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978-1-107-05017-4 - Why is there Philosophy of Mathematics at All?

Ian Hacking

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WHY IS THERE PHILOSOPHY OF MATHEMATICS AT ALL?

This truly philosophical book takes us back to fundamentals – the sheer experience of proof, and the enigmatic relation of mathematics to nature. It asks unexpected questions, such as ‘What makes mathematics mathematics?’, ‘Where did proof come from and how did it evolve?’, and ‘How did the distinction between pure and applied mathematics come into being?’ In a wide-ranging discussion that is both immersed in the past and unusually attuned to the competing philosophical ideas of contemporary mathematicians, it shows that proof and other forms of mathematical exploration continue to be living, evolving practices – responsive to new technologies, yet embedded in permanent (and astonishing) facts about human beings. It distinguishes several distinct types of application of mathematics, and shows how each leads to a different philosophical conundrum. Here is a remarkable body of new philosophical thinking about proofs, applications, and other mathematical activities.

IAN HACKING is a retired professor of the Collège de France, Chair of Philosophy and History of Scientific Concepts, and retired University Professor of Philosophy at the University of Toronto. His most recent books include *The Taming of Chance* (1990), *Rewriting the Soul* (1995), *The Social Construction of What?* (1999), *An Introduction to Probability and Inductive Logic* (2001), *Mad Travelers* (2002), and *The Emergence of Probability* (2006).

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[More information](#)

*In memory of the first reader of this book, 1960
Paul Whittle 1938–2009*

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Frontmatter

[More information](#)

For mathematics is after all an anthropological phenomenon.
Wittgenstein (1978: 399)

Mathematical activity is human activity . . . But mathematical activity produces mathematics. Mathematics, this product of human activity, 'alienates itself' from the human activity which has been producing it. It becomes a living, growing organism.

(Lakatos 1976: 146)

The birth of mathematics can also be regarded as the discovery of a capacity of the human mind, or of human thought – hence its tremendous importance for philosophy: it is surely significant that, in the semilegendary intellectual tradition of the Greeks, Thales is named both as the earliest of the philosophers and the first prover of geometric theorems.

(Stein 1988: 238)

A square can be dissected into finitely many unequal squares, but a cube cannot be dissected into finitely many unequal cubes. *Proof of the latter:*

In a square dissection the smallest square is not at an edge (for obvious reasons). Suppose now a cube dissection does exist. The cubes standing on the bottom face induce a square dissection at that face, and the smallest of the cubes on that face stands on an internal square. The top face of this cube is enclosed by walls; cubes must stand on this top face; take the smallest – the process continues indefinitely.

(Littlewood 1953: 8)

Cambridge University Press

978-1-107-05017-4 - Why is there Philosophy of Mathematics at All?

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Frontmatter

[More information](#)*Contents*

<i>Foreword</i>	<i>page</i> xiii
I A cartesian introduction	I
1 Proofs, applications, and other mathematical activities	I
2 On jargon	2
3 Descartes	3
A Application	4
4 Arithmetic applied to geometry	4
5 Descartes' <i>Geometry</i>	5
6 An astonishing identity	6
7 Unreasonable effectiveness	6
8 The application of geometry to arithmetic	8
9 The application of mathematics to mathematics	9
10 The same stuff?	11
11 Over-determined?	12
12 Unity behind diversity	13
13 On mentioning honours – the Fields Medals	15
14 Analogy – and André Weil 1940	16
15 The Langlands programme	18
16 Application, analogy, correspondence	20
B Proof	21
17 Two visions of proof	21
18 A convention	21
19 Eternal truths	22
20 Mere eternity as against necessity	23
21 Leibnizian proof	23
22 Voevodsky's extreme	25
23 Cartesian proof	26
24 Descartes and Wittgenstein on proof	26
25 The experience of cartesian proof: <i>caveat emptor</i>	28

Cambridge University Press

978-1-107-05017-4 - Why is there Philosophy of Mathematics at All?

