

Index

- abelian, 267
- accumulation point, 56
- action, *see* group action
- active transformation, 69
- ad, 109
- Ad, 109
- adjoint map, *see* map, adjoint
- adjoint matrix, *see* matrix, adjoint
- adjoint representation
 - of a Lie algebra, 109
 - of a Lie group, 109
- adjugate, *see* matrix, adjugate
- affine
 - independence, 141
 - connection, 189
 - hyperplane, *see* hyperplane
 - parameter, 214
 - subspace, 2
- algebra
 - exterior, 41
 - graded, 31
 - symmetric, 52
- algebraic multiplicity, 24
- alternating symbol, *see* Levi-Civita alternating symbol
- alternating tensor, *see* tensor, antisymmetric
- annihilator, 9
- anti-Hermitian matrix, *see* matrix, anti-Hermitian
- anticommutator, 53
- antiisomorphism, 19
- antilinear, *see* map, antilinear
- antipodal map, *see* map, antipodal
- arc length parameterized curve, 214
- atlas, 63
- automorphism
 - general, 267
 - linear, 3
- autoparallel, *see* geodesic
- axial vector, *see* pseudovector

- Baker–Campbell–Hausdorff formula, 110
- ball, 57
- barycentric coordinates, 141

- base space, 177
- basis, 2
 - completion of, 3
 - dual, 8
 - of a topology, *see* topology, basis of
 - of sections, 178
- Betti number, 145
- Bianchi identity, 184
- bijective map, *see* map, bijective
- bilinear form, 14
 - symmetric, 14
- bosonic particle state, 53
- boundary
 - of a manifold, 167
 - of a polytope, 140
 - of a simplex, 143
 - of a simplicial chain, 145
 - of a singular chain, 147
- boundary complex, 152
- boundary operator, 143
- boundary space
 - singular, 147
- bracket-generating distribution, 294
- bridge, 302
- Brouwer fixed point theorem, 133
- bump form, 132
- bundle
 - cotangent, 180
 - tangent, 180
 - tensor product, 180
- bundle projection, 177
- bundle space, *see* total space

- canonical commutation relations, 53
- canonical transformation, 114
- cardinality, 264
- Cartan’s
 - equations of structure, 185
 - lemma, 50
- Cartesian product, 265
- catenoid, 236
- chain

simplicial, 143
 singular, 146
 smooth singular, 158
 chain complex, 150
 chain map, 126
 change of variables theorem, 162
 characteristic class, 189
 characteristic polynomial, *see* matrix, characteristic polynomial of
 Chern–Gauss–Bonnet theorem, 262
 chord, 303
 Chow–Rashevskii theorem, 294
 Christoffel symbol, 202
 class representative, 6
 classification theorem, 155
 closed
 form, 116
 manifold, 155
 set, 56
 closure, 56
 cobasis, 8
 coboundary, 125
 smooth singular, 171
 coboundary operator, *see* connecting homomorphism
 co-closed form, 223
 cocycle, 125
 smooth singular, 170
 cocycle conditions (on a vector bundle), 178
 Codazzi–Mainardi equation, 310
 co-differential operator, 222
 cofactor, *see* matrix, cofactor
 coframe field, 80
 cohomologous closed forms, 123
 cohomology
 of a complex, 125
 smooth singular, 170
 cokernel, 21
 column rank, *see* rank, of a matrix
 commutative diagram, 126
 commutator, 53
 compact set, 56
 complement, 263
 complete manifold, *see* manifold, complete
 complete topological invariant, 58
 completely integrable, 286
 complex
 de Rham, 126
 differential, 125
 component function, 59
 components
 contracted, 12
 contravariant, 11
 covariant, 11
 of a link, 255
 vector, 2
 composition, *see* map, composition of
 compound matrix, *see* matrix, compound
 cone, 118

