

## Index

---

- 2-category, 37
  - of quasi-categories, 320
- 2-functor, 44
- 2-of-3 property, 18
- 2-of-6 property, 18
- acyclic cofibration, *see* trivial cofibration
- acyclic fibration, *see* trivial fibration
- adjunction
  - as Kan extension, 11
  - between quasi-categories, 328–332
    - universal property of, 336
  - change of base, 283, 289, 319
  - doctrinal, 51
  - enriched, 47
    - preservation of weighted (co)limits, 121
    - two-variable, 50, 108
  - interaction with lifting properties, 169–170
  - monadic, 80
  - of quasi-categories
    - homotopy coherent, 336
  - parametrized, 37
  - strong monoidal, 51
  - total derived, 26, 28
  - two-variable, 146
    - and lifting properties, 172
    - deformable, 155
    - derived, 149
    - Quillen, 181
  - $\mathcal{V}$ -deformable, 155
- algebra
  - for a monad, 200
  - for a pointed endofunctor, 199
- $\alpha$ -composite, 168, 193
- anodyne, 176
  - inner, 266
  - left or right, 301
- arrow category
  - enriched, 223
- atomic arrow, 285
- bar  $\mathcal{V}$ -homotopical structure, 156
- bar construction, 61
  - enriched, 138
  - homotopical aspects of, 72
  - simplicial, 60
    - enriched, 137
  - universal property of, 119
- base for enrichment, 34
- based object, 38
  - disjoint basepoint–forgetful adjunction, 52
- bicomplete
  - enriched, 115
- bisimplicial object
  - homotopy colimit, 134
- bisimplicial set
  - Reedy cofibrant, 250
- Borel construction, 89
- Bousfield localization, 197
- Bousfield–Kan map, 117
- cartesian closed, 37
- categorical equivalence, 271
  - vs. equivalence of categories, 273
  - vs. weak homotopy equivalence, 273, 315
- category
  - abelian, 30
  - as a quasi-category, 265
  - connected, 127
  - enriched, 33–35
    - cotensored, 49
  - equivalence of, 47

- category (*cont.*)
  - free, 36, 45
  - tensored, 49
  - underlying category of, 41
- filtered, 131
- free, 286
- locally small, 7, 32
- model structure, 177
- of elements, 102, 103
- of enriched categories
  - tensor and internal hom, 108
- of simplices, 64, 102
- small, 6
- symmetric monoidal, 34
  - closed, 37
- cell complex
  - algebraic, 216–220
- cellular
  - cofibration, 212–213, 220
  - decomposition, 189
- chain complex
  - functorial factorization, 203, 227
  - model structures, 176, 234
- change of base, 40, 46, 51
  - adjunction, 283, 289, 319
  - for weighted (co)limits, 125
- classifying space
  - of a category, 88
  - of a commutative monoid, 133
  - of a group, 61, 88
    - universal cover, 68
  - of  $\mathbb{Z}/2$ , 88
- closed  $\mathcal{V}$ -module, 147
  - cotensor form, 147, 149, 154
  - is a tensored, cotensored,  $\mathcal{V}$ -category, 147
- co-graph factorization, 238
- coalgebra
  - for a comonad, 200
  - for a pointed endofunctor, 199
- cobar  $\mathcal{V}$ -homotopical structure, 156
- cobar construction, 63
  - cosimplicial, 63
  - enriched, 137
  - enriched, 138
  - homotopical aspects of, 72
  - universal property of, 119
- cocomplete
  - enriched, 114
- coend, 7
  - as a weighted colimit, 106
- cofiber sequence, 87
- cofibrant object
  - in a  $\mathcal{V}$ -model category, 140
  - in a homotopical category, 24
  - in a model category, 178
  - in a simplicial model category, 56
- cofibrantly generated
  - model category, 69, 174
  - weak factorization system, 173
- cofibration
  - in a model category, 174
  - closure properties, 213
- cofinal
  - functor, 127
  - sequence, 130
- cohomology, 21
- cokernel pair
  - see kernel pair, 105
- colimit
  - absolute, 127
  - as functor tensor product, 59
  - as Kan extension, 11
  - enriched, 99, 121
  - enriched universal property of, 113
  - universal property of, 100
  - weighted, *see* weighted colimit
- combinatorial model category, 69, 182
- comma
  - category, 7, 111
  - quasi-category, 327
- comonad
  - resolution
    - derived, 203
    - of a category, 286, 294
- compact object, 193, 259
- compactly
  - closed, 78
  - generated, 79
- complete
  - enriched, 114
- completion, 252–257
  - Bousfield–Kan, 254
- cone
  - shape of a, 99, 104
- conical
  - limit or colimit, 112–114
- conservative, 323
- contractible space of choices, 268
- contracting homotopy, 67, 134
- convenient category
  - of topological spaces, 38, 82
- copower, 7
  - as a discrete simplicial tensor, 58
  - as tensor, 50
- coreflective, *see* reflective
- cosimplicial object, *see* simplicial object

