

Nonequilibrium Statistical Physics

Statistical mechanics has been proven to be successful at describing physical systems at thermodynamic equilibrium. Since most natural phenomena occur in nonequilibrium conditions, the present challenge is to find suitable physical approaches for such conditions. This book provides a pedagogical pathway that explores various perspectives. The use of clear language and explanatory figures and diagrams to describe models, simulations, and experimental findings makes it a valuable resource for undergraduate and graduate students and for lecturers teaching at varying levels of experience in the field. Written in three parts, it covers basic and traditional concepts of nonequilibrium physics, modern aspects concerning nonequilibrium phase transitions, and application-orientated topics from a modern perspective. A broad range of topics is covered, including Langevin equations, Lévy processes, directed percolation, kinetic roughening, and pattern formation.

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Nonequilibrium Statistical Physics

A Modern Perspective

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To Claudia and Daniela

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Preface

Since 2006 the authors of this book have been sharing the teaching of an advanced course on nonequilibrium statistical physics at the University of Florence, Italy. This is an advanced course for students in the last year of the master's thesis curriculum in physics, which is attended also by PhD students. If the reader of this Preface is a colleague or a student who already attended a similar course, he or she should not be astonished by the following statement: this book was primarily conceived to organize the contents of our course, because the offer of textbooks on nonequilibrium statistical physics is typically much more limited and specialized than in the case of equilibrium statistical physics. From the very beginning it was clear in our minds that we had to aim at a textbook written for students, neither for experts nor for colleagues. In fact, we believe that a textbook on advanced topics written for the benefit of students can also be useful for colleagues who want to approach these topics, while the contrary does not hold. We dare, indeed, to say that if a book is written devoting special consideration for the understanding of students, it could be beneficial for the whole scientific community. After these preliminary remarks, which should be taken as an omen more than a statement, we want to illustrate and justify the contents of this textbook.

When we started to think about that, the first question that came to our mind was the following: what should it contain? As one could expect, the answer was not unique. It depends not only on our personal taste and interest, but it is also inspired by a sort of "tradition," which changes significantly from university to university as well as from country to country. This is why, after having produced a preliminary list of topics, we asked for the opinion of some Italian and foreign colleagues. Honestly, we did not change that much of the preliminary list after having received their advice, partly because they did not raise strong criticisms and also because we eventually obtained quite incoherent suggestions. As a consequence, if the reader does not agree with our choices, we must admit that we lack strong arguments to tackle their opinion, but we also expect that the reader should not be in the position of raising arguments that are too negative.

A further remark concerns the bibliography. We have decided, as in most textbooks, to limit the number of references. In fact, we have tried to always provide the derivation of the results step by step, and unproved statements are very rare. So, rather than spreading references throughout the book we have preferred to collect them at the end of each chapter, in a section called *Bibliographic Notes*. Here, we give first some general references (either a book or a review), then we give more specific references, always accompanied by a short comment about their relevance to the specific chapter.

Now we come to the very structure of this book. It can be thought as subdivided into three parts. The first two chapters deal with those basic topics that one naturally expects

to find in such a text: Brownian motion, Markov chains, random walks, Langevin and Fokker–Planck equations, linear response theory, Onsager relations, Lévy processes, etc. The third and fourth chapters are devoted to topics that must be present in a *modern* textbook on nonequilibrium physics, namely, nonequilibrium phase transitions. Finally, the last three chapters stem from our personal interests and each one could be sufficient for occupying a volume by itself: kinetic roughening, phenomena of phase-ordering, and pattern formation. Rather than giving here further details about the content of the book, we invite the interested reader to browse the opening to each chapter.

The book is accompanied by a website, <https://sites.google.com/site/nespbook/> that offers additional material. You are invited to report errata or send comments by writing to nespbook@gmail.com.

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Notations and Acronyms

Notations

Throughout this book we assume the Boltzmann constant ($K_B = 1.380658 \cdot 10^{-23} \text{ JK}^{-1}$) to be equal to one, which corresponds to measuring the temperature in joules or an energy in Kelvin.

As for the Fourier transform, we use the same symbol in the real and the dual space, using the following conventions:

$$h(\mathbf{k}) = \int d\mathbf{x} \exp(-i\mathbf{k} \cdot \mathbf{x}) h(\mathbf{x}),$$

$$h(\mathbf{x}) = \frac{1}{(2\pi)^d} \int d\mathbf{k} \exp(i\mathbf{k} \cdot \mathbf{x}) h(\mathbf{k}).$$

Similarly, for the Laplace transform,

$$\omega(s) = \int_0^\infty dt e^{-st} \omega(t),$$

$$\omega(t) = \frac{1}{2\pi i} \text{PV} \int_{a-i\infty}^{a+i\infty} ds e^{st} \omega(s).$$

Here above, PV is the principal value of the integral and $a > a_c$, a_c being the abscissa of convergence.

The friction coefficient of a particle of mass m , i.e. the ratio between the force F_0 acting on it and its terminal drift velocity v_∞ , is indicated by the symbol $\tilde{\gamma} = F_0/v_\infty$, but we frequently use the reduced friction coefficient, $\gamma = \tilde{\gamma}/m$, which has the dimension of the inverse of time. Similar reduced quantities are used in the context of Brownian motion.

The Helmholtz free energy is indicated by the symbol $F = U - TS$, and the free energy density is indicated by f . We often use a free energy functional, also called pseudo free energy functional or Lyapunov functional, and it is indicated by \mathcal{F} . It is the space integral of a function f , $\mathcal{F} = \int d\mathbf{x} f$. The susceptibility, indicated by χ , may be an extensive as well as an intensive quantity, depending on the context.

The space dimension is indicated by d .

Acronyms

ASEP	asymmetric exclusion process
BD	ballistic deposition
BS	Back–Sneppen
BTW	Back–Tang–Wiesenfeld
CA	cellular automaton
CDP	compact directed percolation
CH	Cahn–Hilliard
CIMA	chlorite-iodide-malonic-acid
CTRW	continuous time random walk
DK	Domany–Kinzel
dKPZ	deterministic Kardar–Parisi–Zhang
DLA	diffusion limited aggregation
DLG	driven lattice gas
DP	directed percolation
DyP	dynamical percolation
DW	domain wall
emf	electromotive force
EW	Edwards–Wilkinson
GL	Ginzburg–Landau
GOE	Gaussian orthogonal ensemble
GUE	Gaussian unitary ensemble
HD	high density
KMC	kinetic Monte Carlo
KPZ	Kardar–Parisi–Zhang
LD	low density
LG	lattice gas
LW	Levy walk
MC	maximal current
MF	mean-field
NESS	nonequilibrium steady state
PC	parity conserving
PV	principal value
QFT	quantum field theory
RD	random deposition
RDR	random deposition with relaxation
RG	renormalization group
RSOS	restricted solid on solid
SH	Swift–Hohenberg
SOC	self-organized criticality
TASEP	totally asymmetric exclusion process
TDGL	time-dependent Ginzburg Landau
TW	Tracy–Widom