Physics of Gas–Liquid Flows

Presenting tools for understanding the behavior of gas–liquid flows based on the ways large-scale behavior relates to small-scale interactions, this text is ideal for engineers seeking to enhance the safety and efficiency of natural gas pipelines, water-cooled nuclear reactors, absorbers, distillation columns, and gas lift pumps. The review of advanced concepts in fluid mechanics enables both graduate students and practicing engineers to tackle the scientific literature and engage in advanced research.

The text focuses on gas–liquid flow in pipes as a simple system with meaningful experimental data. This unified theory develops design equations for predicting drop size, frictional pressure losses, and slug frequency, which can be used to determine flow regimes, the effects of pipe diameter, liquid viscosity, and gas density. It describes the effect of wavy boundaries and temporal oscillations on turbulent flows, and explains transition between flow regimes, which is key to understanding the behavior of gas–liquid flows.

Thomas J. Hanratty is Professor Emeritus at the University of Illinois at Urbana-Champaign, and was a leader in establishing industrially important multiphase flow as a new academic discipline, by relating macroscopic behavior to small-scale interactions. His research has been recognized by nine awards from the American Institute of Chemical Engineers (AIChE), the American Society for Engineering Education, Ohio State University, Villanova University, and University of Illinois. He was the inaugural winner of the International Multiphase Flow Prize. Hanratty was named as one of the influential chemical engineers of the modern era at the AIChE centennial celebration in 2008. He has been elected to the National Academy of Engineering, the National Academy of Sciences, and the American Academy of Arts and Sciences.

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"This authoritative and impressive monograph, written by a widely acclaimed pioneer in the field, is an excellent resource for new students as well as seasoned practitioner. It develops the concepts systematically and packages many decades worth of literature in this field lucidly, giving the readers a chance to understand appreciate the evolution of this field."

Sankaran Sundaresan, Princeton University

"Physics of gas-liquid flows" is a must read for graduate students, researchers and engineers seeking a solid basis or wanting to update their knowledge in the dynamics of gas-liquid systems. The book is an authoritative reference, mainly built around the results of the author, a leading expert in the field during several decades. It presents our current experimental and theoretical understanding at both the local and global scales with an original contribution to wave phenomena as they appear in film stratified and annular flows that may be a source of inspiration for researchers and teachers in the years to come."

Jean Fabre, Institut National Polytechnique, Toulouse

"A lifetime of probing research and deep thinking about gas-liquid flows is enclosed between the covers of this book. Starting from simple analyses – the style of which will be familiar to many undergraduates – the author moves gradually to more advanced topics building a succinct yet exhaustive picture of the present understanding of these important flows. Practicing engineers and researchers alike will find many gems in this book."

Andrea Prosperetti, Johns Hopkins University

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Contents

Preface

Gas–liquid flows are ubiquitous in industrial and environmental processes. Examples are the transportation of petroleum products, the cooling of nuclear reactors, the operation of absorbers, distillation columns, gas lift pumps. Quite often corrosion and process safety depend on the configuration of the phases. Thus, the interest in this area should not be surprising.

The goal of this book is to give an account of scientific tools needed to understand the behavior of gas–liquid systems and to read the scientific literature. Particular emphasis is given to flow in pipelines.

The following brief historical account is taken from a plenary lecture by the author at the Third International Conference on Multiphase Flow, Lyon, France, June 8–12, 1998. (*Int. J. Multiphase Flow* 26, 169–190, 2000):

> A symposium held at Exeter (P. M. C. Lacey) in 1965 brought together 160 people with a wide range of interests. Discussions at the 42 presentations indicated, to me, that something special was happening and that future directions of work on multiphase flow were being defined. This thrust was continued in conferences at Waterloo, Canada, in 1968 (E. Rhodes, D. S. Scott) and at Haifa, in 1971 (G. Hetsroni). Intellectual activity in ensuing years is exemplified by more focused conferences on Annular and Dispersed Flows held at Pisa, 1984 (S. Zanelli, P. Andreussi, T. J. Hanratty) and in Oxford, England, in 1987 (G. F. Hewitt, P. Whalley, B. Azzopardi), the Symposium on Measuring Techniques at Nancy (J. M. Delhaye, 1983) and the Conference on Gas Transfer at Heidelberg (Jähne, 1995). However, the 350 papers presented at the Second International Conference on Multiphase Flow in 1995 (A. Serizawa, Y. Tsuji) manifested a new level of activity.

