347
			20.19	hindu-solar-era	347
			20.20	hindu-solar-from-fixed	348
			20.21	fixed-from-hindu-solar	348
			20.22	hindu-lunar-era	349
			20.23	hindu-lunar-from- fixed	349

20.24	fixed-from-hindu-lunar	351	20.48	astro-hindu-lunar-from-fixed	361
20.25	ujjain	351	20.49	fixed-from-astro-hindu-lunar	362
20.26	hindu-location	351	20.50	hindu-solar-longitude-at-or-after	364
20.27	hindu-ascensional-difference	352	20.51	mesha-samkranti	364
20.28	hindu-tropical-longitude	352	20.52	hindu-lunar-day-at-or-after	365
20.29	hindu-solar-sidereal-difference	352	20.53	hindu-lunar-new-year	366
20.30	hindu-daily-motion	353	20.54	hindu-lunar-on-or-before?	367
20.31	hindu-rising-sign	353	20.55	hindu-date-occur	367
20.32	hindu-equation-of-time	354	20.56	hindu-lunar-holiday	368
20.33	hindu-sunrise	354	20.57	diwali	368
20.34	hindu-sunset	355	20.58	hindu-tithi-occur	368
20.35	hindu-standard-from-sundial	355	20.59	hindu-lunar-event	368
20.36	hindu-fullmoon-from-fixed	356	20.60	shiva	369
20.37	fixed-from-hindu-fullmoon	356	20.61	rama	369
20.38	hindu-expunged?	356	20.62	hindu-lunar-station	369
20.39	alt-hindu-sunrise	357	20.63	karana	369
20.40	ayanamsha	359	20.64	yoga	369
20.41	sidereal-start	359	20.65	sacred-wednesdays	370
20.42	astro-hindu-sunset	360	20.66	sacred-wednesdays-in-range	371
20.43	sidereal-zodiac	360	21.1	tibetan-epoch	377
20.44	astro-hindu-calendar-year	360	21.2	tibetan-sun-equation	377
20.45	astro-hindu-solar-from-fixed	360	21.3	tibetan-moon-equation	377
20.46	fixed-from-astro-hindu-solar	361	21.4	fixed-from-tibetan	378
20.47	astro-lunar-day-from-moment	361	21.5	tibetan-from-fixed	379
			21.6	tibetan-leap-month?	381
			21.7	tibetan-leap-day?	381
			21.8	losar	381
			21.9	tibetan-new-year	381

Abbreviations

Abbreviation	Meaning	Explanation
a.d.	ante diem	prior day
A.D.	Anno Domini (= c.e.)	In the year of the Lord
A.H.	Anno Hegiræ	In the year of Mohammed's emigration to Medina
a.m.	ante meridiem	before noon
A.M.	Anno Mundi	In the year of the world since creation
	Anno Martyrum	Era of the Martyrs
A.P.	Anno Persico Anno Persarum	Persian year
A.S.	Anno Samaritanorum	Samaritan year
A.U.C.	Ab Urbe Condita	From the founding of the city of Rome
B.C.	Before Christ (= B.C.E.)	
B.C.E.	Before the Common Era (= B.C.)	
B.E.	Bahá'í Era	
C.E.	Common Era (= A.D.)	
E.E.	Ethiopic Era	
JD	Julian Day number	Elapsed days since noon on Monday, January 1, 4713 B.C.E. (Julian); sometimes J.A.D., Julian Astronomical Day
K.Y.	Kali Yuga	"Iron Age" epoch of the traditional Hindu calendar
m	meters	

continued

Abbreviation	Meaning	Explanation
MJD	Modified Julian Day number	Julian day number minus 2400000.5
p.m.	post meridiem	after noon
R.D.	Rata Die	Fixed date—elapsed days since the onset of Monday, January 1, 1 (Gregorian)
S.E.	Śaka Era	Epoch of the modern Hindu calendar
U.T.	Universal Time	Mean solar time at Greenwich, England (0° meridian), reckoned from midnight; sometimes G.M.T., Greenwich Mean Time
V.E.	Vikrama Era	Alternative epoch of the modern Hindu calendar

Mathematical Notations

Notation	Name	Meaning
$\lfloor x \rfloor$	floor	largest integer not larger than x
$\lceil x \rceil$	ceiling	smallest integer not smaller than x
$\text{round}(x)$	round	nearest integer to x , that is, $\lfloor x + 0.5 \rfloor$
$x \bmod y$	remainder	$x - y\lfloor x/y \rfloor$
$x \bmod [1 \dots y]$	adjusted remainder	y if $x \bmod y = 0$, $x \bmod y$ otherwise
$x \bmod [a \dots b]$	interval mod	x if $a = b$, $a + (x - a) \bmod (b - a)$ otherwise
$\text{gcd}(x, y)$	greatest common divisor	x if $y = 0$, $\text{gcd}(y, x \bmod y)$ otherwise
$\text{lcm}(x, y)$	least common multiple	$xy/\text{gcd}(x, y)$
$ x $	absolute value	unsigned value of x
$\text{sign}(x)$	sign	-1 when x is negative, $+1$ when x is positive, 0 when x is 0
$i^\circ j'k''$	angle	i degrees, j arc minutes, and k arc seconds
$\varphi(n)$	totient function	number of positive integers less than n and relatively prime to it
π	pi	ratio of circumference of circle to diameter
$\sin x$	sine	sine of x , given in degrees
$\cos x$	cosine	cosine of x , given in degrees
$\tan x$	tangent	tangent of x , given in degrees
$\arcsin x$	arc sine	inverse sine of x , in degrees

