CALENDRICAL CALCULATIONS

The Ultimate Edition

An invaluable resource for working programmers, as well as a fount of useful algorithmic tools for computer scientists, astronomers, and other calendar enthusiasts, the Ultimate Edition updates and expands the previous edition to achieve more accurate results and present new calendar variants. The book now includes algorithmic descriptions of nearly forty calendars: the Gregorian, ISO, Icelandic, Egyptian, Armenian, Julian, Coptic, Ethiopic, Akan, Islamic (arithmetic and astronomical forms), Saudi Arabian, Persian (arithmetic and astronomical), Bahá'í (arithmetic and astronomical), French Revolutionary (arithmetic and astronomical), Babylonian, Hebrew (arithmetic and astronomical), Samaritan, Mayan (long count, haab, and tzolkin), Aztec (xihuitl and tonalpohualli), Balinese Pawukon, Chinese, Japanese, Korean, Vietnamese, Hindu (old arithmetic and medieval astronomical, both solar and lunisolar), and Tibetan Phug-lugs. It also includes information on major holidays and on different methods of keeping time. The necessary astronomical functions have been rewritten to produce more accurate results and to include calculations of moonrise and moonset.

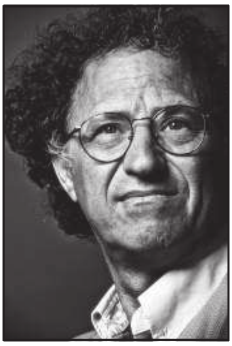
The authors frame the calendars of the world in a completely algorithmic form, allowing easy conversion among these calendars and the determination of secular and religious holidays. Lisp code for all the algorithms is available in machine-readable form.

Edward M. Reingold is Professor of Computer Science at the Illinois Institute of Technology.

Nachum Dershowitz is Professor of Computational Logic and Chair of Computer Science at Tel Aviv University.

About the Authors

Edward M. Reingold was born in Chicago, Illinois, in 1945. He has an undergraduate degree in mathematics from the Illinois Institute of Technology and a doctorate in computer science from Cornell University. Reingold was a faculty member in the Department of Computer Science at the University of Illinois at Urbana-Champaign from 1970–2000; he retired as a Professor Emeritus of Computer Science in December 2000 and moved to the Department of Computer Science at the Illinois Institute of Technology as professor and chair, an administrative post he held until 2006. His research interests are in theoretical computer science—especially the design and analysis of algorithms and data structures. A Fellow of the Association for Computing Machinery since 1996, Reingold has authored or coauthored more than 70 research papers and 10 books; his papers on backtrack search, the generation of combinations, weight-balanced binary trees, and the drawing of trees and graphs are considered classics. He has won awards for his undergraduate and graduate teaching. Reingold is intensely interested in calendars and their computer implementation; in addition to *Calendrical Calculations* and *Calendrical Tabulations*, he is the author and former maintainer of the calendar/diary part of GNU Emacs. In the accompanying photograph he is wearing a tie showing the twelve animal totems of the Chinese calendar.



Beyond his expertise in calendars, Nachum Dershowitz is a leading figure in software verification in general and the termination of programs in particular; he is an international authority on equational inference and term rewriting. Other areas in which he has made major contributions include program semantics, analysis of historical manuscripts, and combinatorial enumeration. Dershowitz has authored or coauthored more than 100 research papers and several books and has held visiting positions at prominent institutions around the globe. He has won numerous awards for his research and teaching, including the Herbrand Award for Distinguished Contributions to Automated Reasoning (2011) and Test-of-Time awards for the IEEE Symposium on Logic in Computer Science (2006), for the International Conference on Rewriting Techniques and Applications (2014), and for the International Conference on Automated Deduction (2015). Born in 1951, his graduate degrees in applied mathematics are from the Weizmann Institute in Israel. He is currently Professor of Computational Logic and Chair of Computer Science at Tel Aviv University and was elected to Academia Europaea in 2013.

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CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India

79 Anson Road, #06–04/06, Singapore 079906

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning, and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781107057623
DOI: 10.1017/9781107415058

First edition 1997
Second edition 2001
Third edition 2008
Fourth edition 2018

Printed in the United Kingdom by TJ International Ltd. Padstow Cornwall

A catalogue record for this publication is available from the British Library.

Library of Congress Cataloging-in-Publication Data
Names: Reingold, Edward M., 1945– author. | Dershowitz, Nachum, author.
Title: Calendrical calculations : the ultimate edition / Edward M. Reingold, Illinois Institute of Technology, Chicago, Nachum Dershowitz, Tel Aviv University, Israel.
Description: Fourth edition. | Cambridge : Cambridge University Press, 2017.
Identifiers: LCCN 2017024295| ISBN 9781107057623 (hardback) | ISBN 9781107683167 (paperback)
Subjects: LCSH: Calendar–Mathematics.
Classification: LCC CE12 .R45 2017 | DDC 529/.3–dc23
LC record available at https://lccn.loc.gov/2017024295

ISBN 978-1-107-05762-3 Hardback
ISBN 978-1-107-68316-7 Paperback

Additional resources for this publication available at www.cambridge.org/calendricalcalculations