- cotensor, 8
  - as weighted limit, 110
  - in an enriched category, 49
- co-Yoneda lemma, 12, 59, 106
- cylinder object
  - for quasi-categories, 271
- décalage, 313
- Day convolution, 300
- deformable
  - adjunction, 26
    - enriched, 154
  - category, 28
  - functor, 24, 72
    - enriched, 153
    - total derived functor as absolute Kan extension, 27
  - two-variable adjunction, 149
- deformation, 23
- $\Delta_+$ 
  - interval representation of opposite category, 66
  - symmetric monoidal structure, 299
  - universal property of, 66
- dense subcategory, 12, 237
- density comonad, 202
  - enriched, 225
- density theorem, 12, 106
- derived functor, 17, 23
  - classical, 29–30
  - composite of, 26
  - enrichment, 153
  - middle, 163
  - of an enriched functor, 163
  - point-set, 23
  - total, 6, 17, 23
  - via a deformation, 25
- diagonal
  - of a simplex, 291
- diagram category
  - homotopical structure of, 82
  - simplicial tensor structure of, 55
- directed graph, 8
- discrete right fibration, 102, 168
- DK-equivalence, 47
- Dold–Kan correspondence, 54
- duality, 10
- edgewise subdivision, 119
- Eilenberg–Zilber
  - lemma, 8, 250
  - property, 251
- end, 8
  - enriched, 109
- endofunctor
  - pointed, 199
  - well pointed, 200
- enriched
  - adjunction, 47
    - deformable, 154
    - preservation of weighted (co)limits, 121
    - two-variable, 50, 108, 155
  - arrow category, 223
  - bar construction, 138
  - bicomplete, 115
  - category, 33–35
    - free, 36, 45
    - homotopy, 151, 152
    - underlying category of, 41
  - cobar construction, 138
  - complete and cocomplete, 114
  - cosimplicial cobar construction, 137
  - density comonad, 225
  - end, 109
  - equivalence of categories, 47
  - functor, 44
    - as lax module map, 147
    - constant, 112
    - derived, 153
    - fully faithful, 47
    - representable, 46
  - functorial factorization, 224–228
  - Kan extension, 116
  - lifting property, 222, 228–235
  - limit or colimit, 99, 121
  - model category, 235–237
  - natural transformation, 45
  - representable functor, 44
  - simplicial bar construction, 137
  - small object argument, 222, 224–228
    - Garner, 233
    - Quillen, 234
  - underlying 2-functor, 46
  - universal property of, 113
  - weak factorization system, 233
  - Yoneda lemma, 109, 110
- epimorphism, 168
- equivalence
  - in a 2-category, 321
  - in a quasi-category, *see* isomorphism of Kan complexes, 306
  - of quasi-categories, 271, 305, 321
- essentially surjective, 47
- evil, 243
- extension, 15
  - extension of scalars, 5, 116

