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Excerpt
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1 INTRODUCTION

1.1 MULTIBODY SYSTEMS

The primary purpose of this book is to develop methods for the dynamic analysis of *multibody systems* that consist of interconnected *rigid* and *deformable* components. In that sense, the objective may be considered as a generalization of methods of structural and rigid body analysis. Many mechanical and structural systems such as vehicles, space structures, robotics, mechanisms, and aircraft consist of interconnected components that undergo large translational and rotational displacements. Figure 1.1 shows examples of such systems that can be modeled as multibody systems. In general, a multibody system is defined to be a collection of subsystems called *bodies*, *components*, or *substructures*. The motion of the subsystems is kinematically constrained because of different types of joints, and each subsystem or component may undergo large translations and rotational displacements.

Basic to any presentation of multibody mechanics is the understanding of the motion of subsystems (bodies or components). The motion of material bodies formed the subject of some of the earliest researches pursued in three different fields, namely, *rigid body mechanics*, *structural mechanics*, and *continuum mechanics*. The term *rigid body* implies that the deformation of the body under consideration is assumed small such that the body deformation has no effect on the gross body motion. Hence, for a rigid body, the distance between any two of its particles remains constant at all times and all configurations. The motion of a rigid body in space can be completely described by using six generalized coordinates. However, the resulting mathematical model in general is highly nonlinear because of the large body rotation. On the other hand, the term *structural mechanics* has come into wide use to denote the branch of study in which the deformation is the main concern. Large body rotations are not allowed, thus resulting in inertia-invariant structures. In many applications, however, a large number of elastic coordinates have to be included in the mathematical model in order to accurately describe the body deformation. From the study of these two

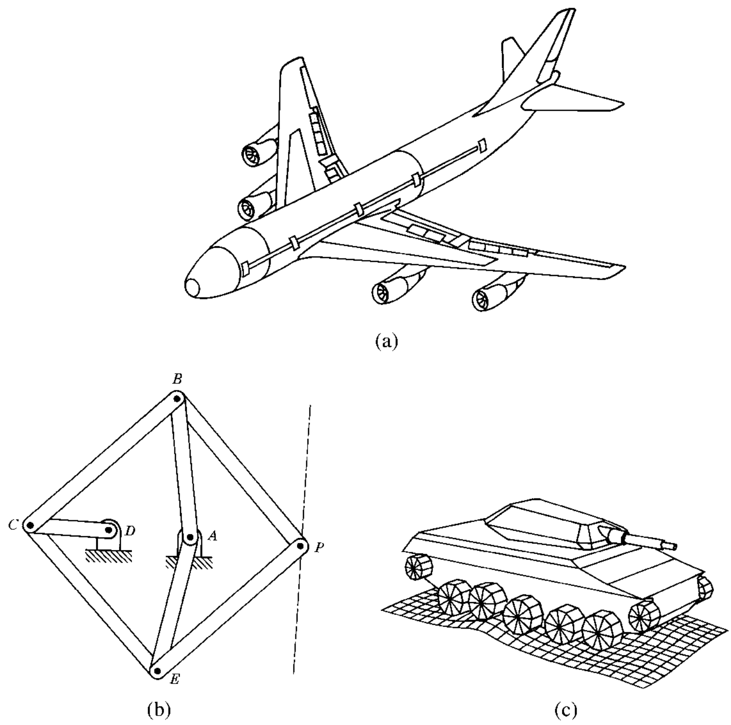

Figure 1.1 Mechanical and structural systems.

subjects, rigid body and structural mechanics, there has evolved the vast field known as *continuum mechanics*, wherein the general body motion is considered, resulting in a mathematical model that has the disadvantages of the previous cases, mainly nonlinearity and large dimensionality. This constitutes many computational problems that will be addressed in subsequent chapters.