continued

Notation	Name	Meaning
$\arccos x$	arc cosine	inverse cosine of x , in degrees
$\arctan x$	arc tangent	inverse tangent of x , in degrees
$[a .. b]$	closed interval	all real numbers x , $a \leq x \leq b$
$(a .. b)$	open interval	all real numbers x , $a < x < b$
$[a .. b)$	half-open interval	all real numbers x , $a \leq x < b$
$(a .. b]$	half-open interval	all real numbers x , $a < x \leq b$
$\neg p$	logical negation	true when p is false and vice versa
$\sum_{i \geq k}^{p(i)} f(i)$	summation	the sum of $f(i)$ for all integers $i = k, k + 1, \dots$, continuing only as long as the condition $p(i)$ holds
$\prod_{i \geq k}^{p(i)} f(i)$	product	the product of $f(i)$ for all integers $i = k, k + 1, \dots$, continuing only as long as the condition $p(i)$ holds
$\sum f(\tilde{x}, \tilde{y}, \dots)$	summation	the sum of $f(\tilde{x}_i, \tilde{y}_i \dots)$ for all like-indexed components of the vectors $\tilde{x}, \tilde{y}, \dots$
$\text{MAX}_{\xi \geq \mu} \{\psi(\xi)\}$	maximum integer value	the largest integer $\xi = \mu, \mu + 1, \dots$ such that $\psi(\mu), \psi(\mu + 1), \dots, \psi(\xi)$ are true
$\text{MIN}_{\xi \geq \mu} \{\psi(\xi)\}$	minimum integer value	the smallest integer $\xi = \mu, \mu + 1, \dots$ such that $\psi(\xi)$ is true
$\text{MIN}_{\xi \in [\mu .. v]}^{p(\mu, v)} \{\psi(\xi)\}$	minimum value	the value ξ such that ψ is false in $[\mu .. \xi)$ and is true in $[\xi .. v]$; see equation (1.35) on page 24 for details
$f^{-1}(y, [a .. b])$	function inverse	approximate x in $[a .. b]$ such that $f(x) = y$
$\boxed{f_1 \mid f_2 \mid f_3 \mid \dots}$	record formation	the record containing fields f_1, f_2, f_3, \dots
$R_{\mathbf{f}}$	field selection	contents of field \mathbf{f} of record R
$\langle x_0, x_1, x_2, \dots \rangle$	list construction	the list containing x_0, x_1, x_2, \dots
$\langle \rangle$	empty list	a list with no elements
$L_{[i]}$	list element	the i th element of list L ; 0-based
$L_{[i..]}$	sublist	a list of the i th, $(i + 1)$ st, and so on elements of list L
$A \parallel B$	concatenation	the concatenation of lists A and B

continued

Notation	Name	Meaning
\vec{x}	vector	indexed list of elements $\langle x_0, x_1, \dots \rangle$
$\{x_0, x_1, x_2, \dots\}$	set formation	the set containing x_0, x_1, x_2, \dots
$x \in S$	set membership	the element x is a member of set S
$x \in \mathbf{Z}$	integer	the number x is an integer
$A \cap B$	set intersection	the intersection of sets A and B
$A \cup B$	set union	the union of sets A and B
$i \dots j$	range of integers	the set $\{i, i + 1, \dots, j\}$
$\langle b_1, \dots, b_k; b_{k+1}, \dots, b_n \rangle$	mixed-radix base	each position i takes values in $[0 \dots b_i)$, with units in position k
$a \xleftarrow{\text{rad}} b$	mixed-radix number	value of a in base b
$x \xrightarrow{\text{rad}} b$	mixed-radix representation	representation of x in base b
$h : m : s$	time of day	h hours, m minutes, and s seconds
$i^{\text{d}} j^{\text{h}} k^{\text{m}} l^{\text{s}}$	duration of time	i days, j hours, k minutes, and l seconds
bogus	error	invalid calendar date or time

Cambridge University Press
978-1-107-05762-3 — Calendrical Calculations
4th Edition
Frontmatter
[More Information](#)

Preface

No one has the right to speak in public before he has rehearsed what he wants to say two, three, and four times, and learned it; then he may speak . . . But if a man . . . puts it down in writing, he should revise it a thousand times, if possible.

Maimonides: *The Epistle on Martyrdom* (circa 1165)

This book has developed over a more than 30-year period during which the calendrical algorithms and our presentation of them have continually evolved. Our initial motivation was an effort by one of us (E.M.R.) to create Emacs-Lisp code that would provide calendar and diary features for GNU Emacs [15]; this version of the code included the Gregorian, Islamic, and Hebrew calendars (the Hebrew implemented by N.D.). A deluge of inquiries from around the globe soon made it clear to us that there was keen interest in an explanation that would go beyond the code itself, leading to our article [3] and encouraging us to rewrite the code completely, this time in Common Lisp [16]. The subsequent addition—by popular demand—of the Mayan and French Revolutionary calendars to GNU Emacs prompted a second article [13]. We received many hundreds of reprint requests for these articles. This response far exceeded our expectations and provided the impetus to write a book in which we could more fully address the multifaceted subject of calendars and their implementation.

