

Contents

	<i>Preface</i>	<i>page xi</i>
	PART ONE ANALYSING PORTFOLIO MORTALITY	1
1	Introduction	3
	1.1 Survival Data	3
	1.2 Software	5
	1.3 Grouped Counts	6
	1.4 What Mortality Ratio Should We Analyse?	6
	1.5 Fitting a Model to Grouped Counts	9
	1.6 Technical Limits for Models for Grouped Data	11
	1.7 The Problem with Grouped Counts	13
	1.8 Modelling Grouped Counts	15
	1.9 Survival Modelling for Actuaries	15
	1.10 The Case Study	17
	1.11 Statistical Notation	19
2	Data Preparation	20
	2.1 Introduction	20
	2.2 Data Extraction	20
	2.3 Field Validation	25
	2.4 Relationship Checking	25
	2.5 Deduplication	26
	2.6 Bias in Rejections	29
	2.7 Sense-Checking	29
	2.8 Derived Fields	32
	2.9 Preparing Data for Modelling and Analysis	36
	2.10 Exploratory Data Plots	41

3	The Basic Mathematical Model	45
3.1	Introduction	45
3.2	Random Future Lifetimes	47
3.3	The Life Table	49
3.4	The Hazard Rate, or Force of Mortality	50
3.5	An Alternative Formulation	53
3.6	The Central Rate of Mortality	54
3.7	Application to Life Insurance and Annuities	55
4	Statistical Inference with Mortality Data	56
4.1	Introduction	56
4.2	Right-Censoring	58
4.3	Left-Truncation	60
4.4	Choice of Estimation Approaches	61
4.5	A Probabilistic Model for Complete Lifetimes	64
4.6	Data for Estimation of Mortality Ratios	67
4.7	Graduation of Mortality Ratios	69
4.8	Examples: the Binomial and Poisson Models	71
4.9	Estimating the Central Rate of Mortality?	72
4.10	Census Formulae for E_x^c	73
4.11	Two Approaches	73
5	Fitting a Parametric Survival Model	75
5.1	Introduction	75
5.2	Probabilities of the Observed Data	76
5.3	Likelihoods for Survival Data	78
5.4	Example: a Gompertz Model	79
5.5	Fitting the Gompertz Model	80
5.6	Data for Single Years of Age	86
5.7	The Likelihood for the Poisson Model	88
5.8	Single Ages <i>versus</i> Complete Lifetimes	90
5.9	Parametric Functions Representing the Hazard Rate	92
6	Model Comparison and Tests of Fit	94
6.1	Introduction	94
6.2	Comparing Models	94
6.3	Deviance	95
6.4	Information Criteria	97
6.5	Tests of Fit Based on Residuals	100
6.6	Statistical Tests of Fit	102
6.7	Financial Tests of Fit	109

<i>Contents</i>		vii
7	Modelling Features of the Portfolio	112
	7.1 Categorical and Continuous Variables	112
	7.2 Stratifying the Experience	115
	7.3 Consequences of Stratifying the Data	120
	7.4 Example: a Proportional Hazards Model	122
	7.5 The Cox Model	124
	7.6 Analysis of the Case Study Data	125
	7.7 Consequences of Modelling the Data	129
8	Non-parametric Methods	132
	8.1 Introduction	132
	8.2 Comparison against a Reference Table	133
	8.3 The Kaplan–Meier Estimator	134
	8.4 The Nelson–Aalen Estimator	140
	8.5 The Fleming–Harrington Estimator	141
	8.6 Extensions to the Kaplan–Meier Estimator	141
	8.7 Limitations and Applications	142
9	Regulation	145
	9.1 Introduction	145
	9.2 Background	145
	9.3 Approaches to Probabilistic Reserving	147
	9.4 Quantile Estimation	148
	9.5 Mortality Risk	150
	9.6 Mis-estimation Risk	151
	9.7 Trend Risk	153
	9.8 Number of Simulations	155
	9.9 Idiosyncratic Risk	155
	9.10 Aggregation	157
PART TWO REGRESSION AND PROJECTION MODELS		161
10	Methods of Graduation I: Regression Models	163
	10.1 Introduction	163
	10.2 Reading Data from the Human Mortality Database into R	165
	10.3 Fitting the Gompertz Model with Least Squares	166
	10.4 Poisson Regression Model	172
	10.5 Binomial Regression Model	173
	10.6 Exponential Family	177
	10.7 Generalised Linear Models	178

