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978-1-107-05374-8 - Applied Computational Aerodynamics: A Modern Engineering Approach

Russell M. Cummings, William H. Mason, Scott A. Morton and David R. McDaniel

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Applied Computational Aerodynamics

This computational aerodynamics (CA) textbook is written at the undergraduate level, based on years of teaching focused on developing the engineering skills required to become an intelligent user of aerodynamic codes, unlike most available books which focus on learning how to write codes. This is done by taking advantage of CA codes that are now freely available and doing projects to learn the basic numerical and aerodynamic concepts required.

The authors believe that new and vibrant ways to interact with CA are important in order to improve understanding of aerodynamics. This book includes a number of unique features to make studying computational aerodynamics more enjoyable. These include:

- The computer programs used in the book's projects are all open source and accessible to students and practicing engineers alike on the book's website, www.cambridge.org/aerodynamics.
- CA Concept Boxes appear throughout the book to make material more relevant and to provide interesting asides from the material at hand.
- Flow Visualization Boxes are used throughout the book to give readers the opportunity to "see" fluid dynamic flows first hand.
- Profiles of both experienced and beginning practitioners of CA are included throughout to add a more personal dimension to the practice of numerical simulations of aerodynamics.
- Best Practices summaries are included at the end of most chapters to provide real-world guidelines for how CA is typically used.
- The website includes access to images, movies, programs, CA codes, additional material, and links to a variety of resources vital to the discussions contained within the book (www.cambridge.org/aerodynamics).

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APPLIED COMPUTATIONAL AERODYNAMICS

A Modern Engineering Approach

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Preface

Aren't there already plenty of excellent books on the topic of CFD? Yes, there are ... if you are a graduate student who wants to learn the intricacies of numerical methods applied to solving the fundamental equations of fluid dynamics. However, we believe that a paradigm shift has taken place in CFD, where the development of algorithms and codes has largely been replaced by people applying well-established codes to real-world applications. While this is a natural progression in any field of science and engineering, we do not believe that the paradigm shift has filtered into the academic world. In academia, undergraduates learning about aerodynamics are still going through theories and applications that were being taught 40 or 50 years ago. We believe that it is time to write a book for people who want to be “intelligent users” of CA, not for those who want to continue developing CA tools. We strongly endorse the perspective of David Darmofal and Earll Murman of MIT (see AIAA Paper 2001–0870):

Within aerodynamics, the need for re-engineering the traditional curriculum is critical. Industry, government, and (to some extent) academia has seen a significant shift away from engineering science and highly specialized research-oriented personnel toward product development and systems-thinking personnel. While technical expertise in aerodynamics is required, it plays a less critical role in the design of aircraft than in previous generations. In addition to these influences, aerodynamics has been revolutionized by the development and maturation of computational methods. These factors cast significant doubt that a traditional aerodynamics curriculum with its largely theoretical approach remains the most effective education for the next generation of aerospace engineers. We believe that change is in order.

We agree completely and believe that CA needs to be brought into the undergraduate classroom as soon as possible. That is why we have written this book!

The target audience for *Applied Computational Aerodynamics* is advanced undergraduates in aerospace engineering who want (or need) to learn CA in

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the broad context of learning to do computational investigations, while also learning engineering methods and aerodynamics. In addition, we believe that working engineers who need to apply CA methods, but who have no CA background, will also find the book valuable.

The educational objectives of the book include: (1) providing a context for computational aerodynamics within aeronautical engineering; (2) learning how to approach and solve computational problems; and (3) providing an entry into the literature by including numerous references and trying to put them in some sort of relevant context. Our overall goal, as mentioned previously, is to educate competent and intelligent users (or even observers) of CA, which will be accomplished through the use of well-defined projects where students will learn how to use the available tools within the context of understanding aerodynamics.