The subject of calendars has always fascinated us with its cultural, historical, and mathematical wealth, and we have occasionally employed calendars as accessible examples in introductory programming courses. Once the book's plan took shape, our curiosity turned into obsession. We began by extending our programs to include other calendars such as the Chinese, Coptic, modern Hindu, and arithmetic Persian. Then, of course, the code for these newly added calendars needed to be rewritten, in some cases several times, to bring it up to the standards of the earlier material. We have long since lost track of the number of revisions, and, needless to say, we could undoubtedly devote another decade to polishing what we have, tracking down minutiae, and implementing and refining additional interesting calendars. As much as we might be tempted to, circumstances do not allow us to follow Maimonides' dictum quoted above.

In this book we give a unified algorithmic presentation for more than three dozen calendars of current and historical interest: the Gregorian (current civil), ISO (International Organization for Standardization), Icelandic, Egyptian (and nearly identical Armenian), Julian (old civil), Coptic and virtually identical Ethiopic, Akan, Islamic (Muslim), including the arithmetic, observational, and Saudi Arabian forms, modern Persian (both the astronomical and arithmetic forms), Bahá'í (both the arithmetic and astronomical forms), French Revolutionary (both the astronomical and arithmetic forms), Babylonian, Hebrew (Jewish) standard and observational, Samaritan, Mayan (long count, haab, and tzolkin) and two almost identical Aztec, Balinese Pawukon, Chinese (and nearly identical Japanese, Korean, and Vietnamese), old Hindu (solar and lunisolar), modern Hindu (solar and lunisolar, traditional and astronomical), and Tibetan. Easy conversion among these calendars is a natural outcome of the approach, as is the determination of secular and religious holidays.

Our goal in this book is twofold: to give precise descriptions of each calendar and to make accurate calendrical algorithms readily available for computer use. The complete workings of each calendar are described in prose and in mathematical/algorithmic form. Working computer programs are included in an appendix and are available on the internet (see following).

Calendrical problems are notorious for plaguing software, as shown by the following examples:

1. Since the early days of computers, when storage was at a premium, programmers—especially COBOL programmers—usually allocated only two decimal digits for the internal storage of years [10]; thus billions of dollars were spent fixing untold numbers of programs to prevent their going awry on New Year's Day of 2000 by interpreting "00" as 1900 instead of 2000. This became known as the "Y2K problem."
2. In a Reuters story dated Monday, November 6, 2006, Irene Klotz wrote:

A computer problem could force NASA to postpone next month's launch of shuttle Discovery until 2007 to avoid having the spaceship in orbit when the clock strikes midnight on New Year's Eve. The shuttle is due to take off from the Kennedy Space Center in central Florida on December 7 on a 12-day mission to continue construction of the half-built International Space Station. But if the launch is delayed for any reason beyond December 17 or 18, the flight likely would be postponed until next year, officials at the U.S. space agency said on Monday. To build in added cushion, NASA may move up the take off to December 6. "The shuttle computers were never envisioned to fly through a year-end changeover," space shuttle program manager Wayne Hale told a briefing. After the 2003 accident involving space shuttle Columbia, NASA started developing procedures to work around the computer glitch. But NASA managers still do not want to launch Discovery knowing it would be in space when the calendar rolls over to January 1, 2007.

The problem, according to Hale, is that the shuttle's computers do not reset to day one, as ground-based systems that support shuttle navigation do. Instead, after December 31, the 365th day of the year, shuttle computers figure January 1 is just day 366.

3. Poorly written calendar software in Notify Technology's code to synchronize mobile devices did not correctly handle monthly recurring events on the 29th, 30th, or 31st of the month because these dates do not occur in all months.
4. The change from daylight saving time to standard time in late 2010 (at various dates around the world) caused the failure of certain repeating iPhone alarms. The alarms failed again on January 1, 2011.
5. Many programs err in, or simply ignore, the century rule for leap years on the Gregorian calendar (every 4th year is a leap year, except for every 100th year, which is not, except for every 400th year, which is):
 - (a) The *New York Times* of March 1, 1997 reported that the New York City Taxi and Limousine Commission chose March 1, 1996, as the start date for a new, higher fare structure for cabs. Meters programmed by one company in Queens ignored the leap day and charged customers the higher rate on February 29.
 - (b) According to the *New Zealand Herald* of January 8, 1997, a computer software error at the Tiwai Point aluminum smelter at midnight on New Year's Eve caused more than A\$ 1 million of damage. The software error was the failure to consider 1996 a leap year; the same problem occurred 2 hours later at Comalco's Bell Bay smelter in Tasmania (which was 2 hours behind New Zealand). The general manager of operations for New Zealand Aluminum Smelters, David Brewer, said, "It was a complicated problem and it took quite some time to find the cause."
 - (c) Early releases of the popular spreadsheet program Lotus[®] 1-2-3[®] treated 2000 as a nonleap year—a problem that was eventually fixed. However, all releases of Lotus[®] 1-2-3[®] take 1900 as a leap year, which is a serious problem with historical data; by the time this error was recognized, the company deemed it too late to correct: "The decision was made at some point that a change now would disrupt formulas which were written to accommodate this anomaly" [17]. Excel[®], part of Microsoft Office[®], suffers from the same flaw; Microsoft acknowledges this error on its "Help and Support" web site, claiming that "the disadvantages of [correcting the problem] outweigh the advantages."
 - (d) According to Reuters (March 22, 2004), the computer display in the 2004 Pontiac Grand Prix shows the wrong day of the week because engineers overlooked the fact that 2004 is a leap year.
 - (e) Similarly, Zune[®], Microsoft's portable media player, failed (according to the *New York Times* of January 1, 2009) because the software did not treat 2008 as a leap year. In fact, Zune's code to compute the present year

from the number of days elapsed since January 1, 1980 would go into an infinite loop on the last day of *any* leap year.

