Contents

List of figures xvi
List of tables xxiii
Acknowledgements xxvi

Part I Our approach in its context 1

1 How this book came about 5
  1.1 An outline of our approach 6
  1.2 Portfolio management as a process 9
  1.3 Plan of the book 10

2 Correlation and causation 13
  2.1 Statistical versus causal explanations 13
  2.2 A concrete example 19
  2.3 Implications for hedging and diversification 22

3 Definitions and notation 23
  3.1 Definitions used for analysis of returns 23
  3.2 Definitions and notation for market risk factors 25

Part II Dealing with extreme events 27

4 Predictability and causality 31
  4.1 The purpose of this chapter 31
  4.2 Is this time different? 32
  4.3 Structural breaks and non-linearities 34
  4.4 The bridge with our approach 37

5 Econophysics 40
  5.1 Econophysics, tails and exceptional events 40
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.2</td>
<td>The scope and methods of econophysics</td>
<td>40</td>
</tr>
<tr>
<td>5.3</td>
<td>‘Deep analogies’</td>
<td>43</td>
</tr>
<tr>
<td>5.4</td>
<td>The invariance of physical and financial ‘laws’</td>
<td>45</td>
</tr>
<tr>
<td>5.5</td>
<td>Where we differ</td>
<td>46</td>
</tr>
<tr>
<td>6</td>
<td>Extreme Value Theory</td>
<td>48</td>
</tr>
<tr>
<td>6.1</td>
<td>A brief description</td>
<td>48</td>
</tr>
<tr>
<td>6.2</td>
<td>Applications to finance and risk management</td>
<td>49</td>
</tr>
<tr>
<td>Part III</td>
<td>Diversification and subjective views</td>
<td>51</td>
</tr>
<tr>
<td>7</td>
<td>Diversification in Modern Portfolio Theory</td>
<td>55</td>
</tr>
<tr>
<td>7.1</td>
<td>Basic results</td>
<td>56</td>
</tr>
<tr>
<td>7.2</td>
<td>Important special cases</td>
<td>58</td>
</tr>
<tr>
<td>7.2.1</td>
<td>Optimal weights with linear constraints</td>
<td>59</td>
</tr>
<tr>
<td>7.2.2</td>
<td>Optimization when a riskless asset is available</td>
<td>62</td>
</tr>
<tr>
<td>7.3</td>
<td>The link with the CAPM – a simple derivation</td>
<td>63</td>
</tr>
<tr>
<td>7.3.1</td>
<td>Derivation of the links between Markowitz and CAPM</td>
<td>64</td>
</tr>
<tr>
<td>7.3.2</td>
<td>Obtaining the familiar $\beta$-formulation</td>
<td>65</td>
</tr>
<tr>
<td>7.4</td>
<td>Reverse-engineering the CAPM</td>
<td>66</td>
</tr>
<tr>
<td>Appendix</td>
<td>Asset allocation in the presence of linear equality constraints</td>
<td>67</td>
</tr>
<tr>
<td>Appendix</td>
<td>Derivation of the stochastic discount factor</td>
<td>69</td>
</tr>
<tr>
<td>8</td>
<td>Stability: a first look</td>
<td>71</td>
</tr>
<tr>
<td>8.1</td>
<td>Problems with the stability of the optimal weights</td>
<td>71</td>
</tr>
<tr>
<td>8.2</td>
<td>Where the instability comes from</td>
<td>72</td>
</tr>
<tr>
<td>8.3</td>
<td>The resampling (Michaud) approach</td>
<td>75</td>
</tr>
<tr>
<td>8.4</td>
<td>Geometric asset allocation</td>
<td>76</td>
</tr>
<tr>
<td>Appendix</td>
<td>Absolute and relative coefficients of risk aversion for power and quadratic utility functions</td>
<td>79</td>
</tr>
<tr>
<td>8.A.1</td>
<td>Local derivatives matching</td>
<td>80</td>
</tr>
<tr>
<td>8.A.2</td>
<td>The coefficient of relative risk aversion</td>
<td>82</td>
</tr>
<tr>
<td>9</td>
<td>Diversification and stability in the Black–Litterman model</td>
<td>83</td>
</tr>
<tr>
<td>9.1</td>
<td>What the Black–Litterman approach tries to achieve</td>
<td>83</td>
</tr>
<tr>
<td>9.2</td>
<td>Views as prior: the Satchell and Scowcroft interpretation</td>
<td>84</td>
</tr>
<tr>
<td>9.3</td>
<td>Doust’s geometric interpretation again</td>
<td>87</td>
</tr>
<tr>
<td>9.4</td>
<td>The link with our approach</td>
<td>90</td>
</tr>
<tr>
<td>10</td>
<td>Specifying scenarios: the Meucci approach</td>
<td>92</td>
</tr>
<tr>
<td>10.1</td>
<td>Generalizing: entropy pooling</td>
<td>95</td>
</tr>
<tr>
<td>10.2</td>
<td>The link with Bayesian nets (and Black–Litterman)</td>
<td>97</td>
</tr>
<tr>
<td>10.3</td>
<td>Extending the entropy-pooling technique</td>
<td>98</td>
</tr>
</tbody>
</table>
Part IV How we deal with exceptional events