A fair question is what happened between 1965 and 1995. My own assessment is that major successes came about, mainly through efforts that relate macroscopic properties of multiphase systems to small-scale behavior. An outcome of this approach is the possible emergence of a new field. This is evidenced in many ways, of which the establishment of the *International Journal of Multiphase Flow* (Gad Hetsroni, 1973) and the Japan Society of Multiphase Flow (A. Akagawa, T. Fukano, 1987) are examples. The following excerpt from a talk by R. T. Lahey at the inauguration of the Japan Society would indicate that my observations are not original: “I believe that this new field will become as widely accepted in the future as other emerging fields . . .”.

Physics of Gas–Liquid Flows addresses both graduate students and practitioners. The treatment is based on a course, taught at the University of Illinois, which required only that the students had taken one undergraduate course in fluid dynamics. As a consequence, attention is given to topics that are usually bulwarks in graduate courses in fluid dynamics, such as ideal flow theory, the Navier–Stokes equations and interfacial waves.

In reporting the results from published research I have retained the units (metric or imperial) as used in the original work.

This work has a kinship to *One Dimensional Two-Phase Flow* by Graham Wallis, published by McGraw-Hill in 1969. One should recognize that much has happened since the publication of this book and that my way of presenting the material could be different from that of that of Dr. Wallis.

Physics of Gas–Liquid Flows leans heavily on contributions by researchers in my laboratory. A verbal summary of these works is contained in an account of the Research of Thomas J. Hanratty which was deposited with the University of Illinois in the Illinois Digital Environment for Access to Learning and Scholarship (IDEALS). It can be visited at http://hdl.handle.net/2142/9132. The illustrations were prepared by Dorothy Loudermilk. Taras V. Pogorelov provided a helping hand in transmitting the manuscripts to Cambridge University Press.

Cover

The photograph featured in the cover was obtained in the laboratory of Thomas J. Hanratty by James B. Young. It captures the trajectories of particles in a turbulent liquid flowing down a vertical pipe. Cross-sections at several locations were illuminated by thin sheets having different colors. Axial viewing photography was used to capture the paths of the particles. The color of a particle gives its axial location.

Symbols

A	Area
A	Measure of the thickness of the viscous wall layer, used by van Driest
A_G	Area of the gas space
A_L	Area of the liquid space
A_{LS}	Area of the liquid in a slug
A_{L1}	Area of the liquid layer in front of a slug
A_{Pi}	Acceleration of a particle in the i-direction
A_{L0}	Critical area for a liquid layer to sustain a stable or growing slug
A_t	Area of a tube or pipe
a	Amplitude of a wave
a_v	Interfacial area per unit volume
B	Height of a rectangular channel, or the height of the gas space
C	Concentration
C_0	Concentration at $y = 0$
C_B	Bulk concentration
C_D	Drag coefficient
C_L	Lift coefficient
C_P	Heat capacity at constant pressure
C_V	Heat capacity at constant volume
C_W	Concentration at the wall
c	Velocity of sound; molecular velocity
c	Complex wave velocity = $c_R + ic_I$
c_0	Wave velocity for stagnant fluids
c_F	Velocity of the liquid at the front of a slug
c_B	Velocity of the bubble behind a slug
c_g	Group velocity, the speed at which wave energy is transmitted
c_G	Sound velocity in the gas phase of a gas–liquid flow
c_{KW}	Kinematic wave velocity, defined by (7.51)
c_L	Sound velocity in the liquid phase of a gas–liquid flow
c^t	Turbulent concentration fluctuation
D	Molecular diffusion coefficient
d_m	Maximum drop diameter
d_t	Pipe diameter

