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1 One-dimensional analysis

1.1 Introduction

The “simplest” models for gas–liquid flow systems are ones for which the velocity is uniform over a cross-section and unidirectional. This includes flows in a long straight pipe and steady flows in a nozzle.

A treatment of pipe flow with a constant cross-section is initiated by reviewing analyses of incompressible and compressible single-phase flows. A simple way to use these results is to describe gas–liquid flows with a homogeneous model that assumes the phases are uniformly distributed, that there is no slip between the phases and that the phases are in thermodynamic equilibrium. The volume fraction of the gas, α, is then directly related to the relative mass flows of the phases. However, the assumption of no slip, $S = 1$, can introduce considerable error. This has prompted a consideration of a separated flow model, where uniform flows of gas and liquid are pictured as moving parallel to one another with different velocities and to be in thermodynamic equilibrium.

The two-fluid model develops equations for the interaction of two interpenetrating streams. It does not require the specification of S or the assumption of equilibrium between the phases. However, it introduces several new variables.

An interesting feature of the single-phase analysis of a compressible fluid is the existence of the choking phenomenon whereby there is a maximum flow which can be realized for a system in which the pressure at the pipe inlet and the pressure in the receiver are controlled. The homogeneous model predicts much smaller choking velocities for gas–liquid flows than would exist for gas flowing alone.

The separated flow model requires equations for the stress at the wall, τ_W, and α (or the slip ratio). The homogeneous model requires only an equation for τ_W. The specification of these quantities is a continuing interest and is a focus in future chapters. A starting point for this pursuit is the widely used Lockhart–Martinelli analysis discussed in this chapter.

Steady flow in a nozzle is different from flow in a straight pipe in that changes in inertia override the importance of wall resistance and of gravity. Equations that use the separated flow model are developed in this chapter for gas–liquid flows in a nozzle.

The materials in Section 1.6 on flow in a nozzle and on the two-fluid model can be ignored in a first reading since they are not needed to follow the main narrative of this book.

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1.2 Single-phase flow

1.2.1 Flow variables

When a single phase is flowing, the field is described by the variables pressure, p, velocity, $\vec{u}$, density, ρ and temperature, T. Four equations are needed: conservation of momentum, conservation of energy, conservation of mass, and a relation among state variables. Examples of the last are the perfect gas law and tables of thermodynamic properties.

1.2.2 Momentum and energy theorem

For flow systems, it is convenient to formulate Newton's second law of motion as a momentum theorem, which focuses on a fixed volume in space rather than on particles moving through a field. A sketch of an arbitrary volume is shown in Figure 1.1. Note that, on the boundary, the velocity vector, $\vec{u}$, can be represented by components perpendicular, u_n, and tangent, u_t, to the boundary. The theorem is stated as follows: "The time rate of change of momentum in a *fixed* volume in space plus the net flow of momentum out of the volume equals the net force acting on the volume."

$$\int \frac{\partial(\rho\vec{u})}{\partial t} dV + \int \rho u_\mathrm{n} \vec{u}\, dA = \vec{F} \tag{1.1}$$

Thermodynamic variables are defined for systems which are at equilibrium. The application of conservation of energy and the equation of state to a flowing system, therefore, usually involves the assumption that the adjustment of molecular properties to a change of the environment occurs much more rapidly than the change of the flow field. (Examples where this assumption might not be valid are flow through a shock wave or the stagnation of flow at a very small impact tube.) The energy of a flowing fluid is defined as

$$e = i + \frac{|u|^2}{2} \tag{1.2}$$

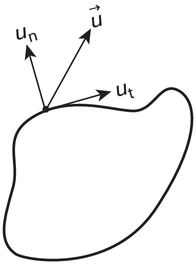

Figure 1.1 An arbitrary volume in space.