- extra degeneracy, *see* contracting homotopy
- factorization system, *see also* orthogonal factorization system
  - proper, 202, *see also* left proper
  - well-copowered, 202
- fat geometric realization, 134, 184
- fibrant object
  - in a  $\mathcal{V}$ -model category, 140
  - in a homotopical category, 26
  - in a model category, 178
  - in a simplicial model category, 56
- fibration
  - in a model category, 174
  - inner, 266
  - Kan, 176
  - left or right, 301
- filler, 265
- filtered category, 131
- final functor, 127, 129
- formal category theory
  - of quasi-categories, 318, 330
- fully faithful
  - enriched, 47
- functor
  - additive, 21, 29
  - continuous, 44
    - non-example, 53
  - cotensor product, 62
  - deformable, 24, 72
  - enriched, 44
    - as lax module map, 147
    - constant, 112
    - fully faithful, 47
    - representable, 44, 46
  - exact, 30
  - hom, *see* functor cotensor product
  - homotopy coherent, 287, 295
  - lax monoidal, 40
  - representable, 43, 103
  - strong monoidal, 40
  - tensor product, 59
- functorial factorization, 190–192
  - enriched, 224–228
  - step-one, 194, 198
- fundamental groupoid, 36, 263
- $G$ -object, 5, 42
- $G$ -space, 39, 41
- geometric realization, 14, 53
  - as functor tensor product, 60
  - fat, 134, 184
  - homotopical properties of, 73
  - in a tensored simplicial category, 55
  - is a deformable functor, 24
  - $n$ -truncated, 66, 240
  - of a split augmented simplicial object, 67
  - preservation of finite products, 78
  - preservation of simplicial tensors, 123
- gluing lemma, 249
- Grothendieck construction, 102
- hom-object, 35
- homology, 21
  - long exact sequence, 30
- homotopical, 17
  - category, 18
    - $\mathcal{V}$ -, 152
    - closed symmetric monoidal, 151
    - minimal, 19
    - saturated, 20
  - functor, 20
- homotopy
  - arising from a natural transformation, 131
  - in a simplicial category, 54
- homotopy category
  - enrichment, 151, 152
  - of a homotopical category, 19
  - of a model category, 178
  - of a quasi-category, 265
  - of a  $\mathcal{V}$ -homotopical category, 158
  - of quasi-categories, 305
  - of spaces, 20, 47, 146, 283
  - stable, 35
- homotopy cofiber, 87
- homotopy coherent
  - diagram, 287, 295
  - natural transformation, 118, 287
  - nerve, 282
- homotopy colimit, 31, 70
  - as a derived functor, 69
  - as functor tensor product, 92
  - as weighted colimit, 116
  - change of base, 124–126
  - in based vs. unbased spaces, 94
  - is not a colimit in the homotopy category, 82
  - local universal property of, 116
  - weighted, 139–144, 155–158
- homotopy commutative diagram, 287
- homotopy equivalence, 160, 162–163
  - is a weak homotopy equivalence, 19
- homotopy final functor, 130–135, 188
- homotopy fixed point, 92, 118, 144
- homotopy group, 21
- homotopy hypothesis, 263
- homotopy initial functor, 130–135, 188

- homotopy Kan extension, 144, 157
- homotopy limit, 31, 70
  - as a derived functor, 69
  - as functor cotensor product, 92
  - as weighted limit, 116
  - change of base, 124–126
  - in based vs. unbased spaces, 92, 122
  - is not a limit in the homotopy category, 82
  - local universal property of, 116
  - of a diagram of quasi-categories, 316
  - weighted, 139–144, 155–158
- homotopy orbit, 89, 144
- homotopy product, 33, 89
- homotopy pullback, 91, 186, 248
  - of quasi-categories, 317
- homotopy pushout, 86, 185–186, 248
  - local universal property of, 117
  - non-example of, 22
- horn filler, 265
- Hurewicz cofibration, 168, 172
- Hurewicz fibration, 168, 215
  