In recent years, greater emphasis has been placed on the design of high-speed, lightweight, precision systems. Generally these systems incorporate various types of driving, sensing, and controlling devices working together to achieve specified performance requirements under different loading conditions. The design and performance analysis of such systems can be greatly enhanced through transient dynamic simulations, provided all significant effects can be incorporated into the mathematical model. The need for a better design, in addition to the fact that many mechanical and structural systems operate in hostile environments, has made necessary the inclusion of many factors that have been ignored in the past. Systems such as engines, robotics, machine tools, and space structures may operate at high speeds and in very high temperature environments. The neglect of the deformation effect, for example, when these systems are analyzed leads to a mathematical model that poorly represents the actual system.

Consider, for instance, the Peaucellier mechanism shown in Fig. 1.1(b), which is designed to generate a straight-line path. The geometry of this mechanism is such

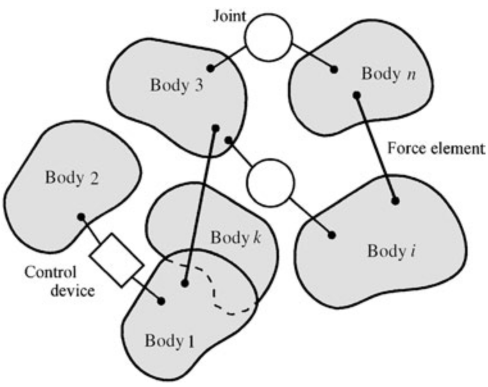

Figure 1.2 Multibody systems.

that $BC = BP = EC = EP$ and $AB = AE$. Points A, C, and P should always lie on a straight line passing through A. The mechanism always satisfies the condition $AC \times AP = c$, where c is a constant called the *inversion constant*. In case $AD = CD$, point C must trace a circular arc and point P should follow an exact straight line. However, this will not be the case when the deformation of the links is considered. If the flexibility of links has to be considered in this specific example, the mechanism can be modeled as a multibody system consisting of interconnected rigid and deformable components, each of which may undergo finite rotations. The connectivity between different components of this mechanism can be described by using revolute joints (turning pairs). This mechanism and other examples shown in Fig. 1.1, which have different numbers of bodies and different types of mechanical joints, are examples of mechanical and structural systems that can be viewed as a multibody system shown in the abstract drawing in Fig. 1.2. In this book, computer-based techniques for the dynamic analysis of general multibody systems containing interconnected sets of rigid and deformable bodies will be developed. To this end, methods for the kinematics and dynamics of rigid and deformable bodies that experience large translational and rotational displacements will be presented in the following chapters. In the following sections of this chapter, however, some of the basic concepts that will be subject of detailed analysis in the chapters that follow are briefly discussed.

1.2 REFERENCE FRAMES

The configuration of a multibody system can be described using measurable quantities such as displacements, velocities, and accelerations. These are vector quantities that have to be measured with respect to a proper *frame of reference* or *coordinate system*. In this text, the term *frame of reference*, which can be represented by three orthogonal axes that are rigidly connected at a point called the *origin* of this reference, will be frequently used. Figure 1.3 shows a frame of reference that consists of the three orthogonal axes $\mathbf{X}_1$, $\mathbf{X}_2$, and $\mathbf{X}_3$. A vector $\mathbf{u}$ in this coordinate system can be

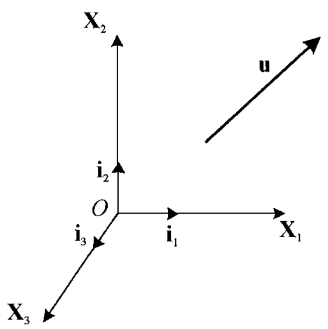

Figure 1.3 Reference frame.

defined by three components u_1, u_2, and u_3, along the orthogonal axes $\mathbf{X}_1$, $\mathbf{X}_2$, and $\mathbf{X}_3$, respectively. The vector $\mathbf{u}$ can then be written in terms of its components as $\mathbf{u} = [u_1 \quad u_2 \quad u_3]^{\mathrm{T}}$, or as $\mathbf{u} = u_1\mathbf{i}_1 + u_2\mathbf{i}_2 + u_3\mathbf{i}_3$, where $\mathbf{i}_1$, $\mathbf{i}_2$, and $\mathbf{i}_3$ are unit vectors along the orthogonal axes $\mathbf{X}_1$, $\mathbf{X}_2$, and $\mathbf{X}_3$, respectively.