conformally flat, 314
 conjugate linear map, *see* map, conjugate linear
 conjugation, 108
 connected component, 123
 connected sum, 154
 connected space, 123
 k -connected space, 136
 connecting homomorphism, 127
 connection, 182
 flat, 209
 Levi-Civita, 200
 matrix, 183
 Riemannian, 200
 constraint, 291
 holonomic, 291
 continuous
 image, 58
 map, *see* map, continuous
 contractible space, 118
 contragredient matrix, *see* matrix, contragredient
 contravariant components, *see* components, contravariant
 convex
 combination, 139
 hull, 139
 set, 139
 coordinate
 chart, 63
 frame, *see* frame field, holonomic
 functions, 60
 map, 62
 neighborhood, *see* coordinate patch
 patch, 62
 coordinates, 60
 geodesic, *see* coordinates, Riemann normal
 isothermal, 314
 local inertial or Lorentz, 278
 locally flat, 278
 Riemann normal, 277
 cotangent bundle, *see* bundle, cotangent
 cotangent space
 as dual of tangent space, 79
 as jet space, 81
 countable set, 264
 covariant
 components, *see* components, covariant
 derivative, 182
 covariant derivative, 182
 covector, 8
 components of, 8
 covector field, *see* differential form, 1-form
 cover, 111
 universal, 111
 covering space, *see* cover
 Cramer's rule, 22
 critical point, 71
 crosspolytope, *see* polytope
 curl, 242

- curvature
 - constant, 233
 - Gauss–Kronecker, 312
 - Gaussian, 233
 - invariant, 207
 - matrix, 186
 - mean, 312
 - operator, 186
 - principal, 312
 - tensor, 188
 - 2-forms, 184
- CW complex, 149
- cycle
 - elementary, 303
 - in a graph, 299
 - of a simplicial complex, 145
- cycle rank, 303
- cycle space
 - of a simplicial complex, 145
 - singular, 147
- cyclic group, 266

- Darboux’s theorem, 113
- de Morgan’s laws, 263
- de Rham
 - cohomology, 123
 - first theorem, 170
 - second theorem, 170
 - theorem, 172
- deformation retraction, 125
- degree
 - of a map, 251
- dense set, 56
- derivation, 75
- derivative
 - Lie, 102
 - of a smooth map, 98
 - of map on \mathbb{R}^n , 59
 - partial, 59
- determinant
 - of a linear map, 43
 - of a matrix, 17
- diagonalizable matrix, *see* matrix, diagonalizable
- diagonalizing a matrix, 271
- diffeomorphic spaces, 69
- diffeomorphism
 - of manifolds, 69
 - on \mathbb{R}^n , 59
- difference (of sets), 263
- differential
 - of a complex, 125
 - of a smooth map, 98
- differential form, 87
 - 1-form, 79
- differential ideal, 290
- dimension
 - of a face of a polytope, 140
 - of a simplicial complex, 142
 - of a vector space, 2
- direct sum, 2
- discrete topology, *see* topology, discrete
- distribution, 284
- divergence, 242
- division ring, *see* skew field
- domain, 263
- dot product, 15
- double cover, *see* cover, double
- double dual, 9
- dual map, *see* map, dual
- dual pairing, 8
- dual space, 8

- E8-space, 145
- edge
 - of a graph, 296
 - of a polytope, 140
- eigenvalue, 24
- eigenvector, 24
- Einstein manifold, *see* manifold, Einstein
- Einstein summation convention, 33
- Einstein tensor, 207
- electromagnetic 2-form, 92
- elementary symmetric function, 25
- elliptic curve, 105
- embedding, 71
- endomorphism
 - general, 267
 - linear, 3
- Enneper’s surface, 316
- epsilon symbol, *see* Levi-Civita alternating symbol
- epsilon tensor, *see* Levi-Civita alternating tensor
- epsilon tensor identities, 240
- equivalence
 - class, 264
 - principle, 278
 - relation, 264
- equivalence mod n , 264
- Euclidean
 - distance, 57
 - group, 228
 - metric, 196
 - space, 15
- Euler
 - characteristic, 149
 - class, 262
- exact form, 117
- exact sequence, 4
 - split, 5
- exponential map
 - on a Lie group, 107
 - on a Riemannian manifold, 276
- extended by linearity, 3
- exterior
 - algebra, *see* algebra, exterior
 - derivative, 89
 - power