- induced representation, 5, 8, 9
- $\infty$ -category, 265
- $(\infty, 1)$ -category, 263
- initial functor, 127, 129
- initial object, *see* terminal object
- injective model structure, 182
- internal hom, 37
- isofibration, 177, 272, 325
- isomorphism
  - in a quasi-category, 298, 304
  
- join, 299
  
- k*-
  - continuous, 79
  - ification, 78–79
  - space, 78
- Kan complex, 81, 176
  - as groupoidal quasi-category, 303
  - equivalence of, 306
  - weak, 265
- Kan extension, 3–10
  - absolute, 27
  - as a weighted (co)limit, 101
  - as functor (co)tensor product, 59
  - colimit of, 65
  - enriched, 116
  - homotopy, 144, 157
  - interaction with weighted (co)limits, 122
  - pointwise, 10, 102
  - preservation of, 9
- $\kappa$ -small, 193
  
- Ken Brown’s lemma, 24, 179
- kernel pair, 100
  
- latching
  - map, 242
  - relative, 245
  - object, 241–247
- lax monoidal
  - functor, 40
- left proper
  - model structure, 249
- Leibniz construction, 170
- Leibniz formula, 170, 264
- levelwise, *see* pointwise
- lift, 168
- lifting function, 209
- lifting problem, 168
  - solution to, 168
- lifting property, 168
  - enriched, 222, 228–235
  - interaction with a two-variable adjunction, 172
  - interaction with an adjunction, 169–170
- limit, 104
  - as Kan extension, 11
  - enriched, 99, 121
  - enriched universal property of, 113
  - universal property of, 99
  - weighted, *see* weighted limit
- local object, 197
- localization, 252–257
- localization functor, 19
  - lax monoidal structure, 159
- locally small, 7, 32
- loop space, 91
  
- mapping cylinder, 84
  - double, 86
  - factorization, 235
- mapping path space, 90
  - factorization, 227, 235
  - of quasi-categories, 316
  - universal property of, 215
- mapping spaces
  - in a quasi-category, 273–281, 296–297
  - are Kan complexes, 303
- mapping telescope, 87, 186
- matching
  - map, 242
  - relative, 245
  - object, 241–247
- mate, 51
- model category, *see also* homotopical category, *see* model structure