d_{h}	Hydraulic diameter = 4A/wetted perimeter $d_{v\mu}$
$d_{v\mu}$	Volume median diameter
d_{qm}	Drop diameter defined by Mugele & Evans
d_{10}	Number mean diameter
d_P	Diameter of a bubble or a drop
d_{32}	Sauter mean diameter
E	Entrainment
E	Energy associated with one wavelength = $P + T$
E_M	Maximum possible entrainment
$E(n)$	Spectral density function
e	Energy per unit mass
e^c	Capture efficiency
$\vec{F}$	Force
F	Mechanical energy lost per unit mass of fluid due to friction
F	Flux of particles
F_D	Resisting force on a particle due to fluid drag
F_D	Flux due to diffusion
F_B	Buoyancy force
$\vec{F}_L$	Lift force
f	Fanning friction factor
f	Factor that accounts for the volume of fluid dragged along by a particle
$f_n(d_P)$	Number distribution function
$f_v(d_P)$	Volume distribution function
f_S	Friction factor for a smooth surface
f_S	Frequency of slugging
f_i	Friction factor for gas flow over a gas–liquid interface
f_{Pi}	Force of the fluid on a particle
G	Mass velocity defined by (1.4)
G_C	Mass velocity at the choking condition
G_L	Mass velocity of the liquid
G_G	Mass velocity of the gas
G_{LE}	Mass velocity of drops entrained in the gas
$\vec{g}$	Acceleration of gravity
$\hat{g}$	Defined by (3.20) and (3.21)
g^+	Equal to $g\upsilon/v^*$
H	Submergence of injector in a gas-lift pump
h	Enthalpy
h	Height above a datum plane
h_H	Enthalpy of a gas–liquid mixture
h^L	Enthalpy of the liquid
h^G	Enthalpy of the gas
h_L	Height of liquid layer in a channel
h_{L1}	Height of the liquid layer in front of a slug

h_L	In a pipe, the length of the bisector of the liquid layer
h_L^+	Dimensionless height of the liquid layer $= h_L v^*/\upsilon$
h_G	Height of the gas layer in a channel
h_{L0}	Critical height of the liquid layer needed for slugs to appear
h_L^c	Location of the centroid of an area
h_{LB}	Height of the liquid at the bottom of a horizontal pipe
h_W	Distance of the top of the waves from the wall
h_{LS}	Height of the liquid layer if the flow is steady
Δh	Wave height
I	Intermittency, fraction of the time that disturbance waves are present
I_1	Integral defined by (12.63)
I_2	Integral defined by (12.64)
i	Internal energy per unit mass
k	Wave number
k_M	Wave number at which wave growth is a maximum
k_S	Sand roughness
k_D	Deposition coefficient
L	Length of pipe or channel
L	Length through which liquid is lifted in a gas lift pup
L_D	Pipe length needed for slugs to develop
L_S	Slug length
L_U	Length of a region of unstable stratified flow
l	Length in the flow direction
ℓ	Mixing length; characteristic viscous length
M	Molecular weight
M_{LG}	Mass transfer rate per unit area from the liquid to the gas
M_{GL}	Mass transfer rate per unit area from the gas to the liquid
m	Average height of the liquid layer around the circumference
m_P	Mass of a particle
m^+	Dimensionless height based on the friction velocity and the viscosity
m_G^+	Ratio of the film height to a gas-phase length scale
m_c^+	Dimensionless film height, based on v_c^* and the viscosity of the liquid
N	Local rate of mass transfer
n	Frequency, cycles per second
P	Perimeter
P	Potential energy associated with one wavelength
P_G	Length of the pipe perimeter in contact with the gas
P_L	Length of the pipe perimeter in contact with the liquid
P_S	Pressure imposed on a solid or liquid surface by a flowing fluid
$\tilde{P}$	Pressure made dimensionless with $\rho_L g d_t$
p	Pressure
p_a	Ambient pressure
p_G	Pressure along a sinusoidal wavy interface $= p_{GR} + ip_{Gi}$

