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QUASICONFORMAL SURGERY IN HOLOMORPHIC DYNAMICS

Since its introduction in the early 1980s, quasiconformal surgery has become a major tool in the development of the theory of holomorphic dynamics, and it is essential background knowledge for any researcher in the field.

In this comprehensive introduction the authors begin with the foundations and a general description of surgery techniques before turning their attention to a wide variety of applications. They demonstrate the different types of surgeries that lie behind many important results in holomorphic dynamics, dealing in particular with Julia sets and the Mandelbrot set. Two of these surgeries go beyond the classical realm of quasiconformal surgery and use trans-quasiconformal surgery. Another deals with holomorphic correspondences, a natural generalization of holomorphic maps.

The book is ideal for graduate students and researchers requiring a self-contained text including a variety of applications. It particularly emphasizes the geometrical ideas behind the proofs, with many helpful illustrations seldom found in the literature.

Bodil Branner is Professor Emerita at the Technical University of Denmark, Lyngby. Her research interests include holomorphic dynamics and complex analysis. She has published in several renowned international journals and given numerous invited talks at conferences, workshops and symposia. Branner has served as Vice-President of the European Mathematical Society, as President of Dansk Matematisk Forening (DMF), and she was one of the founders of European Women in Mathematics. She is an honorary member of DMF, and a Fellow of the AMS.

Núria Fagella is currently Associate Professor at Universitat de Barcelona. Her research is in the area of holomorphic dynamics with an emphasis on the iteration of transcendental functions. She publishes in renowned international journals and with a diverse range of collaborators worldwide. Fagella has been invited to deliver talks and short courses at numerous international conferences and workshops, and has been an organiser of several such events.

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Quasiconformal Surgery in Holomorphic Dynamics

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With contributions by
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Dedicated to the memory of Adrien Douady

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Preface

The firm intention of writing this book was born in the fall of 2003 during the Ecole Thématique du CNRS Chirurgie holomorphe. This workshop was part of a trimester organized by Adrien Douady at the Institut Henri Poincaré in Paris. Douady was one of the fathers of the theory of holomorphic dynamics and of many of the surgery constructions that the workshop addressed. He used surgery as a tool in a number of ways, but especially to obtain a better understanding of different structures in parameter space. As he said: ‘plough in dynamical spaces and harvest in parameter space’.

Douady’s creative and geometric point of view inspired many to explore holomorphic dynamics. Furthermore, he encouraged generous collaboration and believed strongly in the value of sharing ideas. He gathered a large mathematical family around him, and the success of the workshop is a tribute to his influence.

Many of those who had originally developed holomorphic surgery presented lectures. It became clear that the content of these wonderful sessions ought to be the core of a book about surgery. We ourselves felt strongly that the book should be more than a collection of papers, though: our goal became to enlist the help of the speakers in creating a comprehensive study of quasiconformal surgery.

We are delighted that our wishes have come true in the form of this book, which puts together the foundations of surgery and many of its applications, with, as we had hoped, contributions by a number of the workshop participants who themselves had played such an important part in developing the field. We are grateful for their support. They have our sincere and enthusiastic thanks.

Writing and collecting the material and then unifying it into a book was not an easy task for us. Douady’s excitement about the project and his constant encouragement were very important. They kept us going until this very moment. It is to him that we dedicate every word written here, wishing he could have seen the final result.

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There are many others who should also be mentioned. In particular, the participants in the Quasiconformal Surgery course at the Complex Dynamics Seminar at Universitat de Barcelona, Antonio Garijo, Xavier Jarque, Helena Mihaljević-Brandt, Jörn Peter and Jordi Taixés inspired us with questions and suggestions and read parts of early drafts of the book. There are also those who made valuable comments to different parts of later drafts. These include Anna Miriam Benini, Jordi Canela, Matías Carrasco, Jonguk Yang and, very specially, Jonathan Brezin, Albert Clop, Ernest Fontich, Linda Keen, Curtis McMullen and Caroline Series. We heartfully thank all of them, and others who helped in various ways, for their effort. Of course, we take full responsibility for any errors that remain.

Furthermore, we wish to thank our editor, Roger Astley, for his kindness, enthusiasm and dedication during the whole process. On the technical side we are grateful to Christian Mannes for creating *It*, the computer program with which many of the illustrations were made. Our thanks also go to the Institut for Matematik og Computer Science at Danmarks Tekniske Universitet, the Departament de Matemàtica Aplicada i Anàlisi at Universitat de Barcelona, the IMUB (Institut de Matemàtica de la UB) and the CRM (Centre de Recerca Matemàtica) for their support.

And finally, but most especially, our sincere gratitude goes to our families, for their encouragement, support and patience throughout this long, but for us so very exciting, project.

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Kongens Lyngby and Barcelona