The contents of the book include: a brief history of computational aerodynamics and computers (why and how CA is used); engineering problem solving with emphasis on using the computer, but in the broad context of experimental, analytical, and engineering methods; review of the governing equations used in CA; an introduction to aerodynamic concepts; “classical” linear computational aerodynamics methods; the central idea of CFD – the numerical solution of PDEs; geometry and grids; viscosity and turbulence models; the art of CA, including rules of thumb, overall approaches to the simulation of aerodynamics, grid generation, convergence, grid studies, and flow visualization; and projects illustrating both CA and aerodynamics. The book and accompanying website (www.cambridge.org/aerodynamics) will provide access to CA software so that anyone within an academic or industrial environment will be able to accomplish the various projects with readily available computer resources. Finally, we want to reiterate what we are doing and what we are not doing. This is a book designed to teach aerodynamics through the use of modern computational tools. This is not a book for CFD algorithm developers (those books have already been written).

The presentation of material in this book presumes that the reader previously completed a course in fluid mechanics or aerodynamics, although readers without any background in aerodynamics will be able to work through the concepts presented (with a little extra effort). The typical student using this book may still be learning about basic engineering and aerodynamics, but probably does not have well-developed skills in computational problem solving. The book was written with these students in mind. It contains several unique features, including biographies of people who work in computational aerodynamics, as well as concept boxes that help explain certain ideas more clearly. Projects are included for most chapters, as well as some traditional problems where appropriate. We have found that projects work well with the material in this book, and suggest that you perform projects rather than just complete homework problems. We hope that you enjoy our approach

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to computational aerodynamics while you learn! Sample course outlines are also available to aid instructors in presenting the material.

There are a number of biographical sketches throughout this book which we hope will be interesting and possibly inspirational as you look forward to your career. We have included a wide variety of profiles, including those from some “up and coming” young researchers. The response to our requests for profiles was so positive, we actually received many more than we could publish. All of the profiles, however, are available at www.cambridge.org/aerodynamics.

Unique Features

Computational Aerodynamics is such a new and vibrant field, and we believe that new and vibrant ways to interact with the subject matter are important. To that end, we have included a number of unique features that should make studying Computational Aerodynamics more enjoyable. These include:

- Various computer programs are used within the projects contained in the book, all of which are open source and accessible to students and practicing engineers alike.
- CA Concept Boxes appear throughout the book to make material more relevant and to provide interesting asides from the material at hand.
- Flow Visualization Boxes are used throughout the book to give readers the opportunity to “see” fluid dynamic flows first hand.
- Profiles of both experienced and beginning practitioners of Computational Aerodynamics are included throughout the book to give a more personal dimension to the study of this material.
- Summaries of Best Practices are included at the end of most chapters to provide real-world guidelines for how Computational Aerodynamics is typically used.
- Access to a website with images, movies, programs, additional material, and links to a variety of resources vital to the discussions contained within the book (www.cambridge.org/aerodynamics).

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The book contains a number of biographical sketches of people who work in the field of computational aerodynamics – we thank W. Mark Saltzman of Yale University for this wonderful idea. Among those who provided sketches are: Ken Badcock of the University of Liverpool; Tracie Barber of the University of New South Wales; Marsha Berger of New York University; Tuncer Cebeci of Long Beach State University (who provided the material for the sketch of A.M.O. Smith); Kozo Fujii of the Japan Aerospace Exploration Agency; Karen Gundy-Burlet, Larry Erickson, Scott Murman, and Tom Pulliam of NASA Ames Research Center; Zach Hoisington of The Boeing Company; Kerstin Huber of the German Aerospace Center (DLR); Mark Lewis of the University of Maryland; Bob MacCormack and