p_{GR}	Pressure component which has a minimum at the wave crest
p_{GI}	Pressure component which is in phase with the wave slope
p_{Gi}	Gas pressure at the interface
p_{Li}	Liquid pressure at the interface
Q_{A}	Volumetric rate at which a liquid layer is atomizing
Q_{G}	Volumetric flow of gas
Q_{L}	Volumetric flow of the liquid
Q_{sh}	Volumetric flow at which slugs shed liquid
q	Velocity of the fastest-moving particle on the interface of a wave
q	Magnitude of the velocity
q	Volumetric flow per unit breadth $\approx hu_{\mathrm{a}}$
q^2	Equal to $u_x^2 + u_y^2 + u_z^2$
$\dot{q}$	Time rate at which heat is added to a control volume
q_{W}	Rate of heat addition at a wall per unit area
$\dot{q}_{\mathrm{rev}}$	Heat transfer rate if changes are occurring reversibly
q_{WL}	Heat transfer rate per unit area from a wall to a liquid
q_{WG}	Heat transfer rate per unit area from a wall to a gas
q_{GL}	Heat transfer per unit area from a gas to a liquid
q_{LG}	Heat transfer per unit area from a liquid to a gas
R	Wave resistance defined by (4.116) and (4.117)
R	Molar gas constant
R	Radius of curvature of a surface
R_{D}	Mass rate of deposition of particles per unit area
R_{A}	Mass rate of atomization of the wall layer per unit area
$R_{1,2}$	Principal radii of curvature of a surface
R_{ij}	The Reynolds stress divided by the density
R_i^{path}	Correlation coefficient of fluid velocity fluctuations seen by a particle as it moves around in a turbulent field
R_i^{L}	Lagrangian correlation coefficient for turbulent fluid particles
$R_{\mathrm{P}i}^{\mathrm{L}}$	Lagrangian correlation coefficient for particles in a turbulent field
R^{E}	Eulerian correlation coefficient
r_{t}	Radius of a tube or a pipe
r_{P}	Particle or bubble radius
r_{ij}	Wave-induced variation of the R_{ij}
S	Slip ratio, equal to gas velocity divided by the liquid velocity
S	Projected area of a particle
S_{i}	Length of the interface in an idealized stratified flow
s	Entropy
s	Sheltering coefficient, defined by Jeffreys
T	Temperature
T	Kinetic energy in one wavelength
T_{S}	Tangential stress imposed on the interface by a flowing fluid
t	Time

t_C Time constant reflecting the rate of growth of waves
U_P Particle velocity
U_A Velocity of atomizing drops in the flow direction
U_D Velocity of depositing drops in the flow direction
U_B Velocity of a bubble in a stationary liquid; bulk velocity
$U_{S\infty}$ Rise velocity of a bubble in an infinite stagnant liquid
U_1 Convection velocity of a homogeneous, isotropic field
U_C Settling velocity of a suspension of particles
$\vec{U}_S$ Relative velocity between a particle (or a bubble) and a fluid
u One-dimensional velocity
$\vec{u}$ Fluid velocity vector
u_c Fluid velocity at the center of a pipe or channel
u_t^t Turbulent velocity component
u^+ Velocity made dimensionless using the friction velocity
u_n Component of the velocity normal to a surface
u_t Component of the velocity tangent to a surface
u_H Velocity of a homogeneous mixture, defined by (1.48)
u_L Velocity of the liquid
u_{L1} Velocity of the stratified layer in front of a slug
u_{L3} Velocity of the liquid in the body of a slug
u_{slug} Velocity of a slug
u_G Gas velocity
u_{Gc} Critical gas velocity for the initiation of atomization
u_{GM} Maximum value of u_G
u_m Mixture velocity = $u_{GS} + u_{LS}$
u_a Spatially averaged liquid velocity
u_S Liquid velocity at the interface
u_{ga} Spatially averaged gas velocity
u_{GSt} Critical superficial gas velocity at which Kelvin–Helmholtz waves appear
u_{GS} Superficial gas velocity = volumetric flow of gas divided by A
u_{LS} Superficial liquid velocity = volumetric flow of liquid divided by A
u_o Rise velocity of bubbles
$u_{G\infty}$ Rise velocity of single bubbles in infinite media
V Volume
V_D Mean velocity of particles striking a wall
V_{DL} Drift velocity associated with a lift force
V_{Drift} Drift velocity, defined by (8.29)
$\overline{V}_D$ Mean velocity with which particles are depositing
V_W Average velocity with which particles strike a boundary
V_P Volume of a particle or a bubble
V_T Terminal free-fall velocity
$\overline{V}_R$ Average velocity of entrained particle in the radial direction
V_i^0 Velocity with which particles enter the field

