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978-1-107-04231-5 - Introduction to Mathematical Portfolio Theory
Mark S. Joshi and Jane M. Paterson
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Introduction to Mathematical Portfolio Theory

In this concise yet comprehensive guide to the mathematics of modern portfolio theory, the authors discuss mean–variance analysis, factor models, utility theory, stochastic dominance, very long term investing, the capital asset pricing model, risk measures including VAR, coherence, market efficiency, rationality and the modelling of actuarial liabilities. Each topic is clearly explained with assumptions, mathematics, limitations, problems and solutions presented in turn.

Joshi's trademark style of clarity and practicality is here brought to classical financial mathematics. The book is suitable for mathematically trained students in actuarial studies, business and economics as well as mathematics and finance, and it can be used both for self-study and as a course text. The authors' experience as both academics and practitioners brings clarity and relevance to the book, whilst ensuring that the limitations of models are highlighted.

MARK S. JOSHI is a researcher and consultant in mathematical finance, and a Professor at the University of Melbourne. His research focuses on derivatives pricing and interest rate derivatives in particular. He is the author of numerous research articles on quantitative finance and four books.

JANE M. PATERSON obtained a PhD in pure mathematics from the University of Melbourne. She furthered her academic experience with a postdoctoral fellowship at the Mathematical Sciences Research Institute, Berkeley and a research fellowship at the University of Cambridge. More recently she has worked in both the UK and Australia as a director in a variety of specialist and generalist banking roles, including structured finance and economic capital, with organisations including National Australia Bank and ANZ.

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Preface

This book grew out of a lecture course taught at the University of Melbourne over a series of years. The audience was third-year actuarial students who partially gained an exemption from the Faculty and Institute of Actuaries CT8 module if they did well. The nature of the audience and the exemption placed certain constraints on the syllabus and delivery that made it hard to find a suitable textbook. The graduate level texts simply being too hard, whilst the undergraduate and MBA books did not cover the mathematics in sufficient depth. In particular, the students were fairly mathematical but more oriented towards computations than proof. In addition, the choice of topics had to be tuned to the actuarial syllabus and that is reflected in this book.

In terms of mathematical level, we strive to achieve a mid-level where mathematics is not shied away from nor hived off to appendices, but also not so hard as to deter the undergraduate reader. Also, this being a book on mathematical portfolio theory, the mathematics takes centre stage for most of the book: our objective is to study the mathematics of portfolio theory without losing sight of the finance. As both authors have been both practitioners and academics, a theme throughout is that a model is a model and not reality, and we aim to highlight our assumptions and their consequences. We provide a lot of problems with solutions in the belief that this is ultimately how the material is best learnt, and as a consequence of the fact that students always want more problems and more solutions.

We first look at the definitions of risk and return. We then explore Markowitz's portfolio theory. We start with the two-asset case, then add a riskless asset, and finally treat the general case. We derive a couple of different ways to find efficient portfolios in that case. We then move on to seeing how simplifying correlation structures can help to reduce the amount of data needed to estimate the model parameters.

We then make a long excursion into utility theory, looking at both its pros

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and cons. We also look at the offshoots of stochastic dominance and geometric mean maximisation. An important issue in any insurance company or bank is risk control, and we therefore look at risk measures including VAR and conditional expected shortfall. We also examine the coherence axioms.

We then move onto a critical look at the capital asset pricing model and the arbitrage pricing theory. We also discuss market efficiency and rationality. Here we adopt a more discursive viewpoint. We finish by looking at long-term models of stock prices using Brownian motion and the Wilkie model.

Much of this book has been shaped by interactions with our students and from their explicit feedback, and we thank them all for their input. We particularly thank Timothy Hillman for his detailed comments on the manuscript.

For the reader whose appetite has been whetted and wishes to study the material in greater depth, we mention a few books which we have found helpful.

- Elton *et al.*, [6], is very discursive and contains very detailed references but is much less mathematical than this book.
- Pennachi, [15], is a nice text aimed at PhD students.
- Cochrane, [3], has both good discussion and good mathematics and will reward the reader who perseveres.
- Markowitz's original book, *Portfolio Selection,* [11], is still a good read.

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