Analytical Modeling of Heterogeneous Cellular Networks

This self-contained introduction shows how stochastic geometry techniques can be used for studying the behavior of heterogeneous cellular networks (HCNs). The unified treatment of analytic results and approaches, collected for the first time in a single volume, includes the mathematical tools and techniques used to derive them. A single canonical problem formulation encompassing the analytic derivation of the signal to interference plus noise ratio (SINR) distribution in the most widely used deployment scenarios is presented, together with applications to systems based on the 3GPP- LTE standard, and with implications of these analyses on the design of HCNs. An outline of the different releases of the LTE standard and the features relevant to HCNs is also provided.

The book is a valuable reference for industry practitioners looking to improve the speed and efficiency of their network design and optimization workflow, and for graduate students and researchers seeking tractable analytical results for performance metrics in wireless HCNs.

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Analytical Modeling of Heterogeneous Cellular Networks

Geometry, Coverage, and Capacity

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Preface

The ever-rising demand for wireless data means that conventional cellular architectures based on large "macro" cells will soon be unable to support the anticipated density of high-data-rate users. Thus future wireless network standards envisaged by standards bodies like the Third Generation Partnership Project (3GPP), such as LTE Release 12 and later, rely on the following three ways of increasing system capacity: (a) additional spectrum; (b) enhanced spectral efficiency; and (c) offloading from the cellular network onto, say, WiFi.

So-called "small" cells are an attractive method of increasing spectral efficiency by means of spatial reuse of resources. Small cells can also exploit the fact that additional spectrum in the coming years will be freed up at higher frequencies, where path loss is higher than in the frequencies currently employed in macrocellular networks. A dense deployment of small cells can achieve the desired objective of high system capacity. However, such deployments are unlikely to be found outside of high-traffic areas such as major population centers. Thus, basic connectivity and mobility support will continue to be handled by macrocells. In other words, the wireless cellular network of the future is likely to be a *heterogeneous cellular network* (HCN), with more than one class of *base station* (BS).

How HCNs are studied and designed today

One of the most important metrics of network performance and user experience is the *signal to interference plus noise ratio*, or SINR, defined as the ratio of received signal power to the total received power from all sources other than the desired transmitter (i.e. from all *interferers*), plus the thermal noise power at the receiver (which is always present, even in the absence of any interferer). On a given link between a serving BS and a user, the SINR determines the bit error rate on the link, and therefore whether or not the user is *covered* (i.e. SINR exceeds some threshold), and, if it is covered, what the *capacity* (i.e. maximum achievable data rate) of that link is. Thus, a network operator wanting to optimize the operation of an HCN needs to know the *spatial distribution* of the SINR and its dependence on the deployment parameters of the HCN. On the downlink (i.e. a link for transmissions from a BS to a user), for example, this means the distribution of the SINR at an arbitrary user location in the HCN, and its dependence on the relative densities and transmit powers of the different classes, or *tiers*, of BSs in the network.

The usual method used to determine the spatial SINR distribution is extensive simulation. Simulation certainly has the advantage of being able to study any desired scenario to any desired depth of detail. However, this requires one to simulate separately every possible scenario of interest, including every possible choice of deployment parameters. As the number of combinations of deployment parameters rises exponentially in the number of tiers of the HCN, we see that HCN simulation scenarios are much more numerous than single-tier macrocellular network simulation scenarios, and an exhaustive simulation study of all possible scenarios of interest is time-consuming and expensive, if not altogether infeasible. Further, a partial investigation of a limited number of scenarios makes it difficult to draw inferences for new scenarios that have not been studied.

New results in modeling and analysis of HCNs

Since 2011, several theoretical results have contributed greatly to our understanding of the behavior of HCNs. We now have mathematically tractable analytical models, scalable to arbitrary numbers of tiers, that yield important insights into the SINR distribution throughout the network. Assuming the locations of BSs in the tiers are given by points of Poisson point processes (PPPs) in the plane, we can show that the *distribution of the SINR at an arbitrary location in the network can be calculated exactly and with low numerical complexity, and depends only on certain combinations of the network deployment parameters*. Given that, as recently as 2008, the analytic formula for the distribution of SINR in a single-tier network was unknown and this distribution had to be determined via simulation only, it is truly remarkable how far we have come in so short a time.