11 Bayesian nets
11.1 Displaying the joint probabilities for Boolean variables
11.2 Graphical representation of dependence: Bayesian nets
11.3 Influencing and ‘causing’
11.4 Independence and conditional independence
11.5 The link between Bayesian nets and probability distributions
  11.5.1 Screening and Markov parents
  11.5.2 The Master Equation
11.6 Ordering and causation – causal Bayesian nets
11.7 $d$-separation
  11.7.1 Definition
  11.7.2 A worked example
  11.7.3 Hard and soft evidence
  11.7.4 The link between $d$-separation and conditional independence
11.8 Are $d$-separation and conditional independence the same?
11.9 The No-Constraints Theorem
11.10 Why is this so important?
11.11 From Boolean to multi-valued variables

12 Building scenarios for causal Bayesian nets
12.1 What constitutes a root event?
12.2 The leaves: changes in market risk factors
12.3 The causal links: low-resistance transmission channels
12.4 Binary, discrete-valued or continuous?
12.5 The deterministic mapping

Part V Building Bayesian nets in practice

13 Applied tools
13.1 A word of caution
13.2 Why our life is easy (and why it can also be hard)
13.3 Sensitivity analysis
13.4 Assigning the desired dependence among variables
  13.4.1 A worked example: a terrorist attack
13.5 Dealing with almost-impossible combinations of events
13.6 Biting the bullet: providing the full set of master conditional probabilities
13.7 Event correlation
  13.7.1 Evaluation
  13.7.2 Intuitive interpretation
| 14 | More advanced topics: elicitation | 165 |
| 14.1 | The nature of the elicitation problem: what are the problems? | 165 |
| 14.2 | Dealing with elicitation: the Maximum-Entropy approach | 166 |
| 14.3 | Range-only information for canonical probabilities | 168 |
| 14.4 | Dealing with elicitation: Non-canonical-information | 169 |
| 14.4.1 | Definitions | 170 |
| 14.4.2 | An example | 171 |
| 14.4.3 | Unique invertibility, uncertain equivalence | 173 |
| 14.4.4 | Non-unique invertibility, uncertain equivalence | 173 |
| 14.4.5 | A simple example | 174 |
| 14.4.6 | Generalization | 176 |
| 14.5 | Dealing with elicitation: exploiting causal independence | 176 |
| 14.5.1 | Local restructuring of the net | 177 |
| 14.5.2 | Spelling out the implicit assumptions | 180 |
| 14.5.3 | Obtaining the conditional probabilities | 181 |
| 14.5.4 | A few important cases | 182 |
| 14.5.5 | Where do the probabilities of the inhibitors being active come from? | 183 |
| 14.5.6 | A simple example | 184 |
| 14.5.7 | Leak causes | 187 |
| 14.5.8 | Extensions | 187 |
| Appendix 14.A | | 188 |
| 14.A.1 | Knowledge about the range | 188 |
| 14.A.2 | Knowledge about the expectation | 189 |
| 14.A.3 | Knowledge about the expectation and the variance | 191 |
| Appendix 14.B | | 191 |
| 15 | Additional more advanced topics | 195 |
| 15.1 | Efficient computation | 195 |
| 15.1.1 | Pushing sums in | 195 |
| 15.2 | Size constraints: Monte Carlo | 197 |
| 15.2.1 | Obvious improvements | 199 |
| 15.2.2 | More advanced improvements: adapting the Weighted Monte-Carlo Method | 199 |
| 15.3 | Size constraints: joining nets | 201 |
| 16 | A real-life example: building a realistic Bayesian net | 203 |
| 16.1 | The purpose of this chapter | 203 |
| 16.2 | Step-by-step construction in a realistic case | 203 |
| 16.2.1 | Roots, leaves and transmission channels | 203 |
| 16.2.2 | A first attempt | 205 |
| 16.2.3 | Quantifying the horizon and the magnitude of the ‘stress events’ | 206 |
| 16.2.4 | The construction | 208 |
Table of Contents

16.3 Analysis of the joint distribution 229
16.4 Using Maximum Entropy to fill in incomplete tables 233
16.5 Determining the P&L distribution 234
16.6 Sensitivity analysis 235

Part VI Dealing with normal-times returns 239
17 Identification of the body of the distribution 243
17.1 What is ‘normality’? Conditional and unconditional interpretation 243
17.2 Estimates in the ‘normal’ state 247
17.3 Estimates in an excited state 249
17.4 Identifying ‘distant points’: the Mahalanobis distance 251
17.5 Problems with the Mahalanobis distance 254
17.6 The Minimum-Volume-Ellipsoid method 254
  17.6.1 Definition 255
  17.6.2 The intuition 255
  17.6.3 Detailed description 256
  17.6.4 An example and discussion of results 258
17.7 The Minimum-Covariance-Determinant method 267
17.8 Some remarks about the MVE, MCD and related methods 269