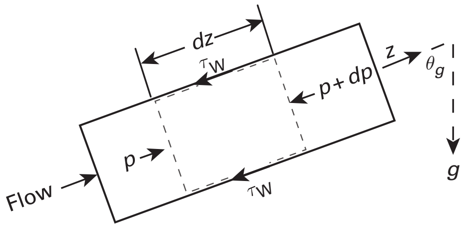

Figure 1.2 Control volume in a pipe.

where i is the internal energy per unit mass and $|u|^2/2$ is the kinetic energy per unit mass.

The first law of thermodynamics is formulated for a flow system as the energy theorem: "The time rate of change of energy in a *fixed volume* in space plus the net flow of energy out of that volume equals the rate at which heat is added, $\dot{q}$, minus the rate at which the fluid in the volume is doing work on the surroundings, $\dot{w}$."

$$\int \frac{\partial(\rho e)}{\partial t} dV + \int \rho u_{\mathrm{n}} e \, dA = \dot{q} - \dot{w} \tag{1.3}$$

where ρe is the energy per unit volume.

1.2.3 Steady flow in a duct of constant area

The equations defining the steady flow in a pipe of constant area, A, can be derived by considering the fixed control volume shown in Figure 1.2. The pipe is inclined at an angle θ_g to the direction of the gravitational vector. The fluid is flowing in the z-direction with an average velocity, u. Equations are to be developed describing the changes of u, p, ρ, and T with z.

Under steady flow conditions, the mass flow, $\rho u A$, does not vary along the pipe. The equation for conservation of mass is

$$AG = u\rho A = \text{constant} \tag{1.4}$$

where term G is called the mass velocity.

The flux of momentum along the pipe is given as GAu. Since the flow is steady, the time rate change in the control volume, indicated by the dashed lines in Figure 1.2, is zero so the momentum theorem states that the net flow of momentum out of a fixed volume equals the net force. The flows of momentum into and out of the control volume are given by $\rho u^2 A$ and $\rho u^2 A + d(\rho u^2 A)$, so the net flow of momentum out is $d(\rho u^2 A)$.

The forces due to pressure acting on the front and back faces are pA and $-(p + dp)A$, so the net force is $-(dp)A$. The force of gravity is $-\rho g \cos\theta_g A dz$. The wall resists the flow with a stress, τ_W, acting on the side walls of the control volume and the net force due to this stress is $-\tau_W P dz$ where P is the perimeter of the duct. The momentum theorem gives

$$d(AGu) = -(dp)A - \tau_W P dz - \rho A g \cos\theta_g dz \tag{1.5}$$

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The wall shear stress is usually represented as

$$\tau_W = \frac{1}{2}\rho u^2 f \tag{1.6}$$

where the Fanning friction factor, f, needs to be determined empirically for turbulent flows. Conservation of mass indicates that $d(GA) = 0$. If this is used in (1.5), the following is obtained after substituting (1.6).

$$G\,du = -dp - \frac{fPG^2}{2A\rho}dz - \rho g \cos\theta_g dz \tag{1.7}$$

From conservation of mass, if A is constant, $du = G\,d(1/\rho)$ so that

$$G^2 d(1/\rho) = -dp - \frac{fPG^2}{2A\rho}dz - \rho g \cos\theta_g\, dz \tag{1.8}$$

This shows that pressure changes result from balancing the frictional drag, the gravitational force and the acceleration of the fluid.

Equation (1.8) can be solved directly for p if ρ is constant, that is, if there is no change in velocity in the direction of flow. If ρ is not constant the relation ρ (p) is needed. This can be obtained from the energy theorem and the equation of state. The rate of energy addition to the control volume can be calculated from the rate of heat addition at the wall per unit area as $q_W P\, dz$. The rate at which the fluid in the control volume does work on the surroundings consists of two terms. The rate at which the fluid at z works to get into the control volume is puA. The rate at which the fluid at $z + dz$ does work to get out of the control volume is $(puA + d(puA))$. The net rate of work on the surroundings due to these effects is $d(pua)$. The fluid in the control volume works against gravity to lift the fluid through a distance $\cos\theta_g\, dz$. This contributes a rate of work of $GAg \cos\theta_g\, dz$.

