Quantum Theory from First Principles

An Informational Approach

Quantum theory is the soul of theoretical physics. It is not just a theory of specific physical systems, but rather a new framework with universal applicability. This book shows how we can reconstruct the theory from six information-theoretical principles, by rebuilding the quantum rules from the bottom up. Step by step, the reader will learn how to master the counterintuitive aspects of the quantum world, and how to efficiently reconstruct quantum information protocols from first principles. Using intuitive graphical notation to represent equations, and with shorter and more efficient derivations, the theory can be understood and assimilated with exceptional ease. Offering a radically new perspective on the field, the book contains an efficient course of quantum theory and quantum information for undergraduates. The book is aimed at researchers, professionals, and students in physics, computer science, and philosophy, as well as the curious outsider seeking a deeper understanding of the theory.

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To our wives and children:

Rosanna and Gilda,
Amy and Francesco,
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More Information

Contents

Preface	*page* xiii
Acknowledgments	xv
1 Introduction	1
1.1 The Quest for Principles: von Neumann	2
1.2 Quantum Information Resurrects the Quest	3
1.3 Quantum Theory as an OPT	4
1.4 The Principles	5
Part I The Status Quo	9
2 Quantum Theory from Hilbert Spaces	11
2.1 Primitive Notions	12
2.2 Hilbert-space Postulates for Quantum Theory	14
2.3 Density Matrices and POVMs	16
2.4 Causality, Convex Structure, Discriminability	20
2.5 Quantum States	25
2.6 Entangled Quantum States and Effects	30
2.7 Compression	33
2.8 Quantum Transformations	35
2.9 Classical Theory as a Restriction of Quantum	49
2.10 Purification	50
2.11 Quantum No Cloning	59
2.12 The von Neumann Postulate: Do We Need It?	61
2.13 Quantum Teleportation	63
2.14 Inverting Transformations	66
2.15 Summary	73
Notes	73
Appendix 2.1 Polar Decomposition	78
Appendix 2.2 The Golden Rule for Quantum Extensions	79
Problems	80
Solutions to Selected Problems and Exercises	85

Part II The Informational Approach 109

3 The Framework 111
3.1 The Operational Language 111
3.2 Operational Probabilistic Theory 115
3.3 States and Effects 116
3.4 Transformations 118
3.5 Coarse-graining and Refinement 119
3.6 Operational Distance Between States 120
3.7 Operational Distances for Transformations and Effects 122
3.8 Summary 124
Notes 124
Problems 126
Solutions to Selected Problems and Exercises 126

4 The New Principles 130
4.1 Atomicity of Composition 130
4.2 Perfect Discriminability 132
4.3 Ideal Compression 133
4.4 A Preview of the Three Main Principles 135
4.5 Summary 137
Notes 138

5 Causal Theories 139
5.1 Causality: From Cinderella to Principle 139
5.2 No Signaling from the Future 141
5.3 Conditioning 142
5.4 A Unique Wastebasket 143
5.5 No Signaling at a Distance 150
5.6 Causality and Space-Time 151
5.7 Theories without Causality 151
5.8 Summary 154
Notes 155
Solutions to Selected Problems and Exercises 156

6 Theories with Local Discriminability 157
6.1 Entanglement and Holism 157
6.2 The Principle 159
6.3 Reconciling Holism with Reductionism 160
6.4 Consequences of Local Discriminability 163
6.5 Different Degrees of Holism 163
6.6 Summary 165

Notes 165
Problems 166
Solutions to Selected Problems and Exercises 167

7 The Purification Principle 168
7.1 A Distinctive and Fundamental Trait 168
7.2 The Purification Principle 171
7.3 Entanglement 172
7.4 Reversible Transformations and Twirling 173
7.5 Steering 175
7.6 Process Tomography 176
7.7 No Information Without Disturbance 177
7.8 Teleportation 178
7.9 A Reversible Picture of an Irreversible World 179
7.10 Displacing the Von Neumann's Cut 181
7.11 The State-transformation Isomorphism 182
7.12 Everything Not Forbidden is Allowed 184
7.13 Purification in a Nutshell 186
7.14 Summary 188
Notes 188
Appendix 7.1 Carathéodory's Theorem 189
Solutions to Selected Problems and Exercises 190

Part III Quantum Information Without Hilbert Spaces 191

8 Encoding Information 193
8.1 Processing Data = Processing Entanglement 193
8.2 Ideal Encodings 195
8.3 Ideal Compression 198
8.4 The Minimal Purification 199
8.5 Sending Information Through a Noisy Channel 200
8.6 The Condition for Error Correction 201
8.7 Summary 204
Solutions to Selected Problems and Exercises 204

9 Three No-go Theorems 206
9.1 No Cloning 207
9.2 No Programming 209
9.3 No Bit Commitment 211
9.4 Summary 215
Notes 215
Solutions to Selected Problems and Exercises 217

10 Perfectly Discriminable States 218
10.1 Perfect Discriminability of States 218
10.2 No Disturbance Without Information 219
10.3 Perfect Discriminability Implies No Disturbance 222
10.4 Orthogonality 225
10.5 Maximal Sets of Perfectly Discriminable States 226
10.6 Summary 227
Solutions to Selected Problems and Exercises 228