Generally, in dealing with multibody systems two types of coordinate systems are required. The first is a coordinate system that is fixed in time and represents a unique standard for all bodies in the system. This coordinate system will be referred to as *global*, or *inertial frame* of reference. In addition to this inertial frame of reference, we assign a *body reference* to each component in the system. This body reference translates and rotates with the body; therefore, its location and orientation with respect to the inertial frame change with time. Figure 1.4 shows a typical body, denoted as body i in the multibody system. The coordinate system $\mathbf{X}_1\mathbf{X}_2\mathbf{X}_3$ is the global inertial frame of reference, and the coordinate system $\mathbf{X}_1^i\mathbf{X}_2^i\mathbf{X}_3^i$ is the body coordinate system. Let

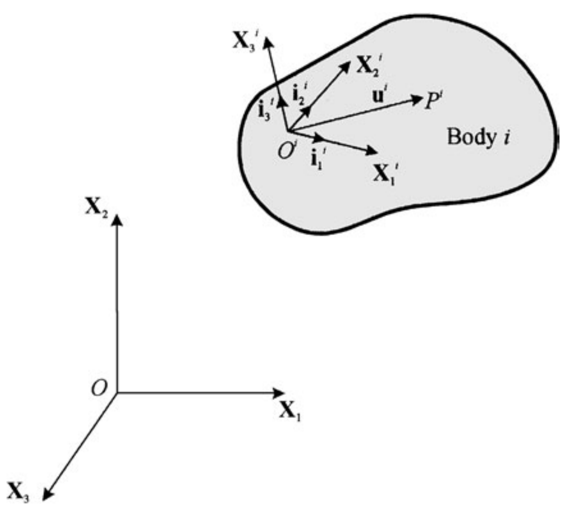

Figure 1.4 Body coordinate system.

$\mathbf{i}_1$, $\mathbf{i}_2$, and $\mathbf{i}_3$ be unit vectors along the axes $\mathbf{X}_1$, $\mathbf{X}_2$, and $\mathbf{X}_3$, respectively, and let $\mathbf{i}_1^i$, $\mathbf{i}_2^i$, and $\mathbf{i}_3^i$ be unit vectors along the body axes $\mathbf{X}_1^i$, $\mathbf{X}_2^i$, and $\mathbf{X}_3^i$, respectively. The unit vectors $\mathbf{i}_1$, $\mathbf{i}_2$, and $\mathbf{i}_3$ are fixed in time; that is, they have constant magnitude and direction, while the unit vectors $\mathbf{i}_1^i$, $\mathbf{i}_2^i$, and $\mathbf{i}_3^i$ have changeable orientations. A vector $\mathbf{u}^i$ defined in the body coordinate system can be written as

$$\mathbf{u}^i = \bar{u}_1^i \mathbf{i}_1^i + \bar{u}_2^i \mathbf{i}_2^i + \bar{u}_3^i \mathbf{i}_3^i \tag{1.1}$$

where $\bar{u}_1^i$, $\bar{u}_2^i$, and $\bar{u}_3^i$ are the components of the vector $\mathbf{u}^i$ in the local body coordinate system. The same vector $\mathbf{u}^i$ can be expressed in terms of its components in the global coordinate system as

$$\mathbf{u}^i = u_1^i \mathbf{i}_1 + u_2^i \mathbf{i}_2 + u_3^i \mathbf{i}_3 \tag{1.2}$$