324

of a map, 42
 of a vector space, 39
 product, 38
 extrinsic curvature tensor, *see* second fundamental form

face
 of a polytope, 140
 of a simplex, 141

facet, 140

Fermi normal coordinates, 214

fermionic particle states, 53

fiber, 177

field, 269
 finite, 269

field strength, 192

first fundamental form, 310

flat space, 209

flow (of a vector field), 101

Fock space, 52

foliation, 289

forest, 302

frame field, 78
 holonomic, 194
 nonholonomic, 194
 orthonormal, 196

Frobenius' theorem, 286

function linearity, 85

functor
 contravariant, 98
 covariant, 98

fundamental group, 136

gauge
 covariant derivative, 192
 field theory, 191
 group, 191
 potential, 191
 transformation, 191

Gauss–Bonnet theorem, 261

Gauss equation, 310

Gauss's formula, 258

Gauss–Kronecker curvature, *see* curvature,
 Gauss–Kronecker

Gauss map, *see* map, Gauss

Gaussian curvature, *see* curvature, Gaussian

general linear group, 105

generator of cohomology, 132

genus, 155

geodesic, 213
 null, 214

geodesic deviation, 215

geodesic deviation equation, 216

geodesic equation, 213

geometric
 manifold, 193
 multiplicity, 24

geometry, 193

Index

graded filtration, 81

gradient, 242

Gram matrix, *see* matrix, Gram

Gram–Schmidt orthonormalization, 28

Grammian, 17

graph, 296

group, 265
 general linear, 105
 orthogonal, 105
 unitary, 108

group action, 268

hairy ball theorem, 253

half-space, 140

Hamiltonian, 114

Hamiltonian vector field, *see* vector field,
 Hamiltonian

harmonic form, 223

Hausdorff topology, *see* topology, Hausdorff

heat equation, 315

hedgehog theorem, *see* hairy ball theorem

Hermitian matrix, *see* matrix, Hermitian

Hilbert space, 52

Hodge
 decomposition theorem, 223
 dual
 of a p vector, 45
 star, *see* Hodge dual
 theorem on harmonic forms, 224

Hodge–de Rham Laplacian, *see* Laplace–de Rham
 operator

holonomy group, 219
 restricted, 219

homeomorphic spaces, 58

homeomorphism, 58

homologous cycles, 145

homology
 singular, 147
 smooth singular, 159

homology group
 simplicial, 145

homomorphism
 group, 267
 linear, 3
 of complexes, 126

homotopic
 maps, 118
 spaces, 118

homotopically trivial map, *see* map, null homotopic

homotopy, 118

homotopy group
 first, *see* fundamental group

homotopy operator, 120

homotopy type, 118

Hopf fibration, 254

hyperbolic space, 233

hyperplane, 140

hypersurface, 310

- idempotent map, 4
- identification, 66
- image, 263
- imbedding, *see* embedding
- immersion, 70
- impedance matrix, *see* matrix, impedance
- incidence matrix, 297
- inclusion map, 5
- inclusion–exclusion, 154
- indefinite inner product, *see* inner product, indefinite
- index
 - of a map, 250
 - of a vector field, 260
- indices
 - contravariant, 11
 - covariant, 11
 - raising and lowering, 197
- injective map, *see* map, injective
- inner product, 14
 - Euclidean, 15
 - indefinite, 15
 - Lorentzian, 15
 - negative definite, 15
 - nonnegative definite, 15
 - nonpositive definite, 15
 - positive definite, 15
- inner product space, 14
- integrable distribution, 284
- integral
 - Riemann, 160
- integral curve, 100
- integral submanifold, 284
- interior
 - of a polytope, 140
 - of a set, 56
- interior product, 93
- intersection, 263
- into map, *see* map, injective
- invariant subspace, 271
- inverse
 - function theorem, 59
 - image, 263
- involution distribution, 285
- isometric geometric manifolds, 193
- isomorphic
 - groups, 267
 - vector spaces, 3
- isomorphism
 - linear, 3
 - of groups, 267
 - of Lie algebras, 107
- isospin, 190
- isothermal coordinates, *see* coordinates, isothermal
- isotopy, 255

