

## Optical Tweezers

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Combining state-of-the-art research with a strong pedagogical approach, this text provides a detailed and complete guide to the theory, practice and applications of optical tweezers. In-depth derivation of the theory of optical trapping and numerical modelling of optical forces are supported by a complete step-by-step design and construction guide for building optical tweezers, with detailed tutorials on collecting and analysing data. Also included are comprehensive reviews of optical tweezers research in fields ranging from cell biology to quantum physics.

Featuring numerous exercises and problems throughout, this is an ideal self-contained learning package for advanced lecture and laboratory courses and an invaluable guide to practitioners wanting to enter the field of optical manipulation.

The text is supplemented by the website [www.opticaltweezers.org](http://www.opticaltweezers.org), a forum for discussion and a source of additional material including free-to-download, customisable, research-grade software (OTS) for calculation of optical forces, digital video microscopy, optical tweezers calibration and holographic optical tweezers.

**Philip H. Jones** is a Reader in Physics at University College London, where he leads the Optical Tweezers research group.

**Onofrio M. Maragò** is a Researcher at the Istituto per i Processi Chimico-Fisici (CNR-IPCF) in Messina, Italy, where he leads the Optical Trapping research group.

**Giovanni Volpe** is an Assistant Professor at Bilkent University, where he is head of the Soft Matter Lab.

Cambridge University Press  
978-1-107-05116-4 - Optical Tweezers: Principles and Applications  
Philip H. Jones, Onofrio M. Maragò and Giovanni Volpe  
Frontmatter  
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## Principles and Applications

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PHILIP H. JONES  
ONOFRIO M. MARAGÒ  
GIOVANNI VOLPE



**CAMBRIDGE**  
UNIVERSITY PRESS

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University Printing House, Cambridge CB2 8BS, United Kingdom

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[www.cambridge.org](http://www.cambridge.org)

Information on this title: [www.cambridge.org/9781107051164](http://www.cambridge.org/9781107051164)

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First published 2015

Printed in the United Kingdom by TJ International Ltd. Padstow Cornwall

*A catalogue record for this publication is available from the British Library*

ISBN 978-1-107-05116-4 Hardback

Additional resources for this publication at [www.cambridge.org/9781107051164](http://www.cambridge.org/9781107051164)

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**To Annie, Becky, Andrew, Antonella, Carmen and Joana  
For their love and patience**

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## Preface

Since the first demonstration of optical tweezers approximately 30 years ago, they have become widespread both as a subject of research in their own right and as an enabling tool in fields as diverse as molecular biology, statistical physics, materials science and quantum physics. Currently the number of active research groups worldwide is in the hundreds – and counting. Furthermore, with the advent of commercially available optical tweezers and low-cost lab kits, optical tweezers experiments can now be found as a common instructional tool in advanced undergraduate and graduate laboratories. This broad interest gives rise to a pressing need for a reference textbook covering the principles and applications of optical tweezers. We began our journey of writing this book with the aim of filling this gap. Therefore we sought to write a textbook with a strong pedagogic approach to both the theory and practice of optical manipulation, supplemented by an overview of the current state of the art in optical manipulation research, and supported by exercises and problems. Eventually, this book saw the light of day.

This book comprises three parts. Part I covers the theory of optical tweezing, providing intuitive and rigorous explanations of the physics behind optical trapping and manipulation, an introduction to the numerical methods most commonly employed in the study of optical forces and torques, and a detailed explanation of the dynamics of optically trapped particles. Part II focuses on the experimental practice of optical manipulation, including both the implementation of a working optical tweezers set-up – complete with detailed step-by-step advice on its construction, on troubleshooting and on the acquisition and analysis of data – and instructions on how to develop more advanced optical manipulation techniques. Parts I and II both include numerous exercises to illustrate the concepts, ideas and techniques discussed, and each chapter ends with problems to solve as a starting point for further investigations. Finally, Part III provides an overview of some of the most exciting applications that optical tweezers have found in various fields, from the study of biological systems to the investigation of the quantum limit for trapped mesoscale objects. Furthermore, we have enhanced this book with an extensive supplementary material, available online for download from the book website at [www.opticaltweezers.org](http://www.opticaltweezers.org). This includes, in particular, the comprehensive OTS – the Optical Tweezers Software toolbox, which we encourage readers to download, use and develop further.

Finally, we wish to thank all the colleagues and friends who have contributed to the writing of this book with their advice, input and encouragement. In particular, our special thanks go to Giuseppe Pesce for his help in writing Chapters 8, 9 and 11, Rosalba Saija for her constant advice on scattering theory and computational issues covered in Chapters 5 and 6, Agnese Callegari and S. Masoumeh Mousavi for their help in developing the OTS toolbox, Giorgio Volpe for his help in writing Chapter 7, and Juan José (Juanjo) Sáenz for

his critical reading of several chapters. We have also received a lot of help and assistance from Ferdinando Borghese, Maria Grazia Donato, Barbara Fazio, Marco Grasso, Pietro Gucciardi, Antonella Iatì, Alessia Irrera, Fatemeh Kalantarifard, Alessandro Magazzù and Mite Mijalkov. We have greatly profited from discussions with Ennio Arimondo, Paolo Denti, Roberto Di Leonardo, Andrea C. Ferrari, Chris Foot, Simon Hanna, Alper Kiraz, Isabel Llorente Garcia, Oliver Morsch, Antonio Alvaro Ranha Neves, Ferruccio Renzoni, Maurizio Righini, Antonio Sasso, Salvatore Savasta, Stephen Simpson and Cirino Vasi. We would also like to acknowledge the students and postdoctoral researchers in our laboratories, whose hard work has permitted us to spare the time needed to write this book. It goes without saying that we claim full ownership of any remaining errors.

Phil Jones  
Onofrio Maragò  
Giovanni Volpe