where u_1^i, u_2^i, and u_3^i are the components of the vector $\mathbf{u}^i$ in the global coordinate system. We have, therefore, given two different representations for the same vector $\mathbf{u}^i$, one in terms of the body coordinates and the other in terms of global coordinates. Since it is easier to define the vector in terms of the local body coordinates, it is useful to have relationships between the local and global components. Such relationships can be obtained by developing the transformation between the local and global coordinate systems. For instance, consider the *planar motion* of the body shown in Fig. 1.5. The coordinate system $\mathbf{X}_1\mathbf{X}_2$ represents the inertial frame and $\mathbf{X}_1^i\mathbf{X}_2^i$ is the body coordinate system. Let $\mathbf{i}_1$ and $\mathbf{i}_2$ be unit vectors along the $\mathbf{X}_1$ and $\mathbf{X}_2$ axes, respectively, and let $\mathbf{i}_1^i$ and $\mathbf{i}_2^i$ be unit vectors along the body axes $\mathbf{X}_1^i$ and $\mathbf{X}_2^i$, respectively. The orientation of the body coordinate system with respect to the global frame of reference is defined by the angle θ^i. Since $\mathbf{i}_1^i$ is a unit vector, its component along the $\mathbf{X}_1$ axis is $\cos\theta^i$, while its component along the $\mathbf{X}_2$ axis is $\sin\theta^i$. One can then write the unit vector $\mathbf{i}_1^i$ in the global coordinate system as

$$\mathbf{i}_1^i = \cos\theta^i \mathbf{i}_1 + \sin\theta^i \mathbf{i}_2 \tag{1.3}$$

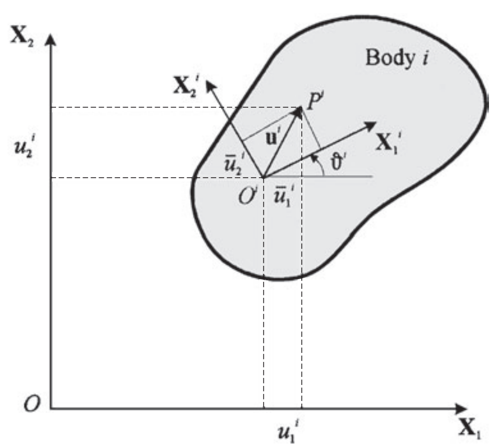

Figure 1.5 Planar motion.

Similarly, the unit vector $\mathbf{i}_2^i$ is given by

$$\mathbf{i}_2^i = -\sin\theta^i \mathbf{i}_1 + \cos\theta^i \mathbf{i}_2 \tag{1.4}$$

The vector $\mathbf{u}^i$ is defined in the body coordinate system as $\mathbf{u}^i = \bar{u}_1^i \mathbf{i}_1^i + \bar{u}_2^i \mathbf{i}_2^i$, where $\bar{u}_1^i$ and $\bar{u}_2^i$ are the components of the vector $\mathbf{u}^i$ in the body coordinate system. Using the expressions for $\mathbf{i}_1^i$ and $\mathbf{i}_2^i$, one gets

$$\begin{aligned}\mathbf{u}^i &= \bar{u}_1^i(\cos\theta^i \mathbf{i}_1 + \sin\theta^i \mathbf{i}_2) + \bar{u}_2^i(-\sin\theta^i \mathbf{i}_1 + \cos\theta^i \mathbf{i}_2)\\ &= u_1^i \mathbf{i}_1 + u_2^i \mathbf{i}_2\end{aligned} \tag{1.5}$$

where u_1^i and u_2^i are the components of the vector $\mathbf{u}^i$ defined in the global coordinate system and given by

$$u_1^i = \bar{u}_1^i \cos\theta^i - \bar{u}_2^i \sin\theta^i, \quad u_2^i = \bar{u}_1^i \sin\theta^i + \bar{u}_2^i \cos\theta^i \tag{1.6}$$

These two equations which provide algebraic relationships between the local and global components in the planar analysis can be expressed in a matrix form as $\mathbf{u}^i = \mathbf{A}^i \bar{\mathbf{u}}^i$, where $\mathbf{u}^i = [u_1^i \quad u_2^i]^{\mathrm{T}}$, $\bar{\mathbf{u}}^i = [\bar{u}_1^i \quad \bar{u}_2^i]^{\mathrm{T}}$, and $\mathbf{A}^i$ is the planar transformation matrix defined as

$$\mathbf{A}^i = \begin{bmatrix}\cos\theta^i & -\sin\theta^i\\ \sin\theta^i & \cos\theta^i\end{bmatrix} \tag{1.7}$$

In Chapter 2 we will study the spatial kinematics and develop the spatial transformation matrix and study its important properties.