לקהֹלוֹת הקדש
שׁמסרו נפשם עֹל קדשת השם
יהי זֹכרם ברוך

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Abbreviations

Abbreviation	Meaning	Explanation
a.d.	ante diem	prior day
A.D.	Anno Domini (= C.E.)	In the year of the Lord
A.H.	Anno Hegiræ	In the year of Mohammed's emigration to Medina
a.m.	ante meridiem	before noon
A.M.	Anno Mundi	In the year of the world since creation
	Anno Martyrum	Era of the Martyrs
A.P.	Anno Persico Anno Persarum	Persian year
A.S.	Anno Samaritanorum	Samaritan year
A.U.C.	Ab Urbe Condita	From the founding of the city of Rome
B.C.	Before Christ (= B.C.E.)	
B.C.E.	Before the Common Era (= B.C.)	
B.E.	Bahá'í Era	
C.E.	Common Era (= A.D.)	
E.E.	Ethiopic Era	
JD	Julian Day number	Elapsed days since noon on Monday, January 1, 4713 B.C.E. (Julian); sometimes J.A.D., Julian Astronomical Day
K.Y.	Kali Yuga	"Iron Age" epoch of the traditional Hindu calendar
m	meters	

continued

Cambridge University Press
978-1-107-05762-3 — Calendrical Calculations
4th Edition
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Abbreviation	Meaning	Explanation
MJD	Modified Julian Day number	Julian day number minus 2400000.5
p.m.	post meridiem	after noon
R.D.	Rata Die	Fixed date—elapsed days since the onset of Monday, January 1, 1 (Gregorian)
S.E.	Śaka Era	Epoch of the modern Hindu calendar
U.T.	Universal Time	Mean solar time at Greenwich, England (0° meridian), reckoned from midnight; sometimes G.M.T., Greenwich Mean Time
V.E.	Vikrama Era	Alternative epoch of the modern Hindu calendar

Mathematical Notations

Notation	Name	Meaning
$\lfloor x \rfloor$	floor	largest integer not larger than x
$\lceil x \rceil$	ceiling	smallest integer not smaller than x
$\text{round}(x)$	round	nearest integer to x, that is, $\lfloor x + 0.5 \rfloor$
$x \bmod y$	remainder	$x - y\lfloor x/y \rfloor$
$x \bmod [1 \,..\, y]$	adjusted remainder	y if $x \bmod y = 0$, $x \bmod y$ otherwise
$x \bmod [a \,..\, b)$	interval mod	x if $a = b$, $a + (x - a) \bmod (b - a)$ otherwise
$\gcd(x, y)$	greatest common divisor	x if $y = 0$, $\gcd(y, x \bmod y)$ otherwise
$\text{lcm}(x, y)$	least common multiple	$xy/\gcd(x, y)$
$\lvert x \rvert$	absolute value	unsigned value of x
$\text{sign}(x)$	sign	-1 when x is negative, $+1$ when x is positive, 0 when x is 0
$i^\circ j' k''$	angle	i degrees, j arc minutes, and k arc seconds
$\varphi(n)$	totient function	number of positive integers less than n and relatively prime to it
π	pi	ratio of circumference of circle to diameter
$\sin x$	sine	sine of x, given in degrees
$\cos x$	cosine	cosine of x, given in degrees
$\tan x$	tangent	tangent of x, given in degrees
$\arcsin x$	arc sine	inverse sine of x, in degrees