11 Identifying Pure States 229
11.1 The State Identification Task 229
11.2 Only One Pure State for Each Atomic Effect 230
11.3 Every Pure State can be Identified 231
11.4 For a Pure State, Only One Atomic Effect 232
11.5 The Dagger 233
11.6 Transposing States 235
11.7 Transposing Effects 237
11.8 Playing with Transposition 239
11.9 Summary 241

12 Diagonalization 242
12.1 Conjugate Systems and Conjugate States 242
12.2 A Most Fundamental Result 244
12.3 The Informational Dimension 246
12.4 The Informational Dimension of a Face 247
12.5 Diagonalizing States 248
12.6 Diagonalizing Effects 251
12.7 Operational Versions of the Spectral Theorem 252
12.8 Operational Version of the Schmidt Decomposition 253
12.9 Summary 255
Solutions to Selected Problems and Exercises 255

Part IV Quantum Theory from the Principles 257

13 Conclusive Teleportation 259
13.1 The Task 259
13.2 The Causality Bound 261
13.3 Achieving the Causality Bound 263
13.4 The Local Discriminability Bound 264
13.5 Achieving the Local Discriminability Bound 266
13.6 The Origin of the Hilbert Space 267
13.7 Isotropic States and Effects 268
13.8 Summary 272
Appendix 13.1 Unitary and Orthogonal Representations 272

14 The Qubit 274
14.1 Two-dimensional Systems 274
14.2 Summary 278
Solutions to Selected Problems and Exercises 278

15 Projections 280
15.1 Orthogonal Complements 280
15.2 Orthogonal Faces 282
15.3 Projections 285
15.4 Projection of a Pure State on Two Orthogonal Faces 292
15.5 Summary 296
Solutions to Selected Problems and Exercises 296

16 The Superposition Principle 297
16.1 The Superposition Principle 297
16.2 Completeness for Purification 298
16.3 Equivalence of Systems with Equal Dimension 299
16.4 Reversible Operations of Perfectly Discriminable Pure States 299
16.5 Summary 300

17 Derivation of Quantum Theory 301
17.1 The Basis 301
17.2 Matrix Representation of States and Effects 304
17.3 Representation of Two-qubit Systems 310
17.4 Positive Matrices 320
17.5 Quantum Theory in Finite Dimensions 324
17.6 Summary 327
Solutions to Selected Problems and Exercises 327

References 329
Index 338

Preface

The book is the result of 20 years of teaching and research by the three authors in the fields of quantum foundations and quantum information, which culminated in two long joint papers [Phys. Rev. A **81** 062348 (2010) and **84** 012311 (2011)] that derive quantum theory from six simple information-theoretical principles. We have now the opportunity of presenting quantum theory in a radically new way, based on a conceptual understanding from the new principles. By "quantum theory" we mean the general theory of physical systems that lies at the core of "quantum mechanics," the latter broadly viewed as the quantum generalization of classical Hamiltonian mechanics. The book will not cover applications to "mechanics," but rather focus on applications to quantum information. For this reason, and with the aim of keeping the center of attention more on conceptual issues, rather than on the mathematical technicalities, we consider finite numbers of finite-dimensional systems, and restrict to probabilities of finite set of events, with some extensions to the infinite/continuous case discussed in the notes at the end of chapters.

The book includes 220 exercises and problems. The exercises are given within the body of each chapter, and selected solutions can be found at the end of that chapter. The exercises represent an integral part of the book and we warmly recommend the reader to solve them (or to check out the solutions), because the results proven therein are often used in our arguments. The problems presented at the end of each chapter build up additional knowledge and problem-solving skills, not strictly needed for the understanding of the arguments, but it is still recommended to solve them (or study the solutions). References, historical comments, and citations are provided in the notes at the end of chapters.

The book can be used for teaching at all levels, ranging from undergraduate, to master, up to PhD, and for pursuing personal research interests. The book is divided into four parts, organized as follows:

Part I *The Status Quo* (Chapter 2) introduces the mathematical structure of quantum theory, deriving it from three simple Hilbert-space postulates (systems, states, and the no-restriction hypothesis), and proving, in the form of theorems, what will later become our six principles for the derivation of quantum theory. The full mathematical structure of quantum theory is derived, including all relevant results in quantum open systems and quantum information. The derivation uses original powerful proving techniques based on tensor operators. In this part, the reader will have the chance to become acquainted with the relevant notions in operational probabilistic theories (OPT) and in convex analysis, and will start using the six principles for deriving results. This entire part can be used for an undergraduate semester course of quantum theory and quantum information, for physicists, mathematicians, and computer scientists.

Part II *The Informational Approach* (Chapters 3–7) presents the framework of operational probabilistic theories and introduces the six principles. Three separate chapters are devoted to the main principles of causality, local discriminability, and purification. Parts I and II together make a complete semester course for a masters-level course.

Part III *Quantum Information Without Hilbert Spaces* (Chapters 8–12) uses all the six principles to derive key results of quantum information theory and general features of quantum theory, including no-go theorems such as the no-cloning and no-bit-commitment theorems. Some parts of these chapters can be incorporated in a masters-level course.

Part IV *Quantum Theory from the Principles* (Chapters 13–17) derives quantum theory from the six principles.

Chapters on causality, local discriminability, and purification are of interest also for philosophers and, more generally, for readers who are seeking for a deeper understanding of these concepts in the light of quantum information.

For possible errors and corrections found after the print of the current edition of the book, the reader is addressed to the webpage: www.qubit.it/errata/homeerrata.html

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