1.3 PARTICLE MECHANICS

Dynamics in general is the science of studying the motion of particles or bodies. The subject of dynamics can be divided into two major branches, *kinematics* and *kinetics*. In kinematic analysis, we study the motion regardless of the forces that cause it, while kinetics deals with the motion and forces that produce it. Therefore, in kinematics attention is focused on the geometric aspects of motion. The objective is, then, to determine the positions, velocities, and accelerations of the system under investigation. In order to understand the dynamics of multibody systems containing rigid and deformable bodies, it is important to understand first the body dynamics. We start with a brief discussion on the dynamics of particles that form the rigid and deformable bodies.

Particle Kinematics A *particle* is assumed to have no dimensions and accordingly can be treated as a point in a three-dimensional space. Therefore, in studying the kinematics of particles, we are concerned primarily with the translation of a point

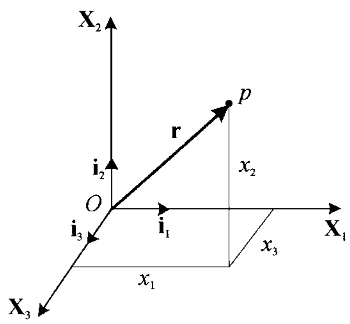

Figure 1.6 Position vector of the particle p.

with respect to a selected frame of reference. The position of the particle can then be defined using three coordinates. Figure 1.6 shows a particle p in a three-dimensional space. The position vector of this particle can be written as

$$\mathbf{r} = x_1\mathbf{i}_1 + x_2\mathbf{i}_2 + x_3\mathbf{i}_3 \tag{1.8}$$

where $\mathbf{i}_1$, $\mathbf{i}_2$, and $\mathbf{i}_3$ are unit vectors along the $\mathbf{X}_1$, $\mathbf{X}_2$, and $\mathbf{X}_3$ axes and x_1, x_2, and x_3 are the Cartesian coordinates of the particle.

The velocity of the particle is defined to be the time derivative of the position vector. If we assume that the axes $\mathbf{X}_1$, $\mathbf{X}_2$, and $\mathbf{X}_3$ are fixed in time, the unit vectors $\mathbf{i}_1$, $\mathbf{i}_2$, and $\mathbf{i}_3$ have a constant magnitude and direction. The velocity vector $\mathbf{v}$ of the particle can be written as

$$\mathbf{v} = \dot{\mathbf{r}} = \frac{d}{dt}(\mathbf{r}) = \dot{x}_1\mathbf{i}_1 + \dot{x}_2\mathbf{i}_2 + \dot{x}_3\mathbf{i}_3 \tag{1.9}$$

where ($\dot{}$) denotes differentiation with respect to time and $\dot{x}_1$, $\dot{x}_2$, and $\dot{x}_3$ are the Cartesian components of the velocity vector. The acceleration of the particle is defined to be the time derivative of the velocity vector, that is,

$$\mathbf{a} = \frac{d}{dt}(\mathbf{v}) = \ddot{x}_1\mathbf{i}_1 + \ddot{x}_2\mathbf{i}_2 + \ddot{x}_3\mathbf{i}_3 \tag{1.10}$$

where $\mathbf{a}$ is the acceleration vector, and $\ddot{x}_1$, $\ddot{x}_2$, and $\ddot{x}_3$ are the Cartesian components of the acceleration vector. Using vector notation, the position vector of the particle in terms of the *Cartesian coordinates* can be written as $\mathbf{r} = [x_1 \quad x_2 \quad x_3]^{\mathrm{T}}$, while the velocity and acceleration vectors are given by

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = [\dot{x}_1 \quad \dot{x}_2 \quad \dot{x}_3]^{\mathrm{T}}, \quad \mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{r}}{dt^2} = [\ddot{x}_1 \quad \ddot{x}_2 \quad \ddot{x}_3]^{\mathrm{T}} \tag{1.11}$$

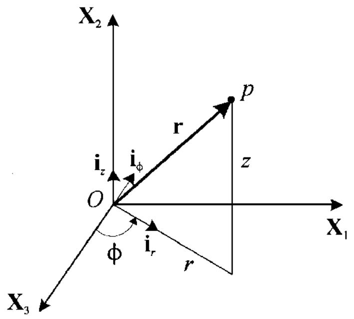

Figure 1.7 Cylindrical coordinates.