Thus, for the steady flow depicted in Figure 1.2, the energy theorem gives

$$AG\,di + AG\,d\frac{|u^2|}{2} = q_W P\; dz - d(pAu) - GAg\;\; \cos\theta_g\, dz \tag{1.9}$$

The enthalpy per unit mass is given as

$$h = i + \frac{p}{\rho} \tag{1.10}$$

where $1/\rho$ is the volume per unit mass. The kinetic energy per unit mass is $|u^2|/2$. If (1.10) and $u = G/\rho$ are substituted into (1.9) and A is constant,

$$dh = \frac{1}{2}G^2 d\left(\frac{1}{\rho^2}\right) + g \cos\theta_g dz = \frac{q_W P}{AG}dz \tag{1.11}$$

Equation (1.11) defines the change of enthalpy. For a given fluid, the thermodynamic state is defined by any three state variables, so

$$h = f(p, \rho) \tag{1.12}$$

Therefore, a table or a diagram representing the thermodynamic variables provides a relationship between ρ and p if $h(z)$ is known. Equation (1.11) gives a relation between p and p along the z-axis. This can be used in (1.8) to eliminate either $p(z)$ or $\rho(z)$.

1.2.4 Choking

Consider a length of pipe for which the fluid enters at a pressure p_1 and discharges into a chamber whose pressure, p_2, is changed. Equation (1.8) can be solved to obtain G for a given p_2/p_1. The solution shows that G initially increases as p_2/p_1 decreases. Eventually it gives a maximum and the impossible result that G decreases with decreasing p_2/p_1. The maximum in G represents a choking condition for which further decreases of the pressure in the discharge chamber have no effect on the mass flow and the flow along the pipe remains unchanged. This can be understood when it is realized that the equations predict that a decrease in p_2 is accompanied by an increase in the velocity and a decrease in the density. These have opposite effects on the mass velocity since $G = \rho u$. Thus, for p_2/p_1 less than the critical value the equations predict that decreases in ρ offset the increases in u.

At choking, the fluid velocity equals the velocity with which small disturbances (sound waves) propagate in a fluid. Thus, information about the change in the pressure at the outlet cannot be transmitted upstream and the behavior in the pipeline is not affected by what is happening in the receiving chamber. The fluid discharging from the pipeline expands supersonically and forms shock waves in the receiver.

This interpretation is illustrated, in a direct way, by considering the momentum balance, equation (1.8). The term on the left side of (1.8) may be written as

$$G^2 d\left(\frac{1}{\rho}\right) = -\frac{G^2}{\rho^2} d\rho = -u^2 \frac{d\rho}{dp} dp \tag{1.13}$$

where $dp/d\rho$ is the change of pressure with density along the pipe. Substituting (1.13) into (1.8) gives

$$\frac{dp}{dz}\left(1 - \frac{u^2}{c^2}\right) = -\frac{fG^2P}{2A\rho} - \rho g \cos\theta_g \tag{1.14}$$

where the velocity of sound, c, is given by

$$c^2 = \frac{dp}{d\rho} \tag{1.15}$$

It is noted that the coefficient of dp/dz changes sign when the Mach number, $\mathrm{Ma} = u/c$, assumes values greater than unity. This suggests that the frictional pressure gradient changes from a negative to a positive value. A positive frictional pressure gradient is an impossibility so Ma cannot change from a value less than unity to a value greater than unity in a pipeline of constant area. The flow chokes when $u = c$ at the outlet.