- Jacobi field, 215
- Jacobi identity, 78
- Jacobi's equation, *see* geodesic deviation equation

- Jacobian
 - determinant, 59
 - matrix, 59
- jet space, 81
- join of simplicial complexes, 152

- kernel, 3
- Killing
 - field, 227
 - form, 110
- Killing's equation, 227
- knot, 255
- Koszul formula, 200
- Kronecker
 - delta, 8
 - symbol, 240
 - product, *see* matrix, Kronecker product

- Laplace–Beltrami operator, 242
- Laplace–de Rham operator, 223
- Laplace expansion, 22
- Laplacian, *see* Laplace–Beltrami operator
- Laplacian matrix, *see* matrix, Laplacian
- leaf, 289
- Lefschetz
 - fixed point theorem, 157
 - number, 157
- Lefschetz–Hopf theorem, 260
- left-invariant vector field, 106
- Legendre polynomial, 28
- Levi-Civita
 - alternating symbol, 238
 - alternating tensor, 240
 - connection, *see* connection, Levi-Civita
- Lie algebra
 - of a Lie group, 106
 - of vector fields, 78
- Lie derivative, *see* derivative, Lie
- Lie group, 105
- Lie subalgebra, 108
- limit point, *see* accumulation point
- line element, 197
- linear
 - functional, 8
 - independence, 2
 - map, *see* map, linear
- link, 255
- linking number
 - of links, 255
 - of manifolds, 258
- Liouville's theorem, 114
- local
 - coordinates, 64
 - trivialization, 177
- locally Euclidean manifold, 62
- locally finite open cover, 56
- locally flat
 - coordinates, *see* coordinates, locally flat

326

manifold, 210
 locally straight integral submanifold, 286
 loop in a manifold, 136

manifold

complete, 233
 Einstein, 238
 isotropic, 232
 Lorentzian, 194
 orientable, 64
 orientation
 by Jacobian, 64
 induced, 167
 orientation class, 275
 parallelizable, 179
 pseudo-Riemannian, 194
 Riemannian, 193
 smooth, 62
 symplectic, 113
 topological, 62
 with boundary, 167

map, 263

adjoint, 20
 antilinear, 14
 antipodal, 252
 bijective, 263
 composition of, 264
 conjugate linear, 19
 continuous, 57
 Gauss, 311
 idempotent, 4
 identity, 264
 inclusion, 5
 injective, 263
 inverse, 264
 linear, 3
 multilinear, 34
 null homotopic, 118
 smooth

 of Euclidean spaces, 59
 of manifolds, 69
 sphere, 311
 surjective, 263

matrix

adjoint, 20
 adjugate, 22
 anti-Hermitian, 108
 change of basis, 10
 characteristic polynomial of, 24
 cofactor, 22
 compound, 43
 congruence of, 18
 contragredient, 10
 diagonalizable, 24
 Gram, 17
 Hermitian, 271
 impedance, 301
 incidence, 297

Index

Kronecker product, 49
 Laplacian, 303
 minor, 22
 rotation, 23
 roto-reflection, 23
 singular, 22
 transpose of, 10
 unitary, 271
 matrix group, 105
 matrix tree theorem, 303
 matter field, 191
 Maurer–Cartan form, 107
 Maxwell’s equations, 91
 Mayer–Vietoris sequence
 long, 130
 short, 129
 mean curvature, *see* curvature, mean
 mean curvature vector, 314
 mesh current, 304
 metric, 193
 bi-invariant, 237
 covariant constancy, 226
 induced, 235
 minimal surface, 315
 Minkowski
 metric, 196
 spacetime, 15
 minor, *see* matrix, minor
 Möbius
 band, 66
 group, 235
 strip, 66
 transformation, 235
 moment curve, 163
 moving frame, *see* frame field
 multigraded algebra, 33
 multi-index notation, 40
 multivalued function, 117

 natural pairing, *see* dual pairing
 negative definite inner product, *see* inner product,
 negative definite
 neighborhood, 55
 Newton’s identities, 26
 nilpotent, 89
 node, *see* vertex, of a graph
 nondegenerate
 dual pairing, 9
 form, 14
 nonnegative definite inner product, *see* inner product,
 nonnegative definite
 nonpositive definite inner product, *see* inner product,
 nonpositive definite
 nonsingular matrix, 22
 normal
 bundle, 308
 space, 308
 vector, 308