Choice of Coordinates The set of coordinates that can be used to define the particle position is not unique. In addition to the Cartesian representation, other sets of coordinates can be used for the same purpose. In Fig. 1.7, the position of particle p can be defined using the three *cylindrical coordinates*, r, ϕ, and z, while in Fig. 1.8, the particle position is identified using the *spherical coordinates* r, θ, and ϕ. In many situations, however, it is useful to obtain kinematic relationships between different sets of coordinates. For instance, if we consider the planar motion of a particle p in a circular path as shown in Fig. 1.9, the position vector of the particle can be written in the fixed coordinate system $\mathbf{X}_1\mathbf{X}_2$ as $\mathbf{r} = [x_1 \quad x_2]^{\mathrm{T}} = x_1\mathbf{i}_1 + x_2\mathbf{i}_2$, where x_1 and x_2 are the coordinates of the particle, and $\mathbf{i}_1$ and $\mathbf{i}_2$ are unit vectors along the fixed axes $\mathbf{X}_1$ and $\mathbf{X}_2$, respectively. In terms of the polar coordinates r and θ, the components x_1 and x_2 are given by $x_1 = r\cos\theta$, $x_2 = r\sin\theta$, and the vector $\mathbf{r}$ can be expressed as

$$\mathbf{r} = r\cos\theta\,\mathbf{i}_1 + r\sin\theta\,\mathbf{i}_2 \tag{1.12}$$

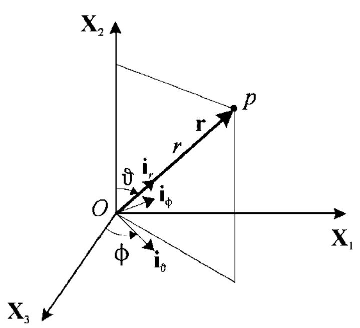

Figure 1.8 Spherical coordinates.

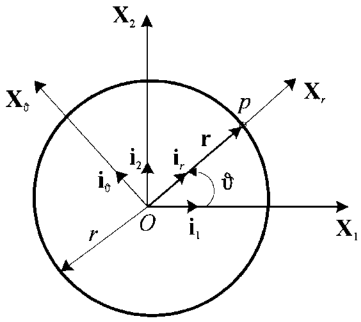

Figure 1.9 Circular motion of a particle.

Since r in this example is constant, and $\mathbf{i}_1$ and $\mathbf{i}_2$ are fixed vectors, the velocity of the particle is given by

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = r\dot{\theta}(-\sin\theta\,\mathbf{i}_1 + \cos\theta\,\mathbf{i}_2) \tag{1.13}$$

and the acceleration vector $\mathbf{a}$ is given by

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = r\ddot{\theta}(-\sin\theta\,\mathbf{i}_1 + \cos\theta\,\mathbf{i}_2) + r(\dot{\theta})^2(-\cos\theta\,\mathbf{i}_1 - \sin\theta\,\mathbf{i}_2) \tag{1.14}$$

One can verify that this equation can be written in the following compact vector form:

$$\mathbf{a} = \boldsymbol{\alpha} \times \mathbf{r} + \boldsymbol{\omega} \times \mathbf{v} \tag{1.15}$$

where $\boldsymbol{\omega}$ and $\boldsymbol{\alpha}$ are the vectors $\boldsymbol{\omega} = \dot{\theta}\,\mathbf{i}_3$, $\boldsymbol{\alpha} = \ddot{\theta}\,\mathbf{i}_3$.