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Abbreviations

ACSYNT	AirCRAFT SYNTHeSis design program
ADI	Alternating Direction Implicit
AF	Approximate Factorization
AFLR	Advancing-Front/Local-Reconnection
AFM	Advancing Front Method
AGARD	Advisory Group for Aeronautics Research and Development (NATO, later replaced by RTO)
AGPS	Aero Grid and Paneling System
AIAA	American Institute of Aeronautics and Astronautics
ALM	Advancing Layer Method
AMR	Adaptive Mesh Refinement
ARC2D	Ames Research Center 2D flow solver
ARC3D	Ames Research Center 3D flow solver
ASME	American Society of Mechanical Engineers
BCFD	Boeing CFD flow solver
CA	Computational Aerodynamics
CAD	Computer Aided Design
CAGD	Computer Aided Geometry Design
CALSPAN	Cornell Aeronautical Laboratory Corporation
CAM	Computer Aided Manufacturing
CART3D	CARTesian 3D flow solver
CASI	Canadian Aeronautics and Space Institute
C ² A ² S ² E	Center for Computer Applications in AeroSpace Science and Engineering
CDC	Control Data Corporation
CFD	Computational Fluid Dynamics
CFL	Courant-Friedrichs-Lewy number
CFL3D	Computational Fluids Laboratory 3D flow solver
CGI	Computer Generated Imagery
CGNS	CFD General Notation System
CPU	Central Processing Unit
CRM	Common Research Model

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Abbreviations

CUBRC	CALSPAN/University of Buffalo Research Center
DATCOM	USAF Stability and Control DATA COMpendium
DDES	Delayed Detached-Eddy Simulation
DES	Detached-Eddy Simulation
DLR	Deutsches Zentrum für Luft- und Raumfahrt (German Aerospace Center)
DNS	Direct Numerical Simulation
DNW	Deutsch-Niederländisches Windkanal (German-Dutch Wind Tunnels)
DOD	Department of Defense
DOE	Department of Energy
DPW	Drag Prediction Workshop
EADS	European Aeronautic Defence and Space Company
EASM	Explicit Algebraic Stress Model
EARSM	Explicit Algebraic Reynolds Stress Model
EFD	Experimental Fluid Dynamics
ENSICA	École Nationale Supérieure d'Ingénieurs de Constructions Aéronautiques (French National Higher School of Aeronautical Construction)
ENSOLV	Euler Navier-Stokes SOLVer
ESDU	Engineering Science Data Unit
EVM	Eddy Viscosity Model
FAST	Flow Analysis Software Toolkit
FDE	Finite Difference Equation
FLOP	Floating Point OPERATION
FLOMANIA	FLOW physics Modelling – AN Integrated Approach
FOI	Totalförsvarets Forskningsinstitut (Swedish Defence Research Agency)
FORTRAN	FORMula TRANslation program language
FTF	Flap Track Fairing
FUN3D	Full Unstructured Navier-Stokes 3D flow solver
GASP	General Aerodynamic Simulation Program
GIGO	Garbage In/Garbage Out
GIS	Grid-Induced Separation
GMGG	Geometry Modeling and Grid Generation
GPU	Graphics Processor Unit
GUI	Graphics User Interface
HPC	High Performance Computing
HSCT	High Speed Civil Transport
IBM	International Business Machines
ICASE	Institute for Computer Applications in Science and Engineering
IDDES	Improved Delayed Detached-Eddy Simulation
I/O	Input/Output