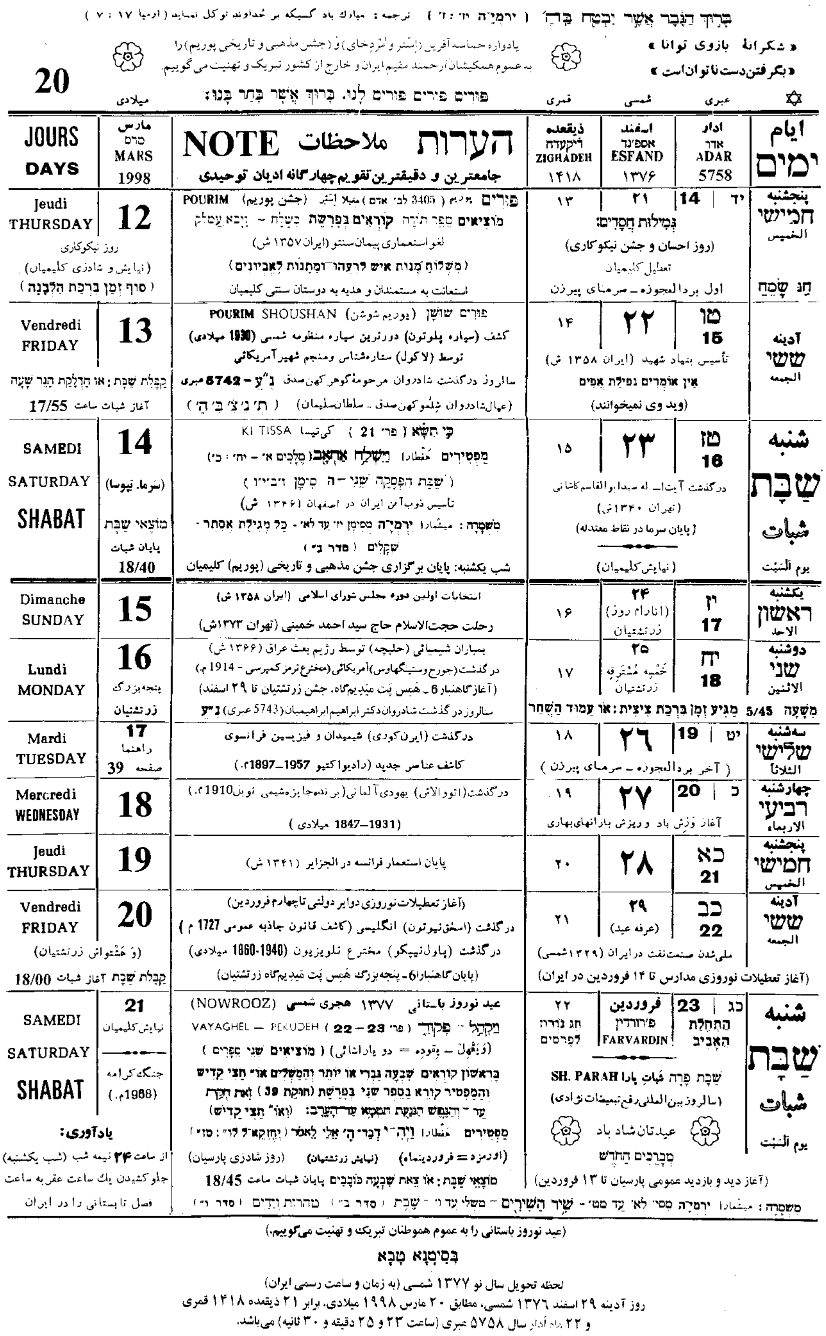
continued

Notation	Name	Meaning
$\arccos x$	arc cosine	inverse cosine of x, in degrees
$\arctan x$	arc tangent	inverse tangent of x, in degrees
$[a\,..\,b]$	closed interval	all real numbers x, $a \leqslant x \leqslant b$
$(a\,..\,b)$	open interval	all real numbers x, $a < x < b$
$[a\,..\,b)$	half-open interval	all real numbers x, $a \leqslant x < b$
$(a\,..\,b]$	half-open interval	all real numbers x, $a < x \leqslant b$
$\neg p$	logical negation	true when p is false and vice versa
$\sum_{i \geqslant k}^{p(i)} f(i)$	summation	the sum of $f(i)$ for all integers $i = k, k+1, \ldots$, continuing only as long as the condition $p(i)$ holds
$\prod_{i \geqslant k}^{p(i)} f(i)$	product	the product of $f(i)$ for all integers $i = k, k+1, \ldots$, continuing only as long as the condition $p(i)$ holds
$\sum f(\tilde{x}, \tilde{y}, \ldots)$	summation	the sum of $f(\tilde{x}_i, \tilde{y}_i \ldots)$ for all like-indexed components of the vectors $\tilde{x}, \tilde{y}, \ldots$
$\underset{\xi \geqslant \mu}{\mathbf{MAX}}\{\psi(\xi)\}$	maximum integer value	the largest integer $\xi = \mu, \mu+1, \ldots$ such that $\psi(\mu), \psi(\mu+1), \ldots, \psi(\xi)$ are true
$\underset{\xi \geqslant \mu}{\mathbf{MIN}}\{\psi(\xi)\}$	minimum integer value	the smallest integer $\xi = \mu, \mu+1, \ldots$ such that $\psi(\xi)$ is true
$\overset{p(\mu,\nu)}{\underset{\xi \in [\mu..\nu]}{\mathbf{MIN}}}\{\psi(\xi)\}$	minimum value	the value ξ such that ψ is false in $[\mu\,..\,\xi)$ and is true in $[\xi\,..\,\nu]$; see equation (1.35) on page 24 for details
$f^{-1}(y, [a\,..\,b])$	function inverse	approximate x in $[a\,..\,b]$ such that $f(x) = y$
$\boxed{f_1 \mid f_2 \mid f_3} \cdots$	record formation	the record containing fields $f_1, f_2, f_3, \ldots$
$R_{\mathbf{f}}$	field selection	contents of field $\mathbf{f}$ of record R
$\langle x_0, x_1, x_2, \ldots \rangle$	list construction	the list containing $x_0, x_1, x_2, \ldots$
$\langle\,\rangle$	empty list	a list with no elements
$L_{[i]}$	list element	the ith element of list L; 0-based
$L_{[i..]}$	sublist	a list of the ith, $(i+1)$st, and so on elements of list L
$A \parallel B$	concatenation	the concatenation of lists A and B

continued

Notation	Name	Meaning
$\tilde{x}$	vector	indexed list of elements $\langle x_0, x_1, \ldots \rangle$
$\{x_0, x_1, x_2, \ldots\}$	set formation	the set containing $x_0, x_1, x_2, \ldots$
$x \in S$	set membership	the element x is a member of set S
$x \in \mathbf{Z}$	integer	the number x is an integer
$A \cap B$	set intersection	the intersection of sets A and B
$A \cup B$	set union	the union of sets A and B
$i \mathinner{..} j$	range of integers	the set $\{i, i+1, \ldots, j\}$
$\langle b_1, \ldots, b_k; b_{k+1}, \ldots, b_n \rangle$	mixed-radix base	each position i takes values in $[0 \mathinner{..} b_i)$, with units in position k
$a \overset{\text{rad}}{\longleftarrow} b$	mixed-radix number	value of a in base b
$x \overset{\text{rad}}{\longrightarrow} b$	mixed-radix representation	representation of x in base b
$h : m : s$	time of day	h hours, m minutes, and s seconds
$i^{\text{d}} j^{\text{h}} k^{\text{m}} l^{\text{s}}$	duration of time	i days, j hours, k minutes, and l seconds
bogus	error	invalid calendar date or time

Cambridge University Press
978-1-107-05762-3 — Calendrical Calculations
4th Edition
Frontmatter
More Information



JOURS DAYS	مارس MARS 1998	NOTE ملاحظات הערות	ذيقعده ZIGHADEH ١۴١٨	اسفند ESFAND ١٣٧۶	ادار אדר ADAR 5758	ايام ימים
Jeudi THURSDAY	12	POURIM (3405)	١٣	٢١	14 יד	חמישי
Vendredi FRIDAY 17/55	13	POURIM SHOUSHAN (1930) 5742	١۴	٢٢	15 טו	ששי
SAMEDI SATURDAY SHABAT	14 18/40	KI TISSA (21)	١۵	٢٣	16 טז	שבת
Dimanche SUNDAY	15		١۶	٢۴	17 יז	ראשון
Lundi MONDAY	16	(1914) (5743)	١٧	٢۵	18 יח	שני
Mardi TUESDAY	17 39	(1897–1957)	١٨	٢۶	19 יט	שלישי
Mercredi WEDNESDAY	18	(1910) (1847–1931)	١٩	٢٧	20 כ	רביעי
Jeudi THURSDAY	19		٢٠	٢٨	21 כא	חמישי
Vendredi FRIDAY 18/00	20	(1727) (1860-1940)	٢١	٢٩	22 כב	ששי
SAMEDI SATURDAY SHABAT	21 (1988)	(NOWROOZ) ١٣٧٧ VAYAGHEL — PEKUDEH (22 – 23) (39) 18/45	٢٢	فروردين FARVARDIN	23 כג	שבת

Page from an Iranian synagogue calendar for mid-March 1998 showing the Gregorian, Hebrew, Persian, and Islamic calendars. (Collection of E.M.R.)