- model structure, 174–176
  - algebraic, 220–221
  - based simplicial, 57
  - cofibrantly generated, 69, 174, 182
  - combinatorial, 69, 182
  - determination by cofibrations and fibrant objects, 270
  - enriched, *see also*  $\mathcal{V}$ -model category, 235–237
  - for quasi-categories, 272
  - homotopy category of, 178
  - left proper, 249
  - mixed, 177
  - monoidal, 140, 151, 181
  - on categories, 177
  - on chain complexes, 176, 234
  - on marked simplicial sets, 310
  - on simplicial categories, 177, 284
  - on simplicial sets, 176
  - on the opposite of a model category, 181
  - on topological spaces, 176, 190, 215
  - projective, 195–197
  - projective or injective, 69, 182
  - Reedy, 245
  - simplicial, 56, 181
  - topological, 57
  - $\mathcal{V}$ -, 57, 140, 152, 182, 236
- module, 44
  - homomorphism, 46
- monad
  - free on a pointed endofunctor, 200
  - idempotent, 80
  - on a quasi-category, 328
  - resolution
    - derived, 203
- monadic adjunction, 80
- monoidal
  - model category, 140, 151, 181
  - product, 34
- monomorphism, 168
  - and the small object argument, 205
  - in simplicial sets, 205, 211
- Moore path, 215
- $n$ -arrow, 284
- $n$ -cell, 38
- $n$ -(co)skeleton, 6, 8
- $n$ -simplex, 12
- natural transformation
  - enriched, 45
  - homotopy coherent, 287
  - set of, 8
- necklace
  - beads of, 292
  - joins of, 292
  - of simplices, 291
  - splitting, 293
  - totally non-degenerate, 292
- nerve, 14, 265
  - homotopy coherent, 282
- nilpotent
  - group, 255
  - space, 255–256
- objectwise, *see* pointwise
- orthogonal  $G$ -spectra, 45
- orthogonal factorization system, 173, 202
- orthogonal spectra, 35
- pasting diagram, 3
- path space, 91
- point-set level, 20
- pointwise, 22, *see also* Kan extension
- power, 8
  - as cotensor, 50
- projective
  - cofibration, 184–189
  - model structure, 182
- pseudofunctor, 44
- pullback-cotensor, 171
- pullback-hom, 171
- pushout-product, 171
- quasi-category, 265
  - 2-category of, 319–321
  - adjunction between, 328–332
    - universal property of, 336
  - arrow, 323
  - associated to a simplicial model category, 295
  - closure under projective cofibrant weighted limits, 268
  - comma, 326
    - vs. slice, 334
  - cylinder object, 271
  - equivalence of, 271, 305, 321
  - formal category theory of, 318, 330
  - homotopy category of, 265, 305
  - homotopy coherent adjunction, 336
  - homotopy limit of, 316
  - homotopy pullback, 326
  - internal hom, 267–268
  - isomorphism in, 298, 304
  - mapping spaces, 273–281, 296–297
  - model structure for, 272
  - monad on, 328
  - of quasi-categories, 305
  - terminal object in, 329, 335

- Quillen  
 adjunction, 177  
   strong monoidal, 57  
 bifunctor, 180–181  
 equivalence, 180  
 functor, 17, 24, 177  
 two-variable adjunction, 181  
 Quillen's Theorem A, 132
- reduction theorem, 124
- Reedy  
 category, 69, 243  
   generalized, 243  
   history of, 243  
 cofibrant, 72, 246  
   cosimplicial object, 333  
 factorization, 243  
 fibrant, 246  
   model structure, 245  
 reflective subcategory, 80  
 relative  $T_1$  inclusion, 258  
 relative cell complex, 212, 218  
 representable functor, 43, 103  
   as free module, 59  
   enriched, 44  
 resolution  
   projective or injective, 29  
 retract  
   argument, 173  
   in an arrow category, 168  
 ring, 36
- $S$ -module, 35  
 saturated homotopical category, 20  
 sequential composite, 168, 193  
 Serre fibration, 168  
 sheafification, 80  
 shuffle, 264, 281  
 simplex  
   diagonal of, 291  
   spine of, 291  
 simplicial  
   category, 47  
   cofibrant, 284–286  
   locally Kan, 284, 295  
   model structure, 177, 284  
   computad, 284–286  
   functor, 48  
   natural transformation, 48  
 simplicial enrichment  
   vs. topological enrichment, 53, 283  
 simplicial homotopy equivalence, 54–56  
 simplicial model category, 31, 56, 181  
   associated quasi-category, 295  
 simplicial object  
   augmented, 66–68, 134  
   colimit, 128  
   homotopy colimit, 94, 117, 134  
   tensor structure, 54  
 simplicial set, 5, *see also* simplicial object  
   as a weighted colimit, 106  
   augmented, 6  
   based  
     simplicial enrichment, 52  
   cartesian closed category of, 14  
   contractible, 130  
   left adjoint from the category of, 12  
   marked, 307–310  
   Quillen model structure, 176  
 simplicial space, 240  
   split, 258  
 slice, 301  
 slice category, 7, 111  
 SM7 axiom, 181  
 small category, 6  
 small object, *see* compact object  
 small object argument  
   algebraic, Garner's  
   algebraic perspective, 198–201  
   corollary, 195  
   enriched, 222, 224–228  
   Garner's, 201–208  
     enriched variant, 224–226, 233, 238  
   permitting the, 193, 202, 224  
   Quillen's, 192–195, 198–201  
     enriched variant, 224–226, 234  
   simplified form of, 204  
 smash product, 38  
   non-associativity of the, 77  
 spine  
   of a simplex, 291  
 stable homotopy category, 35  
 standard  $n$ -simplex, 103  
 stratum, 217  
 strong monoidal  
   adjunction, 51  
   functor, 40  
   subdivision, 15  
   suspension, 86  
 symmetric monoidal category, 34  
   closed, 37, 49  
 symmetric spectra, 35
- tensor, 7  
 as weighted colimit, 110  
 discrete, 58  
 in an enriched category, 49  
 coherence of, 50

