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978-1-107-04231-5 - Introduction to Mathematical Portfolio Theory

Mark S. Joshi and Jane M. Paterson

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