Symbols

$\underset{affine}{\sim}$	affine conjugate
$\underset{hyb}{\sim}$	hybrid equivalent
$\underset{qc}{\sim}$	quasiconformally conjugate
$\underset{top}{\sim}$	topologically conjugate
$\simeq$	conformal equivalence
1_A	The characteristic function takes the value 1 on A and 0 on $\mathbb{C}\setminus A$
$\mathcal{A}_f(\alpha)$	The basin of attraction of an attracting p-cycle $\alpha = \{\alpha_0, \ldots, \alpha_{p-1}\}$ of f
$\mathcal{A}_f^\circ(\alpha)$	The immediate basin of attraction of a cycle α as above
$\mathcal{A}_f(\infty)$	The basin of attraction of infinity of a polynomial f
$\mathbb{A}_r$	Round open annulus $\{r < \|z\| < 1\}$
$\mathbb{A}_{r,R}$	Round open annulus $\{r < \|z\| < R\}$
$\mathcal{B}$	The Bryuno numbers or a Bers' slice
$\mathbb{C}$	The complex plane
$\mathbb{C}^*$	The punctured complex plane $\mathbb{C}\setminus\{0\}$
$\widehat{\mathbb{C}}$	The extended complex plane $\mathbb{C} \cup \{\infty\}$
C_f	Set of critical points of f
$\mathcal{C}^r(U), r \geqslant 1$	The space of r times differentiable maps on U, whose nth derivatives are continuous for all $1 \leqslant n \leqslant r$
$\mathcal{C}^\infty(U)$	The spaces of functions which belong to $\mathcal{C}^r(U)$ for all $r \geqslant 1$
$\mathcal{C}_c^r(U)$	The space of functions in $\mathcal{C}^r(U)$ with compact support, for $1 \leqslant r \leqslant \infty$
$\mathrm{Crit}(f)$	Set of finite critical points of f, i.e. in $\mathbb{C}$
$\partial_z, \partial_{\overline{z}}$	Ordinary partial derivatives with respect to z and $\overline{z}$
$\partial, \overline{\partial}$	Partial derivatives in the sense of distributions with respect to z and $\overline{z}$

Δ	A Siegel disc or a linearizing domain around an attracting periodic point.
$\mathbb{D}$	The open unit disc $\{\lvert z\rvert < 1\}$ in $\mathbb{C}$
$\mathbb{D}^*$	The punctured unit disc $\mathbb{D}\setminus\{0\}$
$\mathbb{D}_r$	The open disc $\{\lvert z\rvert < r\}$
$\mathbb{D}_r(z_0)$	The open disc $\{\lvert z - z_0\rvert < r\}$
$D^+(U, V)$	The set of orientation preserving continuous functions $f : U \to V$ which are differentiable almost everywhere and whose differential $D_u f$ is non-singular almost everywhere and depends measurably on $u \in U$
$D_0^+(U, V)$	The set of functions in $D^+(U, V)$ which are absolutely continuous with resepct to the Lebesgue measure
$d_{\widehat{\mathbb{C}}}(z, w)$	The spherical distance between two points in the Riemann sphere
Ent	The set of entire transcendental maps
Ent*	The set of holomorphic transcendental self-maps of $\mathbb{C}^*$
$\mathrm{ext}(\gamma)$	The domain to the right of γ, an oriented Jordan curve
$\mathcal{F}_f$	The Fatou set of f
$\mathcal{F}_c$	The Fatou set of Q_c
f^n	$f^{\circ n} := f \circ \cdots \circ f$, the map f composed by itself n times
$GL(2, \mathbb{C})$	General linear group $\left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle\vert\, a, b, c, d \in \mathbb{C} \right\}$
$\mathbb{H}$	The upper half plane $\{\mathrm{Im}\, z > 0\}$
$\mathbb{H}_\ell$	The left half plane $\{\mathrm{Re}\, z < 0\}$
$\mathbb{H}_r$	The right half plane $\{\mathrm{Re}\, z > 0\}$
$\mathcal{H}$	The Herman numbers or, in Section Section 8.4, a hyperbolic component
$\mathrm{Im}\, z$	The imaginary part of z
$\mathrm{int}(\gamma)$	The domain to the left of γ, an oriented Jordan curve
int(X)	Interior of the set X
$\mathcal{J}_f$	The Julia set of f
$\mathcal{J}_c$	The Julia set of Q_c
$\mathcal{K}_f$	The filled Julia set of a polynomial or a polynomial-like map f
$\mathcal{K}_c$	The filled Julia set of Q_c
$\mathcal{L}_{p/q}$	The p/q-limb of the Mandelbrot set
$\mathcal{M}$	The Mandelbrot set
Mer	The set of transcendental meromorphic maps with at least one pole which is not omitted

Mer^{∞}	The set of transcendental maps which are meromorphic outside a compact countable set of singularities
mod	modulus
$\mathbb{N}$	The natural numbers $\{1, 2, \ldots\}$
$\mathcal{O}(X)$	The orbit of X, where X is a point or a set
P_f	The postsingular set or the postcritical set
Pol	The set of polynomials of degree at least two
Pol_d	The set of polynomials of degree $d \geqslant 2$
$\mathbb{Q}$	The rational numbers
Q_c	The quadratic polynomial $Q_c(z) = z^2 + c$
$\mathcal{R}$	A Riemann map
$\mathbb{R}$	The real line
$\mathbb{R}^*$	The punctured real line $\mathbb{R}\backslash\{0\}$
$\widehat{\mathbb{R}}$	The extended real line $\mathbb{R} \cup \infty$
Rat	The set of rational maps of degree at least two
Rat_d	The set of rational maps of degree $d \geqslant 2$
$\mathrm{Re}\, z$	The real part of z
R_θ	The rigid rotation by $\theta \in \mathbb{R}$, represented either by $z \mapsto e^{2\pi i \theta} z$, where $z \in \mathbb{S}^1$ or $x \mapsto x + \theta \pmod 1$ where $x \in \mathbb{R}$
$\sigma(z, w)$	The chordal distance between two points in the Riemann sphere
$\mathbb{S}^1$	The unit circle $\{\|z\| = 1\}$
$\mathbb{S}^1_r$	The circle $\{\|z\| = r\}$
$\mathrm{Sing}(f^{-1})$	Set of singularities of an inverse map
$\mathbb{T}$	The quotient space $\mathbb{R}/\mathbb{Z}$
V_f	Set of critical values of f
$\mathbb{Z}$	The integers $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$