V_{tp}	Turbophoretic velocity
V^*_{GS}	Dimensionless group defining the initiation of annular flow (11.1)
v	Specific volume
v_G	Specific volume of a gas
v_L	Specific volume of a liquid
$v*$	Friction velocity, using the shear stress at the wall or the interface
v^*_c	Critical frictional velocity
v^*_G	Friction velocity based on gas density and stress at the interface
v^*_L	Friction velocity based on the liquid density and the stress at the wall
v^*_c	Friction velocity based on liquid density and the characteristic stress
$\vec{v}$	Velocity of a fluid particle
$\vec{v}_P$	Velocity of an entrained particle or bubble
v^t_{Pi}	Turbulent velocity fluctuation
W	Weight rate of flow
W_L	Weight flow rate of liquid
W_{LP}	Mass flow of liquid in the “pool” at the bottom of the pipe
W_G	Weight flow rate of gas
W_W	Mass flow of liquid on the wall
W_{LE}	Mass flow of entrained drops
W_{LF}	Mass flow rate in the wall layer
W_{LFC}	Critical flow below which atomization does not occur
$\dot{w}$	Rate at which fluid in a volume is doing work on surroundings
w	Width of a control volume
$\overline{X^2_P}$	Mean square displacement of particles in a turbulent field
x	Mixture quality, mass fraction that is a gas or vapor
x	Coordinate in the flow direction in a Cartesian coordinate system
y	Coordinate that is perpendicular to a wall
y_{fl}	Location at which particles start a free-flight to the wall
y^+	Distance from the wall made dimensionless with the friction velocity
y_0	Average location of the interface
z	Coordinate in the direction of flow in a pipe flow
z	Distance from reference plane

Greek symbols

α	Volume fraction of gas in a flowing mixture
α	Ratio of the fastest moving particle at the interface to wave velocity
α_W	Ratio of wave height to what would be observed on a deep liquid
α	Parameter representing shape of velocity profile
α	Equals $\left(\tau_i^{path}\right)^{-1}$
α_P	Volume fraction of particles or bubbles
αm^+	Dimensionless quantity defined by (3.28)
β	Reciprocal of the inertial time constant of an entrained particle

$\tilde{\beta}$	Reciprocal time constant for a particle in a liquid (10.50)
Γ	Gamma function
Γ	Volumetric flow per unit length in the spanwise direction
Γ_c	Critical film flow below which atomization does not occur
Γ_c^*	Modified critical film flow (12.74)
Γ_S	A shape factor for the velocity profile (7.6)
γ	Ratio of the heat capacities at constant pressure and constant volume
δ	Thickness of a viscous boundary layer
δ_{ij}	Delta function
ε	Rate of dissipation of mechanical energy per unit volume
ε	Lagrangian turbulent diffusivity of fluid particles
ε	Ratio of the fastest-moving particle in the interface to the wave velocity
ε_h	Diffusivity representing turbulent mixing in the wall layer
ε_L	Liquid holdup = $(1-\alpha)$
ε_P	Lagrangian turbulent diffusivity of entrained particles
ε_P^E	Eulerian diffusivity of entrained particles
η	Displacement of the interface from its average location
θ	Inclination angle to the horizontal
θ	Phase for an oscillating quantity
θ	Angular location on the wall of a pipe, where $\theta = 0$ can be the top or bottom of the pipe
θ_g	Orientation angle of flow to the gravitational vector (Figure 1.2)
Λ	Lagrangian length scale
λ	Wavelength
λ_m	Wavelength which is growing the fastest
λ^T	Lagrangian micro-time-scale
μ	Viscosity
μ_G	Gas viscosity
μ_L	Liquid viscosity
μ_P	Viscosity of the fluid inside a drop or bubble
μ^t	Turbulent viscosity
ρ	Density
ρ_L	Liquid density
ρ_G	Gas density
ρ_P	Particle density
ρ_f	Fluid density
ρ_S	Density of a dissolving solid surface
ρ_H	Density of a homogeneous mixture
ρ_G^*	$\rho_G \coth(kh_G)$
ρ_L^*	$\rho_L \coth(kh_L)$
σ	Surface tension
σ_P	Root-mean-square of the turbulent velocity fluctuations of particles
σ	Root-mean square of a fluctuating quantity

