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Acoustics and Aerodynamic Sound

Music, calm speech, whispering leaves fluttering in a breeze are pleasant and desirable sounds. Noise, howling gales, explosions and screeching traffic are less so. A quantitative understanding of the sources of all such sounds can be obtained by careful analysis of the mechanical equations of motion. This is provided by *Acoustics and Aerodynamic Sound*, which serves as a short, one-semester introduction to acoustics and aerodynamic sound at the advanced undergraduate and graduate levels.

Sound is treated as a branch of fluid mechanics, which is possible because students embarking on an advanced course in acoustics will be familiar with this topic. It is also desirable because an ability to relate acoustic events to hydrodynamic phenomena provides insight into acoustic principles, in particular into the role of vorticity in the mechanics of sound production by vibrating bodies and in the scattering and diffraction of sound.

MICHAEL HOWE has more than 30 years of experience in research on fluid mechanics, acoustics, random vibration, and structural mechanics. He is the author of more than 200 refereed journal articles on these and related subjects. His books include *Acoustics of Fluid-Structure Interactions*; *The Theory of Vortex Sound*; *Hydrodynamics and Sound*; and *Mathematical Methods for Mechanical Sciences*.

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MICHAEL HOWE
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In memoriam Dorothy Howe
1921–2014

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Preface

An introductory account is given of the theory of the production and propagation of sound and its interactions with solid structures. It is intended for a one-semester course on acoustics at the advanced undergraduate or graduate level, and is therefore shorter than most of the standard texts – many important applications are omitted, such as speech and musical acoustics, ultrasonic imaging, and thermoacoustics. Sound is treated as a branch of fluid mechanics, which is possible because most students embarking on an advanced course are likely to have some familiarity with fluid mechanics or be sufficiently mature to assimilate the review material provided in the text. It is also desirable because an ability to relate acoustic events to hydrodynamic phenomena provides valuable insight into acoustic principles, in particular into the role of ‘vorticity’ in the mechanics of sound production by vibrating bodies and in the scattering and diffraction of sound. Any homogeneous fluid that has kinetic energy independent of moving boundaries must possess vorticity, a quantity that propagates by convection and molecular diffusion, and that therefore undergoes relatively little displacement during a typical acoustic cycle; the existence of vorticity signifies the presence of possible sources of sound, and its production occurs when sound is dissipated.

Chapter 1 forms a stand-alone introduction to theoretical acoustics, and is suitable together with material from §§2.1, 2.3, 2.4, 3.1, 3.2, 4.1–4.3, for a ‘short course’ on acoustics and aerodynamic sound. A general discussion of the Kirchhoff integral representation and generalisations is given in Chapter 2, with particular emphasis on sound sources of various types near an acoustically ‘compact’ body or section of a larger solid boundary, including the influence of surface vibration and scattering. Chapter 3 deals with the sound radiated by a baffled piston, Kirchhoff diffraction theory, and interactions of sound with apertures and perforated screens. Lighthill’s theory of aerodynamic sound and its extensions to sound production by ‘vortex-surface interactions’

are presented in Chapter 4. Methods for treating the interaction of sound with small apertures, constrictions and duct open ends are discussed in Chapter 5, including nonlinear jetting and absorption by vorticity production – inspired by Rayleigh’s application of Kelvin’s general ‘minimum energy’ principle to impulsively started fluid motion. Sections numbered with an asterisk may be omitted on a first reading.

No discussion is given of experiments required to validate the theory, nor is there any account of applications to practical noise control problems. The latter are very well treated in several standard engineering handbooks. Similarly, little or no attention is given to specialities such as ray theory and the geometrical theory of diffraction, nonlinear effects of propagation, or the influence on propagation and sound generation of high-speed flows and combusting flows. Electrical analogies involving ‘lumped parameter’ models also are avoided. Nevertheless, the book provides a self-consistent treatment of the more familiar processes of propagation, scattering, diffraction and production of sound.

The reader is assumed to possess a working knowledge of vector calculus including Cartesian tensor notation, and of rudimentary generalised functions (‘delta’ function, etc.). A good appreciation of the subject requires the student to work through the problems at the end of each chapter, which should be within the capabilities of most students working alone. Some of the problems are perhaps better treated as short end-of-term projects. Solutions of ‘starred’ problems are outlined in Chapter 6.

Michael Howe