The entropy of a fluid can be defined by the following thermodynamic relation

$$T ds = dh - \frac{dp}{\rho} \tag{1.16}$$

Thus, the change of entropy along the pipeline can be calculated if $h(z)$, $p(z)$ and $\rho(z)$ are known. This calculation shows that, for an adiabatic flow, s increases with z because of the irreversible effects associated with wall friction. At the choking condition $ds = 0$ so, from (1.15), the disturbance velocity is given as

$$c^2 = \left(\frac{\partial p}{\partial \rho}\right)_s \tag{1.17}$$

This is a thermodynamic quantity which is the classical definition of the velocity of sound. When the flow is not adiabatic, $q_W \neq 0$, this is not the case. For example, if the frictional heating is balanced by heat losses, the temperature might be kept approximately constant so that

$$c^2 = \left(\frac{\partial p}{\partial \rho}\right)_T \tag{1.18}$$

1.2.5 Flow of an ideal gas in a pipe of constant area

An ideal gas is defined with the equations

$$\frac{p}{\rho} = \frac{R}{M} T \tag{1.19}$$

$$di = C_V dT \tag{1.20}$$

$$dh = C_P dT \tag{1.21}$$

where R is the molar gas constant, M is the molecular weight, C_V is the heat capacity at constant volume and C_P is the heat capacity at constant pressure. Thus, for an ideal gas

$$\left(\frac{\partial p}{\partial \rho}\right)_s = \gamma \frac{p}{\rho} \tag{1.22}$$

$$\left(\frac{\partial p}{\partial \rho}\right)_T = \frac{R}{M} T \tag{1.23}$$

where $\gamma = C_P/C_V$ and

$$\frac{R}{M} = C_P - C_V \tag{1.24}$$

The application of the equations developed in Section 1.2.3 will be illustrated by considering the adiabatic flow of an ideal gas in a pipe of constant area. Since a gas is being considered, gravitational effects can be neglected and the momentum balance equation is written as

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$$-\frac{\rho}{G^2}dp + \frac{d\rho}{\rho} - \frac{2f}{d_t}dz = 0 \tag{1.25}$$

where d_t is the pipe diameter. This can be integrated if the dependency of p on ρ is known. Conservation of energy for $q_W = 0$ gives

$$dh + \frac{1}{2}Gd\left(\frac{1}{\rho^2}\right) = 0 \tag{1.26}$$

Since conservation of mass gives G = constant, the equation can be integrated to give

$$(h - h_1) = \frac{G^2}{2{\rho_1}^2} - \frac{G^2}{2\rho^2} \tag{1.27}$$

where subscript 1 indicates inlet conditions. From (1.21),

$$(h - h_1) = C_P(T - T_1) \tag{1.28}$$

If (1.19) and (1.24) are used in (1.28)

$$h - h_1 = \frac{R}{M}\left(\frac{\gamma}{\gamma - 1}\right)(T - T_1) = \left(\frac{\gamma}{\gamma - 1}\right)\left(\frac{p}{\rho} - \frac{p_1}{\rho_1}\right) \tag{1.29}$$

The following relation between p and ρ is obtained if (1.27) is substituted into (1.29):

$$\frac{p}{p_1} = -\frac{(\gamma - 1)}{\gamma}\frac{G^2}{2p_1\rho_1}\frac{\rho_1}{\rho} + \frac{(\gamma - 1)}{\gamma}\frac{G^2}{2p_1\rho_1}\frac{\rho}{\rho_1} + \frac{\rho}{\rho_1} \tag{1.30}$$

Equation (1.30) is used to eliminate p from (1.25). The following relation for ρ is obtained if f is assumed to be constant.

$$\frac{(\gamma + 1)}{\gamma}\frac{1}{2}\ln\frac{\rho}{\rho_1} - \frac{(\gamma - 1)}{\gamma}\frac{1}{4}\left(\frac{\rho^2}{\rho_1^2} - 1\right) - \frac{p_1\rho_1}{2G^2}\left(\frac{\rho^2}{\rho_1^2} - 1\right) - \frac{2f(z - z_1)}{d_t} = 0 \tag{1.31}$$

If the density variation is calculated with (1.31), the pressure variation is obtained from (1.30) and the velocity can be obtained from conservation of mass

$$\frac{u}{u_1} = \frac{\rho_1}{\rho} \tag{1.32}$$

The temperature is obtained from (1.19).

$$\frac{T}{T_1} = \frac{p}{p_1}\frac{\rho_1}{\rho} \tag{1.33}$$

From (1.16), (1.11), (1.19) the following equation is obtained for the entropy

$$ds = \frac{R}{Mp}\left(\frac{G^2}{\rho^2}d\rho - dp\right) \tag{1.34}$$

This can be integrated to obtain $s - s_1$, using the equations for p and ρ developed above.