10.8	Gompertz Model with Poisson Errors	179
10.9	Gompertz Model with Binomial Errors	181
10.10	Polynomial Models	182
11	Methods of Graduation II: Smooth Models	185
11.1	Introduction	185
11.2	Whittaker Smoothing	187
11.3	<i>B</i> -Splines and <i>B</i> -Spline Bases	189
11.4	<i>B</i> -Spline Regression	191
11.5	The Method of <i>P</i> -Splines	193
11.6	Effective Dimension of a Model	198
11.7	Deviance of a Model	199
11.8	Choosing the Smoothing Parameter	201
11.9	Overdispersion	203
11.10	Dealing with Overdispersion	205
12	Methods of Graduation III: Two-Dimensional Models	208
12.1	Introduction	208
12.2	The Lee–Carter Model	210
12.3	The Cairns–Blake–Dowd Model	214
12.4	A Smooth Two-Dimensional Model	216
12.5	Comparing Models	222
13	Methods of Graduation IV: Forecasting	224
13.1	Introduction	224
13.2	Time Series	225
13.3	Penalty Forecasting	232
13.4	Forecasting with the Lee–Carter Model	236
13.5	Simulating the Future	238
13.6	Forecasting with the Cairns–Blake–Dowd Model	243
13.7	Forecasting with the Two-Dimensional <i>P</i> -Spline Model	247
13.8	Model Risk	251
	PART THREE MULTIPLE-STATE MODELS	253
14	Markov Multiple-State Models	255
14.1	Insurance Contracts beyond “Alive” and “Dead”	255
14.2	Multiple-State Models for Life Histories	256
14.3	Definitions	258
14.4	Examples	260
14.5	Markov Multiple-State Models	262
14.6	The Kolmogorov Forward Equations	264

<i>Contents</i>		ix
14.7	Why Multiple-State Models and Intensities?	269
14.8	Solving the Kolmogorov Equations	271
14.9	Life Contingencies: Thiele's Differential Equations	274
14.10	Semi-Markov Models	276
14.11	Credit Risk Models	278
15	Inference in the Markov Model	279
15.1	Introduction	279
15.2	Counting Processes	280
15.3	An Example of a Life History	282
15.4	Jumps and Waiting Times	284
15.5	Aalen's Multiplicative Model	285
15.6	The Likelihood for Single Years of Age	286
15.7	Properties of the MLEs for Single Ages	288
15.8	Estimation Using Complete Life Histories	289
15.9	The Poisson Approximation	290
15.10	Semi-Markov Models	292
15.11	Historical Notes	293
16	Competing Risks Models	294
16.1	The Competing Risks Model	294
16.2	The Underlying Random Future Lifetimes	296
16.3	The Unidentifiability Problem	298
16.4	A Traditional Actuarial Approach	300
16.5	Are Competing Risks Models Useful?	304
17	Counting Process Models	307
17.1	Introduction	307
17.2	Basic Concepts and Notation for Stochastic Processes	308
17.3	Stochastic Integrals	312
17.4	Martingales	314
17.5	Martingales out of Counting Processes	317
17.6	Martingale Central Limit Theorems	319
17.7	A Brief Outline of the Uses of Counting Process Models	320
Appendix A	R Commands	329
A.1	Introduction	329
A.2	Running R	330
A.3	R Commands	330
A.4	Probability Distributions	330
Appendix B	Basic Likelihood Theory	334
B.1	Scalar Parameter Models: Theory	334

x	<i>Contents</i>	
	B.2 The Single-Decrement Model	337
	B.3 Multivariate Parameter Models: Theory	338
	Appendix C Conversion to Published Tables	341
	C.1 Reasons to Use Published Tables	341
	C.2 Equivalent-Reserve Method	341
	C.3 Algorithms for Solving for Percentage of Published Table	343
	Appendix D Numerical Integration	344
	D.1 Introduction	344
	D.2 Implementation Issues	344
	D.3 Numerical Integration over a Grid of Fixed Points	345
	Appendix E Mean and Variance-Covariance of a Vector	347
	Appendix F Differentiation with Respect to a Vector	349
	Appendix G Kronecker Product of Two Matrices	350
	Appendix H R Functions and Programs	351
	H.1 Statistical Inference with Mortality Data	351
	H.2 Model Comparison and Tests of Fit	351
	H.3 Methods of Graduation I: Regression Models	352
	H.4 Methods of Graduation II: Smooth Models	352
	H.5 Methods of Graduation III: Two-Dimensional Models	353
	H.6 Methods of Graduation IV: Forecasting	354
	<i>References</i>	355
	<i>Author Index</i>	361
	<i>Index</i>	363