One may also define the position vector of p in the moving coordinate system $\mathbf{X}_r\mathbf{X}_\theta$. Let, as shown in Fig. 1.9, $\mathbf{i}_r$ and $\mathbf{i}_\theta$ be unit vectors along the axes $\mathbf{X}_r$ and $\mathbf{X}_\theta$, respectively. It can be verified that these two unit vectors can be written in terms of the unit vectors along the fixed axes as $\mathbf{i}_r = \cos\theta\,\mathbf{i}_1 + \sin\theta\,\mathbf{i}_2$, $\mathbf{i}_\theta = -\sin\theta\,\mathbf{i}_1 + \cos\theta\,\mathbf{i}_2$ and their time derivatives can be written as

$$\left.\begin{aligned} \dot{\mathbf{i}}_r &= \frac{d\mathbf{i}_r}{dt} = -\dot{\theta}\sin\theta\,\mathbf{i}_1 + \dot{\theta}\cos\theta\,\mathbf{i}_2 = \dot{\theta}\mathbf{i}_\theta \\ \dot{\mathbf{i}}_\theta &= \frac{d\mathbf{i}_\theta}{dt} = -\dot{\theta}\cos\theta\,\mathbf{i}_1 - \dot{\theta}\sin\theta\,\mathbf{i}_2 = -\dot{\theta}\mathbf{i}_r \end{aligned}\right\} \tag{1.16}$$

The position vector of the particle in the moving coordinate system can be defined as $\mathbf{r} = r\,\mathbf{i}_r$. Using this equation, the velocity vector of particle p is given by

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{dr}{dt}\mathbf{i}_r + r\frac{d\mathbf{i}_r}{dt} \tag{1.17}$$

Since the motion of point p is in a circular path, $dr/dt = 0$, and the velocity vector $\mathbf{v}$ reduces to

$$\mathbf{v} = r\frac{d\mathbf{i}_r}{dt} = r\dot{\theta}\mathbf{i}_\theta \tag{1.18}$$

which shows that the velocity vector of the particle is always tangent to the circular path. The acceleration vector $\mathbf{a}$ is also given by

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = r\ddot{\theta}\mathbf{i}_\theta + r\dot{\theta}\frac{d\mathbf{i}_\theta}{dt} = r\ddot{\theta}\mathbf{i}_\theta - r(\dot{\theta})^2\mathbf{i}_r \tag{1.19}$$

The first term, $r\ddot{\theta}$, is called the *tangential component* of the acceleration, while the second term, $-r(\dot{\theta})^2$, is called the *normal component*.

Particle Dynamics The study of *Newtonian mechanics* is based on Newton's three laws, which are used to study particle mechanics. *Newton's first law* states that a particle remains in its state of rest, or of uniform motion in a straight line if there are no forces acting on the particle. This means that the particle can be accelerated if and only if there is a force acting on the particle. *Newton's third law*, which is sometimes called the *law of action and reaction*, states that to every action there is an equal and opposite reaction; that is, when two particles exert forces on one another, these forces will be equal in magnitude and opposite in direction. *Newton's second law*, which is called the *law of motion*, states that the force that acts on a particle and causes its motion is equal to the rate of change of momentum of the particle, that is, $\mathbf{F} = \dot{\mathbf{P}}$ where $\mathbf{F}$ is the vector of forces acting on the particle, and $\mathbf{P}$ is the linear momentum of the particle, which can be written as $\mathbf{P} = m\mathbf{v}$, where m is the mass, and $\mathbf{v}$ is the velocity vector of the particle. It follows that $\mathbf{F} = d(m\mathbf{v})/dt$. In nonrelativistic mechanics, the mass m is constant and as a consequence, one has

$$\mathbf{F} = m\frac{d\mathbf{v}}{dt} = m\mathbf{a} \tag{1.20}$$

where $\mathbf{a}$ is the acceleration vector of the particle. Equation 20 is a vector equation that has the three scalar components $F_1 = ma_1$, $F_2 = ma_2$, $F_3 = ma_3$, where F_1, F_2, and F_3 and a_1, a_2, and a_3 are, respectively, the components of the vectors $\mathbf{F}$ and $\mathbf{a}$ defined in the global coordinate system. The vector $m\mathbf{a}$ is sometimes called the *inertia* or the *effective* force vector.

1.4 RIGID BODY MECHANICS

Unlike particles, *rigid bodies* have distributed masses. The configuration of a rigid body in space can be identified by using six coordinates. Three coordinates