- tensor product
  - in a monoidal category, *see* monoidal product
  - of functors, *see* functor tensor product
  - of modules, 59
- terminal object, 126–128, 130, 133
  - in a quasi-category, 329, 335
  - in a small category, 68
- topological enrichment
  - vs. simplicial enrichment, 53, 283
- topological group, 139, 144
- topological space
  - as a quasi-category, 265
  - based
    - topological enrichment, 53
  - compactly generated, 79
  - convenient category of, 38, 82
  - enrichment over groupoids, 36
  - failure to be cartesian closed, 77
  - $k$ -space, 78
  - model structures, 176, 215
  - unique closed symmetric monoidal structure, 77
  - weak Hausdorff, 79
- topology of pointwise convergence, 77
- total singular complex, 13
- totalization, 63
  - as a weighted limit, 117
  - of a split augmented cosimplicial object, 67
- transfinite composite, 168, 193
- translation groupoid, 131
- trivial cofibration, 174
- trivial fibration, 174
  - of simplicial sets, 230
- twisted arrow category, 107, 129
- unaugmentable
  - cosimplicial object, 251
- underlying
  - category, functor, natural transformation, 46
- unit object, 34
- universal property, 4
- $\mathcal{V}$ -, *see* enriched
- $\mathcal{V}$ -equivalence, 160–163
- $\mathcal{V}$ -model category, 57, 140, 152, 182, 236–237, *see also* enriched model structure
- weak categorical equivalence, *see* categorical equivalence
- weak equivalence, 18
  - in a  $\mathcal{V}$ -homotopical category, 162–163
  - in the model structure for quasi-categories, *see* categorical equivalence
- weak factorization system, 172
  - algebraic, 208–215
    - closure properties of, 212
    - cofibrantly generated, 210
    - recognition principle for, 214
    - underlying weak factorization system of, 209, 211
  - cofibrantly generated, 173, 192–197
  - enriched, 233
  - role in model category theory, 178
- weak Hausdorff, 79
- weak homotopy equivalence, 19
  - vs. categorical equivalence, 315
- weak Kan complex, 265
- weak limit
  - comma object, 326
  - cotensor, 322
    - uniqueness of, 325
  - homotopy pullback, 326
- weak saturation, 195
- weakly saturated, 168
- weight, 100
  - for homotopy colimit, 188
  - for homotopy limit, 188
- weighted colimit, 109
  - as a functor tensor product, 105, 115
  - bifunctor
    - homotopical properties of, 182
    - derived functor of, 139–144
  - in unenriched category theory, 105–107
    - bifunctor, 107
    - preferred notation for, 110
    - representable definition of, 111
- weighted limit, 109
  - as a functor cotensor product, 101, 115
  - bifunctor
    - homotopical properties of, 182
    - derived functor of, 139–144
  - in unenriched category theory, 100–105
    - bifunctor, 104
    - preferred notation for, 110
    - representable definition of, 111
- wide subcategory, 18
- Yoneda embedding
  - density of, 12
  - is Reedy cofibrant, 251
- Yoneda lemma, 11, 101
  - enriched, 109, 110