The objective of this book is to provide a self-contained exposition of the recent body of results, based on *stochastic geometry*, on SINR distribution in HCNs. Toward this goal, we also develop all the mathematical tools and techniques used to derive these results. It is seen that there is a remarkable unity in the problem of determining the SINR distribution, across all the different kinds of deployments of interest, and that the same basic set of techniques enables analytic treatment of all these scenarios. While it is the author's intention to sum up the "state of the art" as of the time of writing, this is currently a very active area of research, and new scenarios are being investigated every month. It is expected that the reader of this book will be well equipped to understand such future work, and even extend it.

A feasible and efficient workflow for an operator planning a new HCN deployment, or optimizing an existing one, is to use these analytic results to eliminate from consideration a large subset of possible deployments (specified by the deployment parameters) as unable to deliver the desired coverage and/or capacity. Then the few deployment parameter choices that are deemed interesting (based on this analysis) can be investigated exhaustively using simulation. Thus it is hoped that the book will also appeal to industry practitioners looking to apply these new results to improve network performance.

Outline of the book

This book focuses on the *downlink*, i.e. on links where the transmitter is one of the BSs in the system, and the receiver is a user. Thus when we talk of the distribution of SINR, we mean the distribution of SINR at the receiver. The following is a brief description of each chapter.

Introduction The case for the importance of HCNs, and the analytical investigation thereof: the importance of the SINR distribution in design and performance analysis, the infeasibility of studying every scenario via simulation alone, and the benefits of analytic modeling of BS and user locations in the network.

Structure of the SINR calculation problem In this chapter, we define the one mathematical problem that describes all the HCN scenarios that will be studied in the book. This chapter also introduces some key mathematical results from the theory of matrices that apply to the SINR distribution calculation problem. As an example of the application of this problem definition, we show mathematically why analytical treatment of the downlink SINR for the "classical" layout of BSs in a hexagonal lattice is impossible, but fairly straightforward if the BSs are points of a PPP.

Poisson point processes This chapter is intended as a self-contained "crash" course in the basic results from the theory of Poisson point processes (PPPs). The contents should be accessible to anyone with knowledge of calculus and do not require knowledge of measure theory. The most important results from the theory of homogeneous PPPs are easy to grasp intuitively. Rather than mathematically rigorous proofs, we offer heuristic arguments that should let the reader understand when to apply which result to the problem at hand.

SINR analysis for a single tier with fixed power This chapter and the following one comprise the main part of the book. They provide a full exposition of the recent advances in our understanding of the behavior of HCNs when the BS and user locations are modeled as points of independent PPPs. This chapter gives the main results for a single-tier network, so the results are applicable to the macrocellular networks of today.

SINR analysis for multiple tiers with fixed powers In this chapter, the results of the previous chapter are extended to apply to HCNs with multiple tiers. Special attention is given to analyzing several kinds of serving BS selection schemes. The joint distribution of the SINRs from candidate serving BSs, as well as the marginal SINR from the actual serving BS, are derived for different choices of candidate serving BSs from the tiers, and the overall serving BS. Applications include the study of camping and coverage probabilities in HCNs, and the need for inter-cell interference coordination (ICIC) when selection bias is applied across tiers for the selection of the overall serving BS from among the candidate serving BSs.

SINR analysis with power control In this chapter, we extend the ideas and results of the previous chapter to the case where the transmitters employ one of three types of power control: non-adaptive, open-loop, or closed-loop. In particular, we analyze the enhanced ICIC (eICIC) scheme in LTE macro-pico HCNs. We also study SINR distributions with a mix of transmitters employing fixed power and open-loop power control. Finally, we study a simple example of a network employing closed-loop power control and discuss some interesting aspects of overall system behavior.

Spectral and energy efficiency analysis Our results on SINR distributions translate directly to results on spectral efficiency of links to arbitrarily located users in the network. However, the area-averaged spectral efficiency in a cell is that averaged over transmissions to all the users in that cell served by a specific BS, and this is dependent upon the scheduling scheme employed by the BS. We mostly restrict ourselves to the simple round robin scheduler (RRS), but will also indicate how to analyze the spectral efficiency when the proportional-fair scheduler (PFS) is employed. We analyze a macro-pico HCN and show the spectral efficiency advantage of a standalone dense single-tier deployment. Finally, we show how to obtain an analysis of energy efficiency from the spectral efficiency.

Closing thoughts: future heterogeneous networks In this concluding chapter, we present some views on the face of future HCNs, and their key issues, challenges, and technologies. These include device-to-device (D2D) transmissions, cognitive radio, and the role of links conforming to the WiFi (IEEE 802.11) standard in future wireless cellular networks.