- (f) Again according to the *New York Times* (March 1, 2010), Sony Playstation 3[®] code considered 2010 a leap year, an error that caused problems for gamers on March 1—some games would not load, others lost records of trophies, and online connections failed.
6. The calculation of holidays and special dates is a source of confusion:
- (a) According to the *New York Times* of January 12, 1999, for example, Microsoft Windows[®] 95, 98, and NT get the start of daylight saving time wrong for years, like 2001, in which April 1 is a Sunday; in such cases Windows has daylight saving time starting on April 8. An estimated 40 million to 50 million computers were affected, including some in hotels that were used for wake-up calls.
 - (b) Microsoft Outlook[®] 98 had the wrong date for U.S. Memorial Day in 1999, giving it as May 24, 1999, instead of May 31, 1999. It gave wrong dates for U.S. Thanksgiving Day for 1997–2000. Outlook[®] 2000 corrected the Memorial Day error, but compounded the Thanksgiving Day error by giving *two dates* for Thanksgiving for 1998–2000. Their 2015 Web App has incorrect dates for the Hebrew calendar fast days Tzom Tammuz and Tishah be-Av.
 - (c) Various programs calculate the Hebrew calendar by first determining the date of Passover using Gauss's method [6] (see [14]); this method is correct only when sufficient precision is used, and thus such an approach often leads to errors.
 - (d) Delrina Technology's 1994 Daily Planner had three days instead of two for Rosh ha-Shanah.
 - (e) Israeli daylight saving time has ended at various dates over the years, but Microsoft's Windows Vista[®] always ended it on September 2.
7. At least one modern, standard, source for calendrical matters, Parise [12], has many errors, some of which are presumably due not to sloppy editing, but to the algorithms used to produce the tables. For example, the Mayan date 8.1.19.0.0 is given incorrectly as February 14, 80 (Gregorian) on page 290; the dates given on pages 325–327 for Easter for the years 1116, 1152, and 1582 are not Sundays; the epact for 1986 on page 354 is wrongly given as 20; Chinese New Year is wrong for many years; the epoch is wrong for the Ethiopic calendar, and hence that entire table is flawed.
8. Even the Astronomical Applications Department of the U.S. Naval Observatory is not immune to calendrical errors! They gave Sunday, April 9, 2028 and Thursday, March 29, 2029 for Passover on their web site aa.usno.navy.mil/faq/docs/passover.html, instead of the correct dates Tuesday, April 11, 2028 and Saturday, March 31, 2029, respectively. The site was corrected on March 10, 2004.

Finally, the computer world is plagued with unintelligible code that seems to work by magic. Consider the following Unix script for calculating the date of Easter:

```
1 echo $* '[ddsf[1fp[too early]Pq]s@1583>@
2 ddd19%1+sg100/1+d3*4/12-sx8*5+25/5-sz5*4/1x-10-
3 sdlg11+20+lz+lx-30%d[30+]s@0>@d[[1+]s@lg11<@]s@25=@d[1+]
4 s@24=@se44le-d[30+]s@21>@dld+7%-7+
5 [March ]smd[31-[April ]sm]s@31<@psnlmPpsnlz>p]splpx' | dc
```

We want to provide transparent algorithms to replace the gobbledegook that is so common.

Our algorithms are carefully crafted, fully explained, and (in almost all cases) endogenous. They illustrate all the basic features of calendars: fidelity only to solar events (Gregorian, Persian, French), fidelity only to lunar events (Islamic), and fidelity to both solar and lunar events (Hebrew, Chinese, Hindu); intricate cycles disconnected from solar and lunar events (Mayan, Balinese); simultaneous intercalation and extraculation yielding irregular cycles of days of the month and months of the year (Hindu). We hope that in the process of reworking classical calendrical calculations and rephrasing them in the algorithmic language of the computer age we have also succeeded in affording the reader a glimpse of the beauty and individuality of diverse cultures past and present.

The Ultimate Edition

How I labored day and night for almost ten years straight composing this work. Great scholars as yourselves will understand what I have accomplished, having gathered statements that were distant and dispersed among the hills and mountains . . . For these reasons, it is appropriate for one to examine my statements, to scrutinize, and to investigate after me. The reader of this composition should not say, who am I . . . I hereby grant him my permission . . . You, in your wisdom, have done me a great favor. Likewise, anyone who finds a problem and informs me will be rendering me a favor, lest there remain any stumbling block.

Maimonides: Letter to Jonathan ben David Hakohen of Lunel (1199)

After the first edition of the book was published in 1997 we continued to gather material, polish the algorithms, and keep track of errors. Because the second edition was to be published in the year 2000, some wag at Cambridge University Press dubbed it “The Millennium Edition,” and that title got used in prepublication catalogs, creating a *fait accompli*. The Millennium Edition was a comprehensive revision of the first edition, and the third edition was a comprehensive revision of the Millennium Edition. Since the publication of the third edition we have continued to gather new material and polish existing material; this fourth edition is, once again, a comprehensive revision. We have called this “The Ultimate Edition” for several reasons. First, and foremost, we have no intention of ever producing another edition of this book (though minor changes may be made in subsequent printings). Second, because we have strived to be as comprehensive as possible, we are sanguine that we have covered all the world’s calendar types (though not,

of course, all variations). Finally, this material has undergone continuous refinement for over 30 years and diminishing returns have set in: future refinements are unlikely to yield much benefit.