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If an isothermal flow is assumed

$$\frac{p}{p_1} = \frac{\rho}{\rho_1} \tag{1.35}$$

This condition will hold for a high molecular weight gas for which C_V is quite large and $\gamma = (C_V + R)/C_V \approx 1$. If $\gamma = 1$ is substituted into (1.30), it is seen that (1.35) results. However, in general, isothermal conditions can be maintained only if heat is exchanged with the surroundings; that is, the flow would not be adiabatic.

Equation (1.25) can be integrated directly if (1.35) is substituted for ρ

$$-\frac{p_1\rho_1}{2G^2}\left[\left(\frac{p}{p_1}\right)^2 - 1\right] + \ln\frac{p}{p_1} - \frac{2f(z - z_1)}{d_t} = 0 \tag{1.36}$$

For an isothermal flow of an ideal gas, the enthalpy h is constant so the energy equation gives

$$\frac{q_W P}{AG}(z - z_1) = \frac{G^2}{2\rho^2} - \frac{G^2}{2{\rho_1}^2} \tag{1.37}$$

The heat that is added to the flow to maintain a constant temperature must increase with increases in kinetic energy. Since $dh = 0$, equation (1.16), describing the change in entropy, gives

$$ds = \frac{1}{T}\frac{dp}{\rho} \tag{1.38}$$

This can be integrated, using (1.19), to yield

$$s - s_1 = -\frac{R}{M}\ln\frac{p}{p_1} \tag{1.39}$$

The entropy will increase as long as p decreases. It need not attain a maximum at the value of p/p_1 for choking to occur. This difference from the adiabatic case arises since the system cannot be considered as isolated so that a reversible process need not correspond to one for which the entropy is constant.

1.2.6 Mechanical energy balance

The first law of thermodynamics defines a change in internal energy as

$$di = \frac{\dot{q}_{\text{rev}} P}{GA}dz - pd\left(\frac{1}{\rho}\right) \tag{1.40}$$

where $\dot{q}_{\text{rev}}$ is the heat needed for a reversible operation. If this is substituted into the energy balance, equation (1.9),

$$\frac{dp}{\rho} + d\left(\frac{u^2}{2}\right) + g\cos\theta_g dz = (q_W - \dot{q}_{\text{rev}})\frac{P}{AG}dz \tag{1.41}$$

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Define F as the rate of mechanical energy loss per unit mass of fluid, where

$$dF = (\dot{q}_{\mathrm{rev}} - q_{\mathrm{W}}) \tag{1.42}$$

Thus (1.41) can be rewritten as

$$\frac{dp}{\rho} + d\left(\frac{u^2}{2}\right) + g\cos\theta_g dz = -dF \tag{1.43}$$

A comparison of (1.43) with the momentum balance equation gives

$$\frac{dF}{dz} = \frac{\tau_{\mathrm{W}} P}{\rho A} = \frac{4}{d_{\mathrm{t}}}\frac{\tau_{\mathrm{W}}}{\rho} \tag{1.44}$$

1.3 The homogeneous model for gas–liquid or vapor–liquid flow

1.3.1 Basic equations

The simplest approach to flows involving a gas and a liquid is to treat the mixture the same as a single phase. This requires assuming (1) zero slip between the phases, (2) uniform flow and (3) equilibrium between the phases (one can use thermodynamic tables when the vapor and liquid are