18 Constructing the marginals 271
18.1 The purpose of this chapter 271
18.2 The univariate fitting procedure 272
  18.2.1 Other possible approaches 272
18.3 Estimating the vector of expected returns 274
  18.3.1 What shrinkage fixes (and what it does not fix) 276

19 Choosing and fitting the copula 278
19.1 The purpose of this chapter 278
19.2 Methods to choose a copula 278
19.3 The covariance matrix and shrinkage 280
19.4 The procedure followed in this work 281
  19.4.1 The algorithm for Gaussian copula 281
  19.4.2 The algorithm for Student-$t$ copula 282
19.5 Results 282

Part VII Working with the full distribution 291
20 Splicing the normal and exceptional distributions 295
  20.1 Purpose of the chapter 295
  20.2 Reducing the joint probability distribution 295
  20.3 Defining the utility-maximization problem 297
  20.4 Expected utility maximization 298
## Contents

20.5 Constructing the joint spliced distribution 299
  20.5.1 The setting 299
  20.5.2 Building block 1: The excited-events distribution 300
  20.5.3 Building block 2: The ‘compacted’ normal-times distribution for the \( i \)th event 301
  20.5.4 \( i \)th event: the combined distribution 301
  20.5.5 The full spliced distribution 304

20.6 A worked example 305

20.7 Uncertainty in the normalization factor: a Maximum-Entropy approach 308
  20.7.1 Introducing the normalization factor 308
  20.7.2 Introducing uncertainty in the normalization factor 309
  Appendix 20.A 312
  Appendix 20.B 313
  20.B.1 Truncated exponential 314
  20.B.2 Truncated Gaussian 314

21 The links with CAPM and private valuations 316
  21.1 Plan of the chapter 316
  21.2 Expected returns: a normative approach 316
  21.3 Why CAPM? 317
  21.4 Is there an alternative to the CAPM? 318
  21.5 Using the CAPM for consistency checks 319
  21.6 Comparison of market-implied and subjectively assigned second and higher moments 321
  21.7 Comparison with market expected returns 322
  21.8 A worked example 324
  21.9 Private valuation: linking market prices and subjective prices 328
    21.9.1 Distilling the market’s impatience and risk aversion 331
    21.9.2 Obtaining our private valuation 332
    21.9.3 Sanity checks 333
  21.10 Conclusions 334

  Appendix 21.A: Derivation of \( m_{t+1} = a + bct = a - bGR^{MKT} \) 335

Part VIII A framework for choice 339

22 Applying expected utility 343
  22.1 The purpose of this chapter 343
  22.2 Utility of what? 344
  22.3 Analytical representation and stylized implied-behaviour 345
  22.4 The ‘rationality’ of utility theory 347
  22.5 Empirical evidence 348
## Contents

26.3 Analysing the body of the distribution 407
   26.3.1 Correlations and volatilities before culling 407
   26.3.2 Truncation 409
   26.3.3 Correlations and volatilities after culling 409
26.4 Fitting the body of the joint distribution 414
26.5 CAPM and the total moments 416
   26.5.1 Are we using the right betas? 419
26.6 The optimal-allocation results 420
   26.6.1 Results for logarithmic utility function 420
   26.6.2 Sensitivity to different degrees of risk aversion 421
   26.6.3 Conclusions 423
26.7 The road ahead 424

27 Numerical analysis 425
   27.1 How good is the mean-variance approximation? 425
   27.2 Using the weight expansion for the \( k \) dependence 428
      27.2.1 Gaining intuition 429
   27.3 Optimal allocation with uncertain \( k \) via Maximum Entropy: results 430

28 Stability analysis 434
   28.1 General considerations 434
   28.2 Stability with respect to uncertainty in the conditional probability tables 436
      28.2.1 Analytical expressions for the sensitivities 436
      28.2.2 Empirical results 440
   28.3 Stability with respect to uncertainty in expected returns 441
      28.3.1 Sensitivity to stressed returns 442
   28.4 Effect of combined uncertainty 447
   28.5 Stability of the allocations for high degree of risk aversion 447
   28.6 Where does the instability come from? (again) 448

29 How to use Bayesian nets: our recommended approach 453
   29.1 Some preliminary qualitative observations 453
   29.2 Ways to tackle the allocation instability 454
      29.2.1 Optimizing variance for a given return 454
      29.2.2 The Black–Litterman stabilization 455
      29.2.3 The general Bayesian stabilization 455
      29.2.4 Calibrating the utility function to risk and ambiguity aversion 458
   29.3 The lay of the land 459
   29.4 The approach we recommend 460

*Appendix 29.A: The parable of Marko and Micha* 462
Appendix I: The links with the Black–Litterman approach
1 The Black–Litterman ‘regularization’ 465
2 The likelihood function 466
3 The prior 468
4 The posterior 470

References 471
Index 485