In preparing this Ultimate Edition we have corrected all known errors (fortunately, only minor errors were ever reported in the third edition), added much new material, reworked and rearranged some discussions to accommodate the new material, improved the robustness of some functions, added many new references, and made an enormous number of small improvements. Among the new material the reader will find much more use of the mixed-radix notation of [9, sec. 4.1], use of the generalized modulo interval notation of [4], and presentations of Unix dates, Italian time, and the Akan, Icelandic, Saudi Arabian *Umm al-Qura* (an approximation of the Observational Islamic calendar), and Babylonian calendars; there are also expanded treatments of the observational Islamic and Hebrew calendars and brief discussions of the Samaritan and Nepalese calendars. Several of the astronomical functions of Chapter 14 have been rewritten to produce more accurate results (causing occasional changes in astronomically-based calendar computations, such as the Persian and the Chinese). We have added calculations of moonrise and moonset, as well as a function to invert the *molad* in the Hebrew calendar chapter. The sample data in Appendix C has been correspondingly updated and expanded (changes in hardware and software since the preparation of the third edition have caused minor changes in some sample values compared with that edition; the revision of what we called the “Future Bahá’í calendar” has caused significant changes to some of those sample values). Sample dates of many of the holidays we discuss have also been added. A cross reference list for the functions has been added (Appendix B) showing the dependencies among the functions. Despite requests from some readers, we have *not* added oddities such as the World Calendar [1], Star Trek’s stardate [11], Knuth’s Potrzebie calendar [8], the pataphysique calendar [7], or the Martian calendar [5]!

Algorithmically sophisticated readers of the first edition of this book could, with only slight difficulty, jump right into the descriptions of the various calendars, skipping the introductory chapter on “Calendar Basics.” With each successive edition such an omission became more difficult as various commonalities were moved to that chapter and the notations became more specialized. As much as we regret it, failing to read the introduction now may cause even a sophisticated reader bafflement in later chapters. So, for those without the patience to read the introductory chapter, we suggest at least a careful perusing of the “Mathematical Notations” table on pages xxvi–xxviii.

I determined, therefore, to attempt the reformation; I consulted the best lawyers and the most skilled astronomers, and we cooked up a bill for that purpose. But then my difficulty began: I was to bring in this bill, which was necessarily composed of law jargon and astronomical calculations, to both of which I am an utter stranger. However, it was absolutely necessary to make the House of Lords think that I knew something of the matter; and also to make them believe that they knew something themselves, which they do not. For my own part, I could just as soon have talked Celtic or Slavonian to them, as astronomy, and could have understood me full as well; so I resolved . . . to please instead of informing them. I gave them, therefore, only an historical account of calendars, from the Egyptian

down to the Gregorian, amusing them now and then with little episodes . . . They thought I was informed, because I pleased them; and many of them said, that I had made the whole story very clear to them; when, God knows, I had not even attempted it.

Letter from Philip Dormer Stanhope (Fourth Earl of Chesterfield, the man who in 1751 introduced the bill in Parliament for reforming the calendar in England) to his son, March 18, 1751 c.e. (Julian), the day of the Second Reading debate

Calendrical Tabulations

A man who possessed a calendar and could read it was an important member of the village community, certain to be widely consulted and suitably awarded.

K. Tseng: “Balinese Calendar,” *Myths & Symbols in Indonesian Art* (1991)

A companion volume by the authors, *Calendrical Tabulations*, is also available. It contains tables for easy conversion of dates and some holidays on the world’s major calendars (Gregorian, Hebrew, Islamic, Hindu, Chinese, Coptic/Ethiopic, and Persian) for the years 1900–2200. These tables were computed using the Lisp functions from Appendix B of the Millennium Edition and typeset directly from L^AT_EX output produced by driver code. Small changes made to the astronomical code in the interim can cause minor discrepancies in dates and times.¹

The Cambridge University Press Web Site

Exegi monumentum aere perennius. [I have created a monument more lasting than bronze.]

Horace: *Odes*, III, xxx

www.cambridge.org/calendricalcalculations

This web site contains links to files related to this book, including the Lisp code from Appendix D for the calendar functions and the sample data from Appendix C.

The Authors’ Web Site

The author has tried to indicate every known blemish in [2]; and he hopes that nobody will ever scrutinize any of his own writings as meticulously as he and others have examined the ALGOL report.

Donald E. Knuth: “The Remaining Trouble Spots in ALGOL 60,”
Communications of the ACM (1967)

Visit us at

www.calendarists.com

¹ The following minor errors regarding lunar phases in *Calendrical Tabulations* bear noting: First, the dust jacket uses a negative image of the calendar pages; this has the effect of interchanging the full/new moon symbols and the first quarter/last quarter symbols visible in the Gregorian calendar at the middle bottom. Second, when a lunar phase (or equinox or solstice) occurs seconds before midnight, the date is correctly indicated, but the time is rounded up to midnight and shown as 0:00 instead of 24:00. Finally, when two lunar phases occur during the same week, the times given in the right margin are in reverse order.

xxxviii Preface

Among other things, one can find errata for this book at this address. Try as we have, at least one error remains in this book.