Ian Hacking

Frontmatter

[More information](#)

viii

Contents

26	Grothendieck's cartesian vision: making it all obvious	29
27	Proofs <i>and</i> refutations	30
28	On squaring squares and not cubing cubes	32
29	From dissecting squares to electrical networks	34
30	Intuition	35
31	Descartes <i>against</i> foundations?	37
32	The two ideals of proof	38
33	Computer programmes: who checks whom?	40
2	What makes mathematics mathematics?	41
1	We take it for granted	41
2	Arsenic	42
3	Some dictionaries	43
4	What the dictionaries suggest	45
5	A Japanese conversation	47
6	A sullen anti-mathematical protest	48
7	A miscellany	48
8	An institutional answer	51
9	A neuro-historical answer	52
10	The Peirces, father and son	53
11	A programmatic answer: logicism	54
12	A second programmatic answer: Bourbaki	55
13	Only Wittgenstein seems to have been troubled	57
14	Aside on method – on using Wittgenstein	59
15	A semantic answer	60
16	More miscellany	61
17	Proof	62
18	Experimental mathematics	63
19	Thurston's answer to the question 'what makes?'	66
20	On advance	67
21	Hilbert and the Millennium	68
22	Symmetry	71
23	The Butterfly Model	72
24	Could 'mathematics' be a 'fluke of history'?	73
25	The Latin Model	74
26	Inevitable or contingent?	75
27	Play	76
28	Mathematical games, ludic proof	77
3	Why <i>is</i> there philosophy of mathematics?	79
1	A perennial topic	79
2	What is the philosophy of mathematics anyway?	80

Cambridge University Press

978-1-107-05017-4 - Why is there Philosophy of Mathematics at All?

Ian Hacking

Frontmatter

[More information](#)*Contents*

ix

3	Kant: in or out?	81
4	Ancient and Enlightenment	83
A	An answer from the ancients: proof and exploration	83
5	The perennial philosophical obsession . . .	83
6	The perennial philosophical obsession . . . is totally anomalous	85
7	Food for thought (<i>Matière à penser</i>)	86
8	The Monster	87
9	Exhaustive classification	88
10	Moonshine	89
11	The longest proof by hand	89
12	The experience of out-thereness	90
13	Parables	91
14	Glitter	91
15	The neurobiological retort	92
16	My own attitude	93
17	Naturalism	94
18	Plato!	96
B	An answer from the Enlightenment: application	97
19	Kant shouts	97
20	The jargon	98
21	Necessity	99
22	Russell trashes necessity	100
23	Necessity no longer in the portfolio	102
24	Aside on Wittgenstein	103
25	Kant's question	104
26	Russell's version	105
27	Russell dissolves the mystery	106
28	Frege: number a second-order concept	107
29	Kant's conundrum becomes a twentieth-century dilemma: (a) Vienna	108
30	Kant's conundrum becomes a twentieth-century dilemma: (b) Quine	109
31	Ayer, Quine, and Kant	110
32	Logicizing philosophy of mathematics	111
33	A nifty one-sentence summary (Putnam redux)	112
34	John Stuart Mill on the need for a sound philosophy of mathematics	113
4	Proofs	115
1	The contingency of the philosophy of mathematics	115
A	Little contingencies	116
2	On inevitability and 'success'	116
3	Latin Model: infinity	117
4	Butterfly Model: complex numbers	119
5	Changing the setting	121

Cambridge University Press

978-1-107-05017-4 - Why is there Philosophy of Mathematics at All?

Ian Hacking

Frontmatter

[More information](#)

x

Contents

B	Proof	122
6	The discovery of proof	122
7	Kant's tale	123
8	The other legend: Pythagoras	126
9	Unlocking the secrets of the universe	127
10	Plato, theoretical physicist	129
11	Harmonics works	130
12	Why there was uptake of demonstrative proof	131
13	Plato, kidnapper	132
14	Another suspect? Eleatic philosophy	133
15	Logic (and rhetoric)	135
16	Geometry and logic: esoteric and exoteric	136
17	Civilization without proof	137
18	Class bias	138
19	Did the ideal of proof impede the growth of knowledge?	139
20	What gold standard?	140
21	Proof demoted	141
22	A style of scientific reasoning	142
5	Applications	144
1	Past and present	144
A	The emergence of a distinction	144
2	Plato on the difference between philosophical and practical mathematics	144
3	Pure and mixed	146
4	Newton	148
5	Probability – swinging from branch to branch	149
6	<i>Rein</i> and <i>angewandt</i>	150
7	Pure Kant	151
8	Pure Gauss	152
9	The German nineteenth century, told in aphorisms	153
10	Applied <i>polytechniciens</i>	153
11	Military history	156
12	William Rowan Hamilton	158
13	Cambridge pure mathematics	160
14	Hardy, Russell, and Whitehead	161
15	Wittgenstein and von Mises	162
16	SIAM	163
B	A very wobbly distinction	164
17	Kinds of application	164
18	Robust but not sharp	168

Cambridge University Press

978-1-107-05017-4 - Why is there Philosophy of Mathematics at All?