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IGES	Initial Graphics Exchange Specification
ISAE	Institut Supérieur de l'Aéronautique et de l'Espace (merged institute consisting of ENSICA and SUPAERO)
ISAS	Institute of Space and Aeronautical Sciences
JATO	Jet-Assisted Take Off
JAXA	Japan Aerospace Exploration Agency
KTH	Kungliga Tekniska Högskolan (Royal Institute of Technology)
LES	Large Eddy Simulation
LEX	Leading-Edge EXtension
LHS	Left Hand Side
LIC	Line Integral Convolution
LINAIR	LINear AIR vortex lattice code
LU	Lower/Upper
MDO	Multidisciplinary Design Optimization
MTVI	Modular Transonic Vortex Interaction
N+1, N+2, N+3	Next Generation, Second Generation, Third Generation aircraft technology
NACA	National Advisory Committee for Aeronautics
NAL	National Aerospace Laboratory (Japan)
NAS	NASA Advanced Supercomputer division (formerly Numerical Aerodynamic Simulation)
NASA	National Aeronautics and Space Administration
NLR	Nationaal Lucht- en Ruimtevaartlaboratorium (Dutch National Aerospace Lab)
NRC	National Research Council
NSF	National Science Foundation
NSU3D	Navier-Stokes Unstructured 3D flow solver
NTF	National Transonic Facility
ODE	Ordinary Differential Equation
ONERA	Office National d'Etudes et Recherches Aérospatiales (French Aerospace Lab)
OVERFLOW	OVERset grid FLOW solver
PANDA	Program for ANalysis and Design of Airfoils
PC	Personal Computer
PDE	Partial Differential Equation
PIV	Particle Image Velocimetry
PMARC	Panel Method Ames Research Center
PMB	Parallel Multi-Block
PNS	Parabolized Navier-Stokes
PSB	Periodic Suction and Blowing
PSP	Pressure Sensitive Paint
PSC	Personal SuperComputer
RAE	Royal Aeronautical Establishment

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RANS	Reynolds-Averaged Navier-Stokes equations
RHS	Right Hand Side
RISC	Reduced Instruction Set Computing
RMS	Root Mean Square
RTO	Research and Technology Organization (NATO, follow on to AGARD, later replaced by STO)
SLOR	Successive Line Over-Relaxation
SA	Spalart-Allmaras
SARC	Spalart-Allmaras with Rotation Correction
SGS	Sub-Grid Scale
SOR	Successive Over-Relaxation
SSBD	Shaped Sonic Boom Demonstrator
SST	Shear-Stress Transport
STO	Science and Technology Organization (NATO, follow on to RTO)
SUPAERO	École Nationale Supérieure de l'Aéronautique et de l'Espace (French National Higher School of Aeronautics and Space)
TE	Truncation Error
TetrUSS	Tetrahedral Unstructured Software System
TLNS	Thin-Layer Navier-Stokes
TOGW	Take-Off Gross Weight
TSDE	Transonic Small Disturbance Equation
TVD	Total Variation Diminishing
UCAV	Unmanned Combat Air Vehicle
UPACS	Unified Platform for Aerospace Computational Simulation
URANS	Unsteady RANS
USAF	U.S. Air Force
USAFA	U.S. Air Force Academy
USC	University of Southern California
VLES	Very Large Eddy Simulation
VLM	Vortex Lattice Method
VORLAX	VORtex LAttice code
WMLES	Wall Modeled LES

Nomenclature

a	speed of sound (acoustic speed)
\vec{a}	acceleration vector
A	axial force <i>or</i> area
AR	wing aspect ratio, $\equiv b^2 / S$
b	wing span
c	chord <i>or</i> wave speed
c_p	specific heat at constant pressure
c_v	specific heat at constant volume
C_d	section (airfoil) drag coefficient, $\equiv d / q_\infty c$
C_D	drag coefficient, $\equiv D / q_\infty S$
C_{D_0}	zero-lift drag coefficient
C_{D_i}	induced drag coefficient
C_{DES}	constant in DES turbulence model
C_f	local skin friction coefficient, $\equiv \tau / q_\infty$
C_F	total skin friction coefficient, $\equiv D_f / q_\infty S$
C_l	section (airfoil) lift coefficient, $\equiv l / q_\infty c$
$C_{l_{MAX}}$	section (airfoil) maximum lift coefficient
C_{l_α}	section (airfoil) lift curve slope, $\equiv \partial C_l / \partial \alpha$
C_L	lift coefficient, $\equiv L / q_\infty S$
$C_{L_{MAX}}$	maximum lift coefficient
C_{L_α}	lift curve slope, $\equiv \partial C_L / \partial \alpha$
C_m	section (airfoil) pitching moment coefficient, $\equiv m / qc^2$
C_{m_α}	section (airfoil) pitching moment curve slope, $\equiv \partial C_m / \partial \alpha$
C_M	pitching coefficient, $\equiv M / q_\infty S c$
C_{M_α}	pitching moment curve slope, $\equiv \partial C_M / \partial \alpha$
C_p	pressure coefficient, $\equiv (p - p_\infty) / q_\infty$
d	section (airfoil) drag <i>or</i> distance to the wall
D	drag
e	wing span efficiency factor <i>or</i> specific energy
E	total energy
f_x, f_y, f_z	forces in x, y, z directions
\vec{F}	force vector, $\equiv f_x \hat{i} + f_y \hat{j} + f_z \hat{k}$
F, G, H	conserved flux quantities in x, y, z directions