Acknowledgments

It is traditional for the author to magnanimously accept the blame for whatever deficiencies remain. I don't. Any errors, deficiencies, or problems in this book are somebody else's fault, but I would appreciate knowing about them so as to determine who is to blame.

Steven Skiena: *The Algorithm Design Manual* (1997)

Stewart M. Clamen wrote an early version of the Mayan calendar code. Parts of Section 2.3 are based on suggestions by Michael H. Deckers. Chapters 6 and 21 are based in part on the work of Svante Janson.

Our preparation of the fourth edition was aided considerably by the help of Mark D. Bej, Uri Blass, Irvin L. Bromberg, Assaf Cohen, William P. Collins, Craig Dedo, Ben Denckla, Idan Dershowitz, Surya Prasad Dhungel, Tony Finch, Gedalya Gordon, Julian Gilbey, Eysteinn Guðni Guðnason, Peter Zilahy Ingerman, Svante Janson, Kaboel Karso, Eric Kingston, Kwasi Konadu, Stanislav Koncebovski, Kai Kuhlmann, Jonathan Leffler, Yaaqov Loewinger, Zhuo Meng, Susan Milbrath, Josua Müller, Fabrice Orgogozo, Andy Pepperdine, John Powers, Eugene Quah, Lester A. Reingold, Ruth N. Reingold, Dieter Schuh, Matthew Sheby, Enrico Spinielli, Sacha Stern, Sharad Upadhyay, Robert H. van Gent, Nadia Vidro, Steve Ward, and Alan R. White, all of whom pointed out errors, suggested improvements, and helped gather materials. Special thanks go to our copy editor Susan S. Parkinson who went carefully through every every detail of the book and provided many invaluable corrections. We also thank all those acknowledged in the prior editions for their help.

Gerald M. Browne, Sharat Chandran, Shigang Chen, Jeffrey L. Copeland, Idan Dershowitz, Nazli Goharian, Mayer Goldberg, Getatchew Haile, Shiho Inui, Yoshiyasu Ishigami, Howard Jacobson, Subhash Kak, Claude Kirchner, Sakai Kō, Jungmin Lee, Nabeel Naser El-deen, Gerhard A. Nothmann, Trần Đức Ngọc, Sigurður Örn Stefánsson, Fentahun Tiruneh, Roman Waupotitsch, Daniel Yaqob, and Afra Zomorodian helped us with various translations and foreign language fonts. Charles Hoot labored hard on the original program for automatically transforming Lisp code into arithmetic expressions and provided general expertise in Lisp. Mitchell A. Harris helped with fonts, star names, and the automatic translation; Matthew Carroll, Benita Ulisano, and Upendra Gandhi were our system support people; Marla Brownfield helped with various tables. Herbert Voss modified PSTricks several times to enable us to produce various figures. Erga Dershowitz, Idan Dershowitz, Molly Flesner, Schulamith Halevy, Deborah Klapper, Eve Kleinerman, Rachel Mandel, Ruth Reingold, Christine Mumm, and Joyce Woodworth were invaluable in proofreading tens of thousands of dates, comparing our results with published tables. We are grateful to all of them.

Portions of this book appeared, in a considerably less polished state, in our papers [3] and [13]. We thank John Wiley & Sons for allowing us to use that material here.

The second author is grateful to the Institut d'études avancées de Paris for the conducive environment it provided during the last stages of preparation of this edition.

THE END.

This work was completed on the 17th or 27th day of May, 1618; but Book v was reread (while the type was being set) on the 9th or 19th of February, 1619. At Linz, the capital of Austria—above the Enns.

Johannes Kepler: *Harmonies of the World*

I have not always executed my own scheme, or satisfied my own expectations ... [But] I look with pleasure on my book however defective and deliver it to the world with the spirit of a man that has endeavored well ... When it shall be found that much is omitted, let it not be forgotten that much likewise has been performed.

Samuel Johnson: Preface to his *Dictionary*

R.D. 736520
 Chicago, Illinois
 Tel Aviv, Israel

E.M.R.
 N.D.

References

A book without a preface is like a body without a soul.
 Hebrew proverb

- [1] The World Calendar Association, www.theworldcalendar.org.
- [2] A. Birashk, *A Comparative Calendar of the Iranian, Muslim Lunar, and Christian Eras for Three Thousand Years*, Mazda Publishers (in association with Bibliotheca Persica), Costa Mesa, CA, 1993.
- [3] N. Dershowitz and E. M. Reingold, "Calendrical Calculations," *Software—Practice and Experience*, vol. 20, no. 9, pp. 899–928, September 1990.
- [4] N. Dershowitz and E. M. Reingold, "Modulo Intervals: A Proposed Notation," *ACM SIGACT News*, vol. 43, no. 3, pp. 60–64, 2012.
- [5] N. Dershowitz and E. M. Reingold, "A Terrestrial Calendar for Mars (Abstract)," *Program Book of The Founding Convention of the Mars Society*, The University of Colorado at Boulder, pp. 117–118, 1998.
- [6] C. F. Gauss, "Berechnung des jüdischen Osterfestes," *Monatliche Correspondenz zur Beförderung der Erd- und Himmelskunde*, vol. 5 (1802), pp. 435–437. Reprinted in Gauss's *Werke*, Herausgegeben von der Königlichen Gesellschaft der Wissenschaften, Göttingen, vol. VI, pp. 80–81, 1874; republished, Georg Olms Verlag, Hildesheim, 1981.
- [7] A. Jarry, *Ubu à l'Anvers*, Rossaert, Antwerp, 1997.