Preface

No one has the right to speak in public before he has rehearsed what he wants to say two, three, and four times, and learned it; then he may speak . . . But if a man . . . puts it down in writing, he should revise it a thousand times, if possible.
Maimonides: *The Epistle on Martyrdom* (circa 1165)

This book has developed over a more than 30-year period during which the calendrical algorithms and our presentation of them have continually evolved. Our initial motivation was an effort by one of us (E.M.R.) to create Emacs-Lisp code that would provide calendar and diary features for GNU Emacs [15]; this version of the code included the Gregorian, Islamic, and Hebrew calendars (the Hebrew implemented by N.D.). A deluge of inquiries from around the globe soon made it clear to us that there was keen interest in an explanation that would go beyond the code itself, leading to our article [3] and encouraging us to rewrite the code completely, this time in Common Lisp [16]. The subsequent addition—by popular demand—of the Mayan and French Revolutionary calendars to GNU Emacs prompted a second article [13]. We received many hundreds of reprint requests for these articles. This response far exceeded our expectations and provided the impetus to write a book in which we could more fully address the multifaceted subject of calendars and their implementation.

The subject of calendars has always fascinated us with its cultural, historical, and mathematical wealth, and we have occasionally employed calendars as accessible examples in introductory programming courses. Once the book's plan took shape, our curiosity turned into obsession. We began by extending our programs to include other calendars such as the Chinese, Coptic, modern Hindu, and arithmetic Persian. Then, of course, the code for these newly added calendars needed to be rewritten, in some cases several times, to bring it up to the standards of the earlier material. We have long since lost track of the number of revisions, and, needless to say, we could undoubtedly devote another decade to polishing what we have, tracking down minutiæ, and implementing and refining additional interesting calendars. As much as we might be tempted to, circumstances do not allow us to follow Maimonides' dictum quoted above.

In this book we give a unified algorithmic presentation for more than three dozen calendars of current and historical interest: the Gregorian (current civil), ISO (International Organization for Standardization), Icelandic, Egyptian (and nearly identical Armenian), Julian (old civil), Coptic and virtually identical Ethiopic, Akan, Islamic (Muslim), including the arithmetic, observational, and Saudi Arabian forms, modern Persian (both the astronomical and arithmetic forms), Bahá'í (both the arithmetic and astronomical forms), French Revolutionary (both the astronomical and arithmetic forms), Babylonian, Hebrew (Jewish) standard and observational, Samaritan, Mayan (long count, haab, and tzolkin) and two almost identical Aztec, Balinese Pawukon, Chinese (and nearly identical Japanese, Korean, and Vietnamese), old Hindu (solar and lunisolar), modern Hindu (solar and lunisolar, traditional and astronomical), and Tibetan. Easy conversion among these calendars is a natural outcome of the approach, as is the determination of secular and religious holidays.

Our goal in this book is twofold: to give precise descriptions of each calendar and to make accurate calendrical algorithms readily available for computer use. The complete workings of each calendar are described in prose and in mathematical/algorithmic form. Working computer programs are included in an appendix and are available on the internet (see following).

Calendrical problems are notorious for plaguing software, as shown by the following examples:

1. Since the early days of computers, when storage was at a premium, programmers—especially COBOL programmers—usually allocated only two decimal digits for the internal storage of years [10]; thus billions of dollars were spent fixing untold numbers of programs to prevent their going awry on New Year's Day of 2000 by interpreting "00" as 1900 instead of 2000. This became known as the "Y2K problem."

2. In a Reuters story dated Monday, November 6, 2006, Irene Klotz wrote:

> A computer problem could force NASA to postpone next month's launch of shuttle Discovery until 2007 to avoid having the spaceship in orbit when the clock strikes midnight on New Year's Eve. The shuttle is due to take off from the Kennedy Space Center in central Florida on December 7 on a 12-day mission to continue construction of the half-built International Space Station. But if the launch is delayed for any reason beyond December 17 or 18, the flight likely would be postponed until next year, officials at the U.S. space agency said on Monday. To build in added cushion, NASA may move up the take off to December 6. "The shuttle computers were never envisioned to fly through a year-end changeover," space shuttle program manager Wayne Hale told a briefing. After the 2003 accident involving space shuttle Columbia, NASA started developing procedures to work around the computer glitch. But NASA managers still do not want to launch Discovery knowing it would be in space when the calendar rolls over to January 1, 2007.

The problem, according to Hale, is that the shuttle's computers do not reset to day one, as ground-based systems that support shuttle navigation do. Instead, after December 31, the 365th day of the year, shuttle computers figure January 1 is just day 366.

3. Poorly written calendar software in Notify Technology's code to synchronize mobile devices did not correctly handle monthly recurring events on the 29th, 30th, or 31st of the month because these dates do not occur in all months.

4. The change from daylight saving time to standard time in late 2010 (at various dates around the world) caused the failure of certain repeating iPhone alarms. The alarms failed again on January 1, 2011.