g	gravity
G	amplification factor
h	enthalpy, $\equiv e + p / \rho$
H	helicity density, $\equiv \vec{V} \cdot \vec{\omega}$
i, j, k	structured grid indices
$\hat{i}, \hat{j}, \hat{k}$	unit vectors in x, y, z directions
J	structured grid transformation Jacobian
k	thermal conductivity coefficient <i>or</i> turbulent kinetic energy
k_t	turbulent thermal conductivity
l	section (airfoil) lift <i>or</i> characteristic length
\vec{l}	vector along a curve
L	lift
m	mass <i>or</i> section (airfoil) pitching moment <i>or</i> doublet strength
M	Mach number, $\equiv V / a$ <i>or</i> pitch moment
M_{crit}	critical Mach number
M_{DD}	drag divergence Mach number
n	time index
N	normal force
p	pressure
Pr	Prandtl number, $\equiv \mu c_p / k$
q	pitch rate <i>or</i> dynamic pressure, $\equiv \rho V^2 / 2 = \gamma p M^2 / 2$
Q	Q criterion, $\equiv \left(\vec{\xi} ^2 - \vec{S} ^2 \right)$ <i>or</i> conserved flow variables <i>or</i> heat
r	radius
R	universal gas constant, $= c_p - c_v$
Re	Reynolds number, $\equiv \rho V l / \mu$
S	reference area <i>or</i> surface area <i>or</i> strain rate
\vec{S}	rate of shearing
t	thickness <i>or</i> physical time
T	temperature
u, v, w	velocity components in x, y, z directions
V	velocity magnitude
\vec{V}	velocity vector, $\equiv u\hat{i} + v\hat{j} + w\hat{k}$
∇	volume
W	work
\vec{x}	position vector
x, y, z	Cartesian coordinates
y^+	wall unit distance normal to surface
<i>Greek</i>	
α	angle of attack
$\dot{\alpha}$	plunge rate
β	angle of sideslip
δ	thickness

δ_{ij}	Kronecker delta
Δ	grid spacing in DES turbulence model
ε	turbulent dissipation
ϕ	linearized velocity potential
Φ	velocity potential <i>or</i> viscous dissipation
γ	ratio of specific heats, $\equiv c_p / c_v$ <i>or</i> local circulation
Γ	circulation <i>or</i> vortex strength
η	Kolmogorov length microscale
λ	taper ratio, c_t / c_r <i>or</i> stability parameter
Λ	wing sweep <i>or</i> source strength
μ	fluid viscosity <i>or</i> Mach angle
μ_t	turbulent viscosity
ν	kinematic viscosity, $\equiv \mu / \rho$ <i>or</i> Courant number, $\equiv c\Delta t / \Delta x$
ν_t	turbulent eddy viscosity
ρ	fluid density
τ	shear stress <i>or</i> computational domain time <i>or</i> Kolmogorov temporal microscale
ω	vorticity
ξ	rotational velocity
ξ, η, ζ	computational domain spatial coordinates
ψ	stream function
<i>Subscripts or Superscripts</i>	
f	friction
LE	leading edge
o	stagnation/total property
r	root
ref	reference value
t	tip <i>or</i> turbulent
w	wave
wet	wetted (in contact with the fluid)
*	nondimensional quantity
∞	freestream condition