Intended readership of the book

This book tries to address two groups that overlap only partially:

(1) graduate students and researchers seeking tractable analytical results for topics in wireless HCNs;
(2) industry practitioners looking at improving the speed and efficiency of their network design and optimization workflow by moving away from the present exclusive reliance on simulations to a mix of analysis followed by simulation of a selected subset of scenarios deemed to be of interest from the analysis.

As already mentioned, the level of mathematical knowledge assumed for the reader is calculus and probability at the senior undergraduate level. A list of the probability distributions used in the book is provided in Appendix A. In particular, knowledge of measure theory is not required. Knowledge of 3GPP-LTE is not required, but familiarity with the LTE standard will help the reader to understand the applications of the results presented in the book. An outline of the different releases in the LTE standard and the features relevant to HCNs in each one is provided in Appendix B.

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If I had any doubts as to the feasibility of pulling off this book-writing project, they were put to rest by the enthusiasm with which my tentative proposal for the book was greeted by my director, Watanabe-san, my editor at Cambridge University Press, Phil Meyler, and by the DII general counsel, Jeremy Tucker, Esq., who negotiated the final contract with Cambridge. My heartfelt thanks to all of them.

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Notation

$\sim$	distributed as
${}_2F_1(a,b;c;z)$	Gaussian hypergeometric function
α	slope of path-loss model (path-loss exponent)
$\mathit{\Gamma}$	random variable denoting SINR
γ	SINR threshold
$\Gamma(z)$	gamma function
$\gamma(z,a)$	lower incomplete gamma function
Φ, Ψ	PPP of BS locations
$\tilde{\Phi}, \tilde{\Psi}$	PPP of received powers from the BSs of the PPP Φ or Ψ
x	scalar
$\boldsymbol{x}$	vector
$\mathbf{A}$	matrix
X	random variable
$f_X(x)$	PDF of X evaluated at x
$\boldsymbol{X}$	random vector
$f_{\boldsymbol{X}}(\boldsymbol{x})$	joint PDF of $\boldsymbol{X}$ evaluated at $\boldsymbol{x}$
j	$\sqrt{-1}$
K	intercept of path-loss model
b	BS belonging to a network
H_b	fade attenuation on link from BS b to user
Y_b	received power at user from BS b
n_{tier}	total number of tiers in the HCN
n_{open}	number of accessible (open) tiers in the HCN
I	serving tier
U_i	received power at user from candidate serving BS in tier i
V_i	total received power at user from all BSs in tier i other than the candidate serving BS
W	total received power at user from all BSs in HCN
$\mathcal{A}$	set or event
$1_{\mathcal{A}}(\cdot)$	indicator function of the set $\mathcal{A}$
$\mathbb{E}[X]$	expectation of the random variable X
$\mathbb{P}(\mathcal{A})$	probability of the event $\mathcal{A}$, also equal to $\mathbb{E}[1_{\mathcal{A}}]$
$\mathbb{R}$	the set of real numbers, also written $(-\infty,\infty)$
$\mathbb{R}_+$	the set of non-negative reals, also written $[0,\infty)$
$\mathbb{R}_{++}$	the set of positive reals, also written $(0,\infty)$

Acronyms and abbreviations

ABS	almost-blank subframe
BS	base station
(C)CDF	(complementary) cumulative distribution function
CDMA	code division multiple access
CLPC	closed-loop power control
CRE	cell range expansion
CSG	closed subscriber group
CSMA	carrier-sensed multiple access
CSR	complete spatial randomness
D2D	device-to-device
EM	expectation-maximization
FDD	frequency division duplexing
feICIC	further enhanced ICIC
HCN	heterogeneous cellular network
HeNB	home enhanced NodeB
ICIC	inter-cell interference coordination
ISD	inter-site distance
LTE	long-term evolution
LTE-A	LTE-Advanced
MHP	Matérn hard-core process
MIMO	multiple-input multiple-output
MMSE	minimum mean squared error
MTC	machine-type communication
OA	open access
OFDM	orthogonal frequency division multiplexing
OFDMA	orthogonal frequency division multiple access
OLPC	open-loop power control
P2P	peer-to-peer
PDF	probability density function
PFS	proportional-fair scheduler
PMF	probability mass function
PPP	Poisson point process
PSK	phase shift keying
REB	range expansion bias

RRH	remote radio head
RRS	round robin scheduler
RSRP	reference symbol received power
RSRQ	reference symbol received quality
SINR	signal to interference plus noise ratio
SIR	signal to interference ratio
TDD	time division duplexing
3GPP	Third Generation Partnership Project
TTI	transmit time interval
UE	user equipment
UMTS	universal mobile telecommunications system