5. Many programs err in, or simply ignore, the century rule for leap years on the Gregorian calendar (every 4th year is a leap year, except for every 100th year, which is not, except for every 400th year, which is):

   (a) The *New York Times* of March 1, 1997 reported that the New York City Taxi and Limousine Commission chose March 1, 1996, as the start date for a new, higher fare structure for cabs. Meters programmed by one company in Queens ignored the leap day and charged customers the higher rate on February 29.

   (b) According to the *New Zealand Herald* of January 8, 1997, a computer software error at the Tiwai Point aluminum smelter at midnight on New Year's Eve caused more than A$ 1 million of damage. The software error was the failure to consider 1996 a leap year; the same problem occurred 2 hours later at Comalco's Bell Bay smelter in Tasmania (which was 2 hours behind New Zealand). The general manager of operations for New Zealand Aluminum Smelters, David Brewer, said, "It was a complicated problem and it took quite some time to find the cause."

   (c) Early releases of the popular spreadsheet program Lotus®1-2-3® treated 2000 as a nonleap year—a problem that was eventually fixed. However, all releases of Lotus® 1-2-3® take 1900 as a leap year, which is a serious problem with historical data; by the time this error was recognized, the company deemed it too late to correct: "The decision was made at some point that a change now would disrupt formulas which were written to accommodate this anomaly" [17]. Excel®, part of Microsoft Office®, suffers from the same flaw; Microsoft acknowledges this error on its "Help and Support" web site, claiming that "the disadvantages of [correcting the problem] outweigh the advantages."

   (d) According to Reuters (March 22, 2004), the computer display in the 2004 Pontiac Grand Prix shows the wrong day of the week because engineers overlooked the fact that 2004 is a leap year.

   (e) Similarly, Zune®, Microsoft's portable media player, failed (according to the *New York Times* of January 1, 2009) because the software did not treat 2008 as a leap year. In fact, Zune's code to compute the present year from the number of days elapsed since January 1, 1980 would go into an infinite loop on the last day of *any* leap year.

(f) Again according to the *New York Times* (March 1, 2010), Sony Playstation 3® code considered 2010 a leap year, an error that caused problems for gamers on March 1—some games would not load, others lost records of trophies, and online connections failed.

6. The calculation of holidays and special dates is a source of confusion:

   (a) According to the *New York Times* of January 12, 1999, for example, Microsoft Windows® 95, 98, and NT get the start of daylight saving time wrong for years, like 2001, in which April 1 is a Sunday; in such cases Windows has daylight saving time starting on April 8. An estimated 40 million to 50 million computers were affected, including some in hotels that were used for wake-up calls.

   (b) Microsoft Outlook® 98 had the wrong date for U.S. Memorial Day in 1999, giving it as May 24, 1999, instead of May 31, 1999. It gave wrong dates for U.S. Thanksgiving Day for 1997–2000. Outlook® 2000 corrected the Memorial Day error, but compounded the Thanksgiving Day error by giving *two dates* for Thanksgiving for 1998–2000. Their 2015 Web App has incorrect dates for the Hebrew calendar fast days Tzom Tammuz and Tishah be-Av.

   (c) Various programs calculate the Hebrew calendar by first determining the date of Passover using Gauss's method [6] (see [14]); this method is correct only when sufficient precision is used, and thus such an approach often leads to errors.

   (d) Delrina Technology's 1994 Daily Planner had three days instead of two for Rosh ha-Shanah.

   (e) Israeli daylight saving time has ended at various dates over the years, but Microsoft's Windows Vista® always ended it on September 2.

7. At least one modern, standard, source for calendrical matters, Parise [12], has many errors, some of which are presumably due not to sloppy editing, but to the algorithms used to produce the tables. For example, the Mayan date 8.1.19.0.0 is given incorrectly as February 14, 80 (Gregorian) on page 290; the dates given on pages 325–327 for Easter for the years 1116, 1152, and 1582 are not Sundays; the epact for 1986 on page 354 is wrongly given as 20; Chinese New Year is wrong for many years; the epoch is wrong for the Ethiopic calendar, and hence that entire table is flawed.

8. Even the Astronomical Applications Department of the U.S. Naval Observatory is not immune to calendrical errors! They gave Sunday, April 9, 2028 and Thursday, March 29, 2029 for Passover on their web site `aa.usno.navy.mil/faq/docs/passover.html`, instead of the correct dates Tuesday, April 11, 2028 and Saturday, March 31, 2029, respectively. The site was corrected on March 10, 2004.

Finally, the computer world is plagued with unintelligible code that seems to work by magic. Consider the following Unix script for calculating the date of Easter:

```
echo $* '[ddsf[lfp[too early]Pq]s@1583>@
ddd19%1+sg100/1+d3*4/12-sx8*5+25/5-sz5*4/lx-10-
sdlg11*20+lz+lx-30%d[30+]s@0>@d[[1+]s@lg11<@]s@25=@d[1+]
s@24=@se44le-d[30+]s@21>@dld+7%-7+
[March ]smd[31-[April ]sm]s@31<@psnlmPpsn1z>p]splpx' | dc
```

We want to provide transparent algorithms to replace the gobbledegook that is so common.

Our algorithms are carefully crafted, fully explained, and (in almost all cases) endogenous. They illustrate all the basic features of calendars: fidelity only to solar events (Gregorian, Persian, French), fidelity only to lunar events (Islamic), and fidelity to both solar and lunar events (Hebrew, Chinese, Hindu); intricate cycles disconnected from solar and lunar events (Mayan, Balinese); simultaneous intercalation and extraculation yielding irregular cycles of days of the month and months of the year (Hindu). We hope that in the process of reworking classical calendrical calculations and rephrasing them in the algorithmic language of the computer age we have also succeeded in affording the reader a glimpse of the beauty and individuality of diverse cultures past and present.