- vector field, 308
- nucleon, 190
- null homotopic map, *see* map, null homotopic
- null space, *see* kernel
- nullity, 3

- 1-form, *see* differential form, 1-form
- one-parameter group of diffeomorphisms, *see* flow (of a vector field)
- one-parameter subgroup, 106
- one-point map, 118
- one-to-one map, *see* map, injective
- onto map, *see* map, surjective
- open cover, 56
- operator
 - Hermitian, 271
 - linear, 3
- order of a tensor, 30
- order of vanishing, 81
- ordered set, 265
- orientable manifold, *see* manifold, orientable
- orientation class, *see* manifold, orientation class
- oriented simplicial complex, *see* simplicial complex, oriented
- orthogonal
 - complement, 17
 - group, 23, 105
 - matrix, 23
 - transformation, 23
 - vectors, 16
- orthonormal set of vectors, 16

- parallel transport map, 213
- parallel transport or translation, 212
- parallelizable manifold, *see* manifold, parallelizable
- parameterization of a manifold, 70
- parity transformation, 239
- partition of a set, 265
- partition of unity, 129
- passive transformation, 69
- path
 - identity, 135
 - in a graph, 299
 - in a manifold, 134
 - inverse, 135
 - product, 135
- path-connected space, 136
- path homotopy, 135
- path ordered exponential, 218
- path ordering operator, 218
- Pauli matrices, 111
- period (integral), 170
- permutation, 266
- permutation group, *see* symmetric group
- Pfaffian
 - of a matrix, 261
 - system, 289
- Plateau's problem, 315

- Platonic solid, 152
- Poincaré
 - conjecture, 208
 - duality, 224
 - lemma, 120
- Poisson bracket, 114
- polar vector, 239
- polygon, 152
- polyhedron, 140
- polytope, 139
 - crosspolytope, 152
 - simplicial, 152
- positive definite inner product, *see* inner product, positive definite
- power sum symmetric function, 25
- preimage, *see* inverse image
- principal
 - bundle, 191
 - curvatures, *see* curvature, principal
 - directions, 312
- product (of sets), *see* Cartesian product
- projection operator, 4
- projection, canonical, 6
- projective general linear group, 235
- proper
 - distance, 214
 - face, 140
 - time, 214
- pseudoscalar, 239
- pseudotensor, 239
- pseudovector, 239
- pullback, 95
- punctured Euclidean space, 125
- pushforward, 97

- quaternions, 269
- quotient
 - of two topological spaces, *see* topology, quotient
 - of two vector spaces, 6

- range, 263
- rank
 - of a linear map, 3
 - of a matrix, 8
 - of a tensor, 32
 - of a vector bundle, 177
- rank-nullity theorem, 5
- refinement (of a cover), 56
- regular point, 71
- regular value, 71
- regular value theorem, 72
- representation
 - of a group, 108
 - of a Lie algebra, 109
 - of a linear map, 7
- retraction, 125
- Ricci
 - curvature scalar, 207

328

flow, 208
 identity, 226
 tensor, 206
 Riemann curvature tensor, 204
 Riesz's lemma, 19, 29
 ring, 268
 rotation matrix, *see* matrix, rotation
 roto-reflection matrix, *see* matrix, roto-reflection
 row rank, *see* rank, of a matrix