Ian Hacking

Frontmatter

[More information](#)*Contents*

xi

19	Philosophy and the <i>Apps</i>	169
20	Symmetry	171
21	The representational–deductive picture	172
22	Articulation	174
23	Moving from domain to domain	174
24	Rigidity	176
25	Maxwell and Buckminster Fuller	176
26	The maths of rigidity	179
27	Aerodynamics	181
28	Rivalry	182
29	The British institutional setting	184
30	The German institutional setting	186
31	Mechanics	187
32	Geometry, ‘pure’ and ‘applied’	188
33	A general moral	188
34	Another style of scientific reasoning	189
6	In Plato’s name	191
1	Hauntology	191
2	Platonism	191
3	<i>Webster’s</i>	193
4	Born that way	193
5	Sources	194
6	Semantic ascent	195
7	Organization	196
A	Alain Connes, Platonist	197
8	Off-duty and off-the-cuff	197
9	Connes’ archaic mathematical reality	198
10	Aside on incompleteness and platonism	201
11	Two <i>attitudes</i> , structuralist and Platonist	202
12	What numbers could not be	203
13	Pythagorean Connes	205
B	Timothy Gowers, anti-Platonist	206
14	A very public mathematician	206
15	Does mathematics need a philosophy? No	207
16	On becoming an anti-Platonist	208
17	Does mathematics need a philosophy? Yes	209
18	Ontological commitment	211
19	Truth	212
20	Observable and abstract numbers	213
21	Gowers versus Connes	215

Cambridge University Press

978-1-107-05017-4 - Why is there Philosophy of Mathematics at All?

Ian Hacking

Frontmatter

[More information](#)

xii

Contents

22	The 'standard' semantical account	216
23	The famous maxim	218
24	Chomsky's doubts	220
25	On referring	221
7	Counter-platonisms	223
1	Two more platonisms – and their opponents	223
A	Totalizing platonism as opposed to intuitionism	224
2	Paul Bernays (1888–1977)	224
3	The setting	225
4	Totalities	227
5	Other totalities	228
6	Arithmetical and geometrical totalities	230
7	Then and now: different philosophical concerns	231
8	Two more mathematicians, Kronecker and Dedekind	232
9	Some things Dedekind said	233
10	What was Kronecker protesting?	235
11	The structuralisms of mathematicians and philosophers distinguished	236
B	Today's platonism/nominalism	238
12	Disclaimer	238
13	A brief history of nominalism now	238
14	The nominalist programme	239
15	Why deny?	241
16	Russellian roots	242
17	Ontological commitment	244
18	Commitment	245
19	The indispensability argument	246
20	Presupposition	248
21	Contemporary platonism in mathematics	250
22	Intuition	252
23	What's the point of platonism?	253
24	Peirce: The only kind of thinking that has ever advanced human culture	254
25	Where do I stand on today's platonism/nominalism?	256
26	The last word	256
	<i>Disclosures</i>	258
	<i>References</i>	262
	<i>Index</i>	281

Foreword

This is a book of philosophical thoughts about proofs, applications, and other mathematical activities.

Philosophers tend to emphasize mathematical ‘knowledge’, but as G. H. Hardy said on the first page of his *Apology* (1940), ‘the function of a mathematician is to *do* something, to prove new theorems, to add to mathematics’. I have emphasized the ‘do’. Hardy was writing not only an *Apologia pro vita sua*, but also a mathematician’s *Lament* that he was now too old to create much more mathematics. He also, notoriously, wanted to keep mathematics pure, whereas I believe that the uses, ‘the applications’, are as important as the theorems proved. Neither proof nor application is, however, as clear and distinct an idea as might be hoped.

To reflect on the doing of mathematics, on mathematics as activity, is not to practise the sociology of mathematics. Happily that is now a burgeoning field, from which one can learn much, but what follows is philosophizing, moved by old-fashioned questions – to which I add my title question, why *do* these questions arise perennially, from Plato to the present day?

This book began as the René Descartes Lectures at Tilburg University in the Netherlands, in early October, 2010. (I started writing out the talks on the summer solstice of that year.) The format was three lectures, each followed by comments from two different scholars. The original intention was that the lectures and comments would be published immediately.