The Ultimate Edition

> *How I labored day and night for almost ten years straight composing this work. Great scholars as yourselves will understand what I have accomplished, having gathered statements that were distant and dispersed among the hills and mountains ... For these reasons, it is appropriate for one to examine my statements, to scrutinize, and to investigate after me. The reader of this composition should not say, who am I ... I hereby grant him my permission ... You, in your wisdom, have done me a great favor. Likewise, anyone who finds a problem and informs me will be rendering me a favor, lest there remain any stumbling block.*
>
> Maimonides: Letter to Jonathan ben David Hakohen of Lunel (1199)

After the first edition of the book was published in 1997 we continued to gather material, polish the algorithms, and keep track of errors. Because the second edition was to be published in the year 2000, some wag at Cambridge University Press dubbed it "The Millennium Edition," and that title got used in prepublication catalogs, creating a *fait accompli*. The Millennium Edition was a comprehensive revision of the first edition, and the third edition was a comprehensive revision of the Millennium Edition. Since the publication of the third edition we have continued to gather new material and polish existing material; this fourth edition is, once again, a comprehensive revision. We have called this "The Ultimate Edition" for several reasons. First, and foremost, we have no intention of ever producing another edition of this book (though minor changes may be made in subsequent printings). Second, because we have strived to be as comprehensive as possible, we are sanguine that we have covered all the world's calendar types (though not, of course, all variations). Finally, this material has undergone continuous refinement for over 30 years and diminishing returns have set in: future refinements are unlikely to yield much benefit.

In preparing this Ultimate Edition we have corrected all known errors (fortunately, only minor errors were ever reported in the third edition), added much new material, reworked and rearranged some discussions to accommodate the new material, improved the robustness of some functions, added many new references, and made an enormous number of small improvements. Among the new material the reader will find much more use of the mixed-radix notation of [9, sec. 4.1], use of the generalized modulo interval notation of [4], and presentations of Unix dates, Italian time, and the Akan, Icelandic, Saudi Arabian *Umm al-Qura* (an approximation of the Observational Islamic calendar), and Babylonian calendars; there are also expanded treatments of the observational Islamic and Hebrew calendars and brief discussions of the Samaritan and Nepalese calendars. Several of the astronomical functions of Chapter 14 have been rewritten to produce more accurate results (causing occasional changes in astronomically-based calendar computations, such as the Persian and the Chinese). We have added calculations of moonrise and moonset, as well as a function to invert the *molad* in the Hebrew calendar chapter. The sample data in Appendix C has been correspondingly updated and expanded (changes in hardware and software since the preparation of the third edition have caused minor changes in some sample values compared with that edition; the revision of what we called the "Future Bahá'í calendar" has caused significant changes to some of those sample values). Sample dates of many of the holidays we discuss have also been added. A cross reference list for the functions has been added (Appendix B) showing the dependencies among the functions. Despite requests from some readers, we have *not* added oddities such as the World Calendar [1], Star Trek's stardate [11], Knuth's Potrzebie calendar [8], the pataphysique calendar [7], or the Martian calendar [5]!

Algorithmically sophisticated readers of the first edition of this book could, with only slight difficulty, jump right into the descriptions of the various calendars, skipping the introductory chapter on "Calendar Basics." With each successive edition such an omission became more difficult as various commonalities were moved to that chapter and the notations became more specialized. As much as we regret it, failing to read the introduction now may cause even a sophisticated reader bafflement in later chapters. So, for those without the patience to read the introductory chapter, we suggest at least a careful perusing of the "Mathematical Notations" table on pages xxvi–xxviii.

> *I determined, therefore, to attempt the reformation; I consulted the best lawyers and the most skilled astronomers, and we cooked up a bill for that purpose. But then my difficulty began: I was to bring in this bill, which was necessarily composed of law jargon and astronomical calculations, to both of which I am an utter stranger. However, it was absolutely necessary to make the House of Lords think that I knew something of the matter; and also to make them believe that they knew something themselves, which they do not. For my own part, I could just as soon have talked Celtic or Sclavonian to them, as astronomy, and could have understood me full as well; so I resolved . . . to please instead of informing them. I gave them, therefore, only an historical account of calendars, from the Egyptian*

down to the Gregorian, amusing them now and then with little episodes . . . They thought I was informed, because I pleased them; and many of them said, that I had made the whole story very clear to them; when, God knows, I had not even attempted it.

Letter from Philip Dormer Stanhope (Fourth Earl of Chesterfield, the man who in 1751 introduced the bill in Parliament for reforming the calendar in England) to his son, March 18, 1751 C.E. (Julian), the day of the Second Reading debate

Calendrical Tabulations

A man who possessed a calendar and could read it was an important member of the village community, certain to be widely consulted and suitably awarded.

K. Tseng: "Balinese Calendar," *Myths & Symbols in Indonesian Art* (1991)

A companion volume by the authors, *Calendrical Tabulations*, is also available. It contains tables for easy conversion of dates and some holidays on the world's major calendars (Gregorian, Hebrew, Islamic, Hindu, Chinese, Coptic/Ethiopic, and Persian) for the years 1900–2200. These tables were computed using the Lisp functions from Appendix B of the Millennium Edition and typeset directly from LaTeX output produced by driver code. Small changes made to the astronomical code in the interim can cause minor discrepancies in dates and times.[1]

The Cambridge University Press Web Site

Exegi monumentum aere perennius. [*I have created a monument more lasting than bronze.*]

Horace: *Odes,* III, xxx

`www.cambridge.org/calendricalcalculations`

This web site contains links to files related to this book, including the Lisp code from Appendix D for the calendar functions and the sample data from Appendix C.