- [8] D. E. Knuth, “The Potrzebie System of Weights and Measures,” *MAD Magazine*, vol. 1, no. 33, pp. 36–37, June 1957. Reprinted in Knuth’s *Selected Papers on Fun & Games*, Center for the Study of Language and Information, Stanford University, Stanford, CA, 2011.
- [9] D. E. Knuth, *The Art of Computer Programming, vol. 2: Seminumerical Algorithms*, 3rd edn., Addison-Wesley Publishing Company, Reading, MA, 1998.
- [10] P. G. Neumann, “Inside Risks: The Clock Grows at Midnight,” *Communications of the ACM*, vol. 34, no. 1, p. 170, January 1991.
- [11] M. Okuda, and D. Okuda, *Star Trek Chronology: The History of the Future*, revised edn., Pocket Books, NY, 1996.
- [12] F. Parise, ed., *The Book of Calendars*, Facts on File, New York, 1982.
- [13] E. M. Reingold, N. Dershowitz, and S. M. Clamen, “Calendrical Calculations, Part II: Three Historical Calendars,” *Software—Practice and Experience*, vol. 23, no. 4, pp. 383–404, April 1993.
- [14] I. Rhodes, “Computation of the Dates of the Hebrew New Year and Passover,” *Computers & Mathematics with Applications*, vol. 3, pp. 183–190, 1977.
- [15] R. M. Stallman, *GNU Emacs Manual*, 13th edn., Free Software Foundation, Cambridge, MA, 1997.
- [16] G. L. Steele, Jr., G. L. Steele, Jr., *COMMON LISP: The Language*, 2nd edn., Digital Press, Bedford, MA, 1990.
- [17] K. Wilkins, Letter to Nachum Dershowitz from a Customer Relations Representative, Lotus Development Corporation, Cambridge, MA, April 21, 1992.

La dernière chose qu'on trouve en faisant un ouvrage, est de savoir celle qu'il faut mettre la première. [The last thing one settles in writing a book is what one should put in first.]

Blaise Pascal: *Pensées sur l'esprit et le style* (1660)

Credits

Whoever relates something in the name of its author brings redemption to the world.

Midrash Tanhuma (Numbers, 27)

Photograph of Edward M. Reingold on the dust jacket is by Photography by Rick & Rich (Northbrook, IL, 2014); used with permission.

Photograph of Nachum Dershowitz on the dust jacket is by Olivier Toussaint (Nancy, 2011); used with permission.

Quote on page xxxi from *Epistles of Maimonides: Crisis and Leadership*, A. Halkin, trans., Jewish Publication Society, 1993; used with permission.

Translation of Scaliger's comment on the Roman calendar on page 75 is from A. T. Grafton, *Joseph Scaliger: A Study in the History of Classical Scholarship, vol. II, Historical Chronography*, Oxford University Press, Oxford, 1993; used with permission.

Translation of Ptolemy III's *Canopus Decree* on page 92 is from page 90 of R. Hannah, *Greek & Roman Calendars*, Gerald Duckworth & Co., London, 2005; used with permission.

Translation on page 114 of Scaliger's comment on the Hebrew calendar (found on page 294 of Book 7 in the 1593 Frankfort edition of *De Emendatione Temporum*) is by H. Jacobson; used with permission.

Translation of "The Synodal Letter" on page 143 (found in Gelasius, *Historia Concilii Nicæni*, book II, Chapter xxxiii) is from J. K. Fotheringham, "The Calendar," in *The Nautical Almanac and Astronomical Ephemeris*, His Majesty's Stationery Office, London, 1931–1934; revised 1935–1938; abridged 1939–1941.

Translation of the extract from Canon 6 of Gregorian reform on page 145 is by M. H. Deckers; used with permission.

Translation of the Quintus Curtius Rufus quotation on page 257 is from J. C. Rolfe, *History of Alexander*, Harvard University Press, Cambridge, MA, 1946.

Translation of Ovid quotation on page 259 is from J. G. Frazer, *Ovid's Fasti*, Harvard University Press, Cambridge, MA, 1931.

Letter on page 273 reprinted with permission.

License and Limited Warranty and Remedy

The Functions (code, formulas, and calendar data) contained in this book and/or provided on the publisher's web site for this book were written by Nachum Dershowitz and Edward M. Reingold (the "Authors"), who retain all rights to them except as granted in the License and subject to the warranty and liability limitations below. These Functions are subject to this book's copyright.

In case there is cause for doubt about whether a use you contemplate is authorized, please contact the Authors.