τ	Magnitude of a shear stress
τ_c	Time constant characterizing the rate of growth of waves
τ_c	Characteristic stress in a liquid layer = $(2/3)\tau_W + (1/3)\tau_i$
τ_P	Inertial time constant of a particle
τ_P	Effective stress due to atomization and deposition
τ_i^{path}	Time constant characterizing R_i^{path}
τ^L	Lagrangian time constant
τ^E	Eulerian time-scale
τ_{PB}	Volume-average inertial time constant
τ_{PS}	Time constant for a particle obeying Stokes law
τ_W	Shear stress at a wall
τ_{WG}	Resisting stress at the wall on a gas
τ_{WL}	Resisting stress at the wall on a liquid
τ_i	Shear stress at an interface
τ_{ij}	Component of a stress tensor
τ_{ij}^t	Component of a turbulent stress tensor
τ_{DA}	Net momentum flux due to atomization and deposition
υ	Kinematic viscosity
υ_L	Kinematic viscosity of the liquid
υ_G	Kinematic viscosity of the gas
υ^t	Turbulent kinematic viscosity
ϕ	Potential function describes velocity for an irrotational field
ϕ_G^2	Dimensionless frictional pressure gradient defined by (1.100)
ϕ_L^2	Dimensionless frictional pressure gradient defined by (1.101)
ω	Circular frequency = kc
$\vec{\omega}_P$	Angular rotation of a particle or bubble
ψ	Stream function defined by (6.20)

Dimensionless groups

$\tilde{\beta}$	$= 3C_D\rho_f\|\vec{u} - \vec{v}_P\|/4r_P(2\rho_P + \rho_f)$
Bo	Bond number = $\dfrac{d_h}{[\sigma/(\rho_L - \rho_G)g]^{1/2}}$
F	$= \dfrac{\gamma(\mathrm{Re}_{LF})}{\mathrm{Re}_G^{0.9}}\dfrac{\nu_L}{\nu_G}\sqrt{\dfrac{\rho_L}{\rho_G}}$ is a parameter in (3.41)
F_H	F for a horizontal pipe (12.48)
Fr	Froude number = $u_{GS}/(gd_t)^{1/2}$
Ma	Mach number = ratio of the fluid velocity to the velocity of sound
Re	Reynolds number
Re_G	Gas-phase Reynolds number = $d_tW_G/A_t\mu_G$ for a pipe
Re_{LF}	Film Reynolds number = $4\Gamma/\upsilon_L$
Re_{L0}	Liquid Reynolds number based on the velocity at the interface

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Re_P	Particle Reynolds number = $d_\mathrm{P}\rho U_\mathrm{S}/\mu$
Sc	Schmidt number
θ	Number reflecting the effect of viscosity on wave growth
We	Weber number
X	Martinelli parameter

Other symbols

$\langle\ \rangle$	Spatial average, ensemble average or phase average
$(\ )_\mathrm{i}$	Quantity evaluated at the interface
$\lvert\ \rvert$	Signifies an absolute value
$\dfrac{D}{Dt}$	Substantial derivative. Time change seen in a framework of a moving fluid particle (defined by equation (4.13))
$\overline{N}$	Overbar indicates a time-average
N'	Prime indicates a fluctuating quantity
N^t	A turbulent fluctuation
$\hat{N}$	Indicates complex amplitude of a fluctuating quantity induced by sinusoidal waves
$\tilde{N}$	Tilde indicates $\hat{N}$ divided by the wave amplitude, a
$\tilde{\tilde{N}}$	Difference between the phase-average and the time-average
$\hat{N}$	Defined by (3.20), (3.21)
$\left(\dfrac{dP}{dz}\right)_\mathrm{F}$	Frictional contribution to the pressure gradient
$\left(\dfrac{dP}{dz}\right)_\mathrm{GS}$	Frictional pressure gradient for gas flowing along a pipe
$\left(\dfrac{dP}{dz}\right)_\mathrm{LS}$	Frictional pressure gradient for liquid flowing along a pipe