 scalar, 1, 268
 Schläfli symbol, 153
 Schur's theorem, 233
 Schwarzschild line element, 229
 second
 fundamental form, 310
 fundamental tensor, 310
 section
 of a bundle, 178
 of linear map, 5
 sectional curvature, 232
 self-adjoint matrix, *see* matrix, Hermitian
 semicolon notation, 225
 semisimple Lie algebra, 110
 sesquilinear form, 14
 set, 263
 shape operator, 311
 short exact sequence
 of complexes, 127
 of vector spaces, 4
 sign (of a permutation), 266
 signature, 18
 similarity
 invariant, 24
 transformation, 24
 simplex
 geometric, 141
 open, 141
 oriented, 142
 singular, 146
 smooth singular, 158
 standard, 159
 simplicial complex, 142
 oriented, 143
 simplicial polytope, *see* polytope, simplicial
 simply connected topological space, 208
 singular matrix, *see* matrix, singular
 singularity, *see* zero (of a vector field)
 skew field, 269
 skew symmetric tensor, *see* tensor, antisymmetric
 skew-Hermitian matrix, *see* matrix, anti-Hermitian
 smooth function
 on \mathbb{R}^n , 59
 on manifold, 69
 smooth map
 of Euclidean spaces, *see* map, smooth
 of manifolds, *see* map, smooth
 space, *see* topological, space

Index

spanning set of vectors, 2
 spanning tree, *see* tree, spanning
 spectral theorem, 271
 sphere map, *see* map, sphere
 spherical harmonic, 247
 spinor, 111
 stack of records theorem, 249
 standard basis, 2
 standard simplex, *see* simplex, standard
 stereographic projection, 64
 Stokes' theorem
 chain version, 164
 manifold version, 167
 stress energy tensor, 207
 structure
 constants, 111
 functions, 202
 submanifold, 70
 submersion, 71
 subset, 263
 subspace
 of a vector space, 2
 topological, 58
 support
 of a form, 164
 of a function, 129
 supporting hyperplane, 140
 surjective map, *see* map, surjective
 suspension, 119
 suspension isomorphism, 138
 Sylvester–Franke theorem, 50
 Sylvester's law of inertia, 18
 symmetric
 algebra, *see* algebra, symmetric
 group, 266
 symplectic
 form, 113
 map, 114
 symplectomorphism, 114

 tangent
 bundle, *see* bundle, tangent
 space, 75
 vector, 75
 Tellegen's theorem, 301
 tensor, 30
 antisymmetric, 37
 inertia, 32
 of type (r, s) , 33
 order vs. rank, 32
 symmetric, 37
 tensor algebra, 31
 tensor density, 239
 tensor field, 84
 tensor product, 30
 of linear maps, 49
 symmetric, 51
 tensor product bundle, *see* bundle, tensor product

- theorema egregium, 312
- topological
 - space, 55
- topological invariant, 58
- topology
 - basis of, 57
 - coarser, 58
 - discrete, 55
 - Euclidean, 57
 - finer, 58
 - Hausdorff, 56
 - induced, 58
 - on a set, 55
 - product, 57
 - quotient, 58
 - standard
 - on \mathbb{R} , 57
 - on \mathbb{R}^n , 57
 - subspace, *see* topology, induced
- torsion
 - form, 201
 - of a connection, 199
 - of an abelian group, 145
 - tensor, 199
- total space, 177
- trace
 - of a matrix, 24
 - of an endomorphism, 26
- transition function
 - of a bundle, 178
 - of a manifold, 63
- translate (subspace), 2
- transpose, *see* matrix, transpose of
- transposition, 266
- tree, 302
 - number, 303
 - spanning, 302
- triangulable topological space, 145
- triangulation
 - continuous, 144
 - smooth, 159
- trivial bundle, 178
- uncountable set, 264
- union, 263
- unit vector, 16
- unitary
 - group, 108
 - matrix, *see* matrix, unitary
- universal coefficient theorem, 172
- universal cover, *see* cover, universal
- unknot, 255
- unlink, 255
- valuation, 154
- vector, 1
 - p -vector, 39
- vector bundle, 176
- vector field, 77
 - Hamiltonian, 114
- vector potential, 122
- vector space, 270
- vertex
 - of a graph, 296
 - of a polytope, 140
- volume form
 - in general, 274
 - on a geometric manifold, 221
- wedge
 - product, *see* exterior product
 - sum, 138
- Weingarten map, *see* shape operator
- Weitzenböck formula, 245
- Whitney
 - approximation theorem, 120
 - embedding theorem, 71
 - sum, 189
- winding number, 252
- Yang–Mills equations, 192
- zero of a vector field, 259
- zero set, 289