1. LICENSE. The Authors grant you a license for personal use. This means that for strictly personal use you may copy and use the code and keep a backup or archival copy also. The Authors grant you a license for re-use within non-commercial, non-profit software provided prominent credit is given and the Authors' rights are preserved. Any other uses, including, without limitation, allowing the code or its output to be accessed, used, or available to others, are not permitted.
2. WARRANTY.
 - (a) *The Authors and Publisher provide no warranties of any kind, either express or implied, including, without limiting the generality of the foregoing, any implied warranty of merchantability or fitness for a particular purpose.*
 - (b) *Neither the Authors nor Publisher shall be liable to you or any third parties for damages of any kind, including without limitation, any lost profits, lost savings, or other incidental or consequential damages arising out of, or related to, the use, inability to use, or accuracy of calculations of the code and functions contained herein, or the breach of any express or implied warranty, even if the Authors or Publisher have been advised of the possibility of those damages.*
 - (c) *The foregoing warranty may give you specific legal rights which may vary from state to state in the U.S.A.*
3. LIMITATION OF LICENSEE REMEDIES. You acknowledge and agree that your exclusive remedy (in law or in equity), and Authors' and Publisher's entire liability with respect to the material herein, for any breach of representation or for any inaccuracy shall be a refund of the price of this book. *Some States in the U.S.A. do not allow the exclusion or limitation of liability for incidental or consequential damages, and thus the preceding exclusions or limitation may not apply to you.*

4. **DISCLAIMER.** Except as expressly set forth above, the Authors and Publisher:
- (a) make no other warranties with respect to the material and expressly disclaim any others;
 - (b) do not warrant that the functions contained in the code will meet your requirements or that their operation shall be uninterrupted or error free;
 - (c) license this material on an “as is” basis, and the entire risk as to the quality, accuracy, and performance herein is yours should the code or functions prove defective (except as expressly warranted herein). You alone assume the entire cost of all necessary corrections.

MENSIVM DIVISIO.

MENSES ENNEADECAETERICI.

IVDE BYN	SYROCHAL DANORVM	SYROGR. CUEYM	HAGARE NOBYN	CALIPPI ET MILTONI.	SAXONYM
TISRI	Tisri prior	Dys	RADIK prior	Pynepsion	Wintyrfylth
Marche	Tisri alter	Apelleus	Robie alter	Manslerion	Blathmonath
Schwan	Canon prior	Adynese	Gumadi prior	Popidon	Giula prior
Cadex	Canon alter	Peritus	Gumadi alter	Ganction	Giula posterior
Tebeth	Afobae	Oyprus	Regiaba	Antheforion	Sabmonath
Schebat	Adar	Xanthicus	Sababena	Elaphebolion	Reimonath
Adar prior	Nisan	Artemisius	Ramadhan	Monyebion	Chofwmonath
Adar posterior	Iyar	Dafnus	Schewal	Thargelion	Tymidehi
Nisan	Sivan	Pancenus	Dulhaida	Scirrhophorion prior	Lida posterior
Iyar	Tammuz	Gorpius	Muhai am	Scirrhophorion alter	Lida embolimus
Sivan	Ab	Diogonus emb.	Tzephbar	HECATOM. B E O N	Wendenmonath
Tammuz	Elul	Tzephbar emb.	Tzephbar emb.	HECATOM. B E O N	HECATOM. N A T H.
Elul		RETAPS		Metagintion	Boedromion

MENSES AEQVABILES VAGI

ÆGYPTIO BYN	ARMENO BYN	PER.SARVM
THOTH	Newgarmi	Behemen
Pachti	Hawi	Alphander
Asyri	Adstereka	Mistereka
Chobie	SARAMI	PHRYDIN
Tybi	Tberi	Adisphagchih
Atebir	Cagets	Chardad
Phamonth	Harats	Thir
Pharmuthi	Mashie	Mardad
Pachon	Atich	Sebeheriz
Payni	Abeli	Isobar
Epiphi	Mariri	Aban
Magi	Marcats	Adar
Epagomene	Harwats	Di

MENSES AEQVABILES TETRAETERICI

ATTICO. BYN	MACEDO. BYN	THEBANO. BYN
GAMELTON	Dylirus	BYCATIVS
Antheforion	Xanthicus	Icrmenus
Elaphebolion	Artemisius	* *
Monyebion	Dafnus prior	* *
Thargelion	Dafnus posterior	* *
Scirrhophorion	Antepylus	* *
Iteccombon	PA N E M Y S	Hippodromius
Metagintion	Lous	Pancenus
Boedromion	Gorpius	* *
Pynepsion	Hyperberceus	Damatris
Manslerion	Dne	Alakmenius
Popidon prior	Apelleus	* *
Popidon alter	Adynusius	Embolimus
Peritus	Antepylus	Antepylus

MEN-

MENSES

ROMANO BYN.	ATHENIEN- BYN.	SYRO- BYN.
IANUARIVS	Pynepsion	Antepylus
Februarivs	Manslerion	Peritus
Martivs	Popidon	Dafnus
Aprilis	Gantion	Xanthicus
Maius	Antheforion	Artemisius
Junivs	Elaphebolion	Dafnus
Julivs	Monyebion	Pancenus
Augustus	Thargelion	Lous
September	Scirrhophorion	Gorpius
October	HECATOM. B E O N	HECATOM. B E O N
November	HECATOM. B E O N	HECATOM. B E O N
December	Metagintion	Boedromion

MISCELA

MENSES VA- GI SYRO- BYN.	MENSES VA- GI SYRO- BYN.	MENSES VA- GI SYRO- BYN.
MYHARAB	MARTIVS	TI
Tzephbar	Aprilis	TI
Rabie prior	Maius	TI
Rabie alter	Junivs	TI
Gumadi prior	Quintilis	TI
Gumadi alter	Sextilis	TI
Regiaba	September	TI
Sababena	October	TI
Schewal	November	TI
Dulhaida	December	TI
Dulhaidia	Januarius	TI
	Februarius	TI
	Mercedonius	TI

Two pages of Joseph Scaliger's, *De Emendatione Temporum* (Frankfort edition, 1593), giving month names of Illinois, Urbana, IL.)