The Authors' Web Site

The author has tried to indicate every known blemish in [2]*; and he hopes that nobody will ever scrutinize any of his own writings as meticulously as he and others have examined the ALGOL report.*

Donald E. Knuth: "The Remaining Trouble Spots in ALGOL 60," *Communications of the ACM* (1967)

Visit us at

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[1] The following minor errors regarding lunar phases in *Calendrical Tabulations* bear noting: First, the dust jacket uses a negative image of the calendar pages; this has the effect of interchanging the full/new moon symbols and the first quarter/last quarter symbols visible in the Gregorian calendar at the middle bottom. Second, when a lunar phase (or equinox or solstice) occurs seconds before midnight, the date is correctly indicated, but the time is rounded up to midnight and shown as 0:00 instead of 24:00. Finally, when two lunar phases occur during the same week, the times given in the right margin are in reverse order.

Among other things, one can find errata for this book at this address. Try as we have, at least one error remains in this book.

Acknowledgments

It is traditional for the author to magnanimously accept the blame for whatever deficiencies remain. I don't. Any errors, deficiencies, or problems in this book are somebody else's fault, but I would appreciate knowing about them so as to determine who is to blame.

Steven Skiena: *The Algorithm Design Manual* (1997)

Stewart M. Clamen wrote an early version of the Mayan calendar code. Parts of Section 2.3 are based on suggestions by Michael H. Deckers. Chapters 6 and 21 are based in part on the work of Svante Janson.

Our preparation of the fourth edition was aided considerably by the help of Mark D. Bej, Uri Blass, Irvin L. Bromberg, Assaf Cohen, William P. Collins, Craig Dedo, Ben Dencka, Idan Dershowitz, Surya Prasad Dhungel, Tony Finch, Gedalya Gordon, Julian Gilbey, Eysteinn Guðni Guðnason, Peter Zilahy Ingerman, Svante Janson, Kaboel Karso, Eric Kingston, Kwasi Konadu, Stanislav Koncebovski, Kai Kuhlmann, Jonathan Leffler, Yaaqov Loewinger, Zhuo Meng, Susan Milbrath, Josua Müller, Fabrice Orgogozo, Andy Pepperdine, John Powers, Eugene Quah, Lester A. Reingold, Ruth N. Reingold, Dieter Schuh, Matthew Sheby, Enrico Spinielli, Sacha Stern, Sharad Upadhyay, Robert H. van Gent, Nadia Vidro, Steve Ward, and Alan R. White, all of whom pointed out errors, suggested improvements, and helped gather materials. Special thanks go to our copy editor Susan S. Parkinson who went carefully through every every detail of the book and provided many invaluable corrections. We also thank all those acknowledged in the prior editions for their help.

Gerald M. Browne, Sharat Chandran, Shigang Chen, Jeffrey L. Copeland, Idan Dershowitz, Nazli Goharian, Mayer Goldberg, Getatchew Haile, Shiho Inui, Yoshiyasu Ishigami, Howard Jacobson, Subhash Kak, Claude Kirchner, Sakai Kō, Jungmin Lee, Nabeel Naser El-deen, Gerhard A. Nothmann, Trần Đức Ngọc, Sigurður Örn Stefánsson, Fentahun Tiruneh, Roman Waupotitsch, Daniel Yaqob, and Afra Zomorodian helped us with various translations and foreign language fonts. Charles Hoot labored hard on the original program for automatically transforming Lisp code into arithmetic expressions and provided general expertise in Lisp. Mitchell A. Harris helped with fonts, star names, and the automatic translation; Matthew Carroll, Benita Ulisano, and Upendra Gandhi were our system support people; Marla Brownfield helped with various tables. Herbert Voss modified PSTricks several times to enable us to produce various figures. Erga Dershowitz, Idan Dershowitz, Molly Flesner, Schulamith Halevy, Deborah Klapper, Eve Kleinerman, Rachel Mandel, Ruth Reingold, Christine Mumm, and Joyce Woodworth were invaluable in proofreading tens of thousands of dates, comparing our results with published tables. We are grateful to all of them.

Portions of this book appeared, in a considerably less polished state, in our papers [3] and [13]. We thank John Wiley & Sons for allowing us to use that material here.

The second author is grateful to the Institut d'études avancées de Paris for the conducive environment it provided during the last stages of preparation of this edition.

THE END.
This work was completed on the 17th or 27th day of May, 1618; but Book v was reread (while the type was being set) on the 9th or 19th of February, 1619. At Linz, the capital of Austria—above the Enns.

Johannes Kepler: *Harmonies of the World*

I have not always executed my own scheme, or satisfied my own expectations . . . [But] I look with pleasure on my book however defective and deliver it to the world with the spirit of a man that has endeavored well . . . When it shall be found that much is omitted, let it not be forgotten that much likewise has been performed.

Samuel Johnson: Preface to his *Dictionary*

R.D. 736520
Chicago, Illinois — E.M.R.
Tel Aviv, Israel — N.D.

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A book without a preface is like a body without a soul.
Hebrew proverb

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La dernière chose qu'on trouve en faisant un ouvrage, est de savoir celle qu'il faut mettre la première. [*The last thing one settles in writing a book is what one should put in first.*]

Blaise Pascal: *Pensées sur l'esprit et le style* (1660)

Credits

Whoever relates something in the name of its author brings redemption to the world.