I began to realize at the end of the week the extent to which the material needed to mature. The commentators generously agreed to keep their comments. So my first duty is to thank them deeply for their hard work. Hard work? Typically they received, late in the day, some 20,000 words per lecture, of which only 7,000 would be spoken, and they did not even know which ones.

For the first talk, ‘Why Is There Philosophy of Mathematics?’: Mary Leng and Hannes Leitgeb.

For the second talk, ‘Meaning and Necessity – and Proof’: James Conant and Martin Kusch.

For the third talk, ‘Roots of Mathematical Reasoning’: Marcus Giaquinto and Pierre Jacob.

Thank you all.

I originally proposed ‘Proof’ as the series title. That was the title of a thesis, which, together with some work in modal logic, was awarded a PhD by Cambridge University in 1962. It was dominated by my reading of Wittgenstein’s recently published *Remarks on the Foundations of Mathematics*, although much influenced by what was to become Imre Lakatos’ *Proofs and Refutations*, which was being completed in Cambridge as a doctoral dissertation when I began mine.

I have published very little about the philosophy of mathematics, but it has always been at the back of my mind, so the Descartes Lectures were a chance to finish the job. The title ‘Proof’ would give no idea of what the talks would be about, so Stephan Hartmann, the organizer of the events (to whom many thanks), and I hit on ‘Proof, Calculation, Intuition, and A Priori Knowledge’.

Very soon after the Descartes Lectures, in late October 2010, I gave three similar talks at the University of California, Berkeley, beginning with the Howison Lecture, ‘Proof, Truth, Hands and Mind’. Here is how I explained the title, after indulgently admiring my choice of words of one syllable:

Why this title? First, because *proof* has been an essential part of Western mathematics ever since Plato. And Plato thought that mathematics was the sure guide to *truth*. I want also to think of how we do mathematics, in a material way that Plato would hardly have acknowledged. We think with our *hands*, our whole bodies. We communicate with one another not only by talking and writing but also by gesticulating. If I am thinking mathematically I may draw a diagram to take you through a series of thoughts, and in this way pass my thoughts in my *mind* over to yours.

After California I put this material aside while teaching on other topics at the University of Cape Town, and intensely experiencing all too little of that amazing land and its peoples. In January 2011 I did attend the annual meetings of the Philosophical Society of Southern Africa, and the corresponding Society for the Philosophy of Science, near Durban. There I presented, respectively, abridged forms of the first two Descartes/Howison Lectures (Hacking, 2011a, 2011b). I may mention also a contribution to a conference in Israel in honour of Mark Steiner, in December 2011, which began with

Cambridge University Press

978-1-107-05017-4 - Why is there Philosophy of Mathematics at All?

Ian Hacking

Frontmatter

[More information](#)*Foreword*

xv

Pythagoras and ended with P. A. M. Dirac (Hacking, 2012b). Then in November 2012 I did part of the third Descartes Lecture as the Henry Myers Lecture for the Royal Anthropological Institute, London.

In March and April of 2012 I gave six Gaos Lectures at the National Autonomous University of Mexico, at the invitation of Carlos López Beltrán and Sergio Martínez, to whom again many thanks. The title was *The Mathematical Animal*, but in fact the first five lectures covered only the first Descartes Lecture. And so it has come to pass that this book is not the entire set of lectures given in Tilburg, but only the first.

The connection between the present book and my dissertation of 1962 will not be obvious, but *plus ça change*. My title here is, *Why Is There Philosophy of Mathematics At All?* I was astonished, in preparing the present book for the press, to reread the brief preface to my dissertation of 1962: 'We must return to simple instances to see what is surprising, to discover, in fact, why there are philosophies of mathematics at all.' And I may mention that my choice of topics comes from the first edition of Wittgenstein's *Remarks on the Foundations of Mathematics* (1956). The two significant nouns most often used in that edition (to which I prepared my own index) are *Beweis* and *Anwendung*, 'proof' and 'application'.

I thank the Social Science and Humanities Research Council of Canada for awarding me its annual Gold Medal for Research. The cash coming with the medal is rightly dedicated to further research, and much of it was used in preparing this book. I thank James Davies in Toronto and Kaave Lajevardi in Teheran for a lot of help in the home stretch. The final threads were tied up in March 2013 during a blissful time at the Stellenbosch Institute for Advanced Study.