Midrash Tanḥuma (Numbers, 27)

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Cambridge University Press
978-1-107-05762-3 — Calendrical Calculations
4th Edition
Frontmatter
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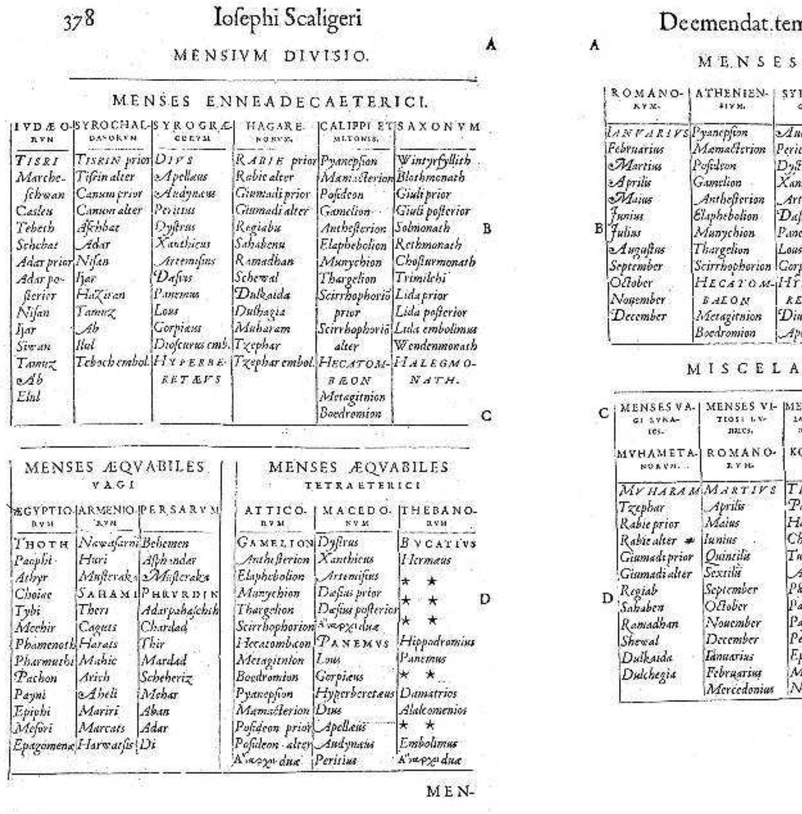

378 Iosephi Scaligeri

MENSIVM DIVISIO.

MENSES ENNEADECAETERICI.

IVDÆORVM	SYROCHALDÆORVM	SYROGRÆCORVM	HAGARENORVM	CALIPPI ET METONIS	SAXONVM
TISRI	TISRIN prior	DIVS	RABIE prior	Pyanepsion	Wintyrfyllith
Marche-schwan	Tisrin alter	Apellæus	Rabie alter	Mæmacterion	Blothmonath
Casleu	Canun prior	Audynæus	Giumadi prior	Posideon	Giuli prior
Tebeth	Canun alter	Peritius	Giumadi alter	Gamelion	Giuli posterior
Schebat	Aschbat	Dystrus	Regiabu	Anthesterion	Solmonath
Adar prior	Adar	Xanthicus	Sahabenu	Elaphebolion	Rethmonath
Adar posterior	Nisan	Artemisius	Ramadhan	Munychion	Chostu rmonath
Nisan	Ijar	Dæsius	Schewal	Thargelion	Trimilchi
Ijar	HaZiran	Panemus	Dulkaida	Scirrhophoriō prior	Lida prior
Siwan	Tamuz	Lous	Dulhagia	Scirrhophoriō alter	Lida posterior
Tamuz	Ab	Gorpiæus	Muharam	HECATOMBÆON	Lida embolimus
Ab	Ilul	Dioscurus emb.	Tzephar	Metagitnion	Wendenmonath
Elul	Tebeth embol.	HYPERBERETÆVS	Tzephar embol.	Boedromion	HALEGMONATH.

MENSES ÆQVABILES VAGI

ÆGYPTIORVM	ARMENIORVM	PERSARVM
THOTH	Newasardi	Behemen
Paophi	Hori	Asphandar
Athyr	Mehekan	Mehekan
Choiac	SAHAMI	PHRVRDIN
Tybi	Thori	Adarpahaschith
Mechir	Caghots	Chardad
Phamenoth	Harats	Thir
Pharmuthi	Mahic	Mardad
Pachon	Arich	Scheheriz
Payni	Aheli	Mehar
Epiphi	Mariri	Aban
Mesori	Marcats	Adar
Epagomenæ	Harwatsis	Di

MENSES ÆQVABILES TETRAETERICI

ATTICORVM	MACEDONVM	THEBANORVM
GAMELION	Dystrus	BVCATIVS
Anthesterion	Xanthicus	Hermæus
Elaphebolion	Artemisius	* *
Munychion	Dæsius prior	* *
Thargelion	Dæsius posterior	* *
Scirrhophorion	[illegible] duæ	Hippodromius
Hecatombæon	PANEMVS	Panemus
Metagitnion	Lous	* *
Boedromion	Gorpiæus	Damatrios
Pyanepsion	Hyperberetæus	Alalcomenios
Mæmacterion	Dius	* *
Posideon prior	Apellæus	Embolimus
Posideon alter	Audynæus	[illegible] duæ
[illegible] duæ	Peritius	

MEN-

De emendat. tem

MENSES

ROMANORVM	ATHENIENSIVM
IANVARIVS	Pyanepsion
Februarius	Mæmacterion
Martius	Posideon
Aprilis	Gamelion
Maius	Anthesterion
Iunius	Elaphebolion
Iulius	Munychion
Augustus	Thargelion
September	Scirrhophorion
October	HECATOMBÆON
Nouember	Metagitnion
December	Boedromion

MISCELA

MENSES VAGI LVNARES. MVHAMETANORVM.	MENSES VITIOSI LVNARES. ROMANORVM.
MVHARAM	MARTIVS
Tzephar	Aprilis
Rabie prior	Maius
Rabie alter	Iunius
Giumadi prior	Quintilis
Giumadi alter	Sextilis
Regiab	September
Sahaben	October
Ramadhan	Nouember
Shewal	December
Dulkaida	Ianuarius
Dulchegia	Februarius
	Mercedonius

Two pages of Joseph Scaliger’s, *De Emendatione Temporum* (Frankfort edition, 1593), giving month names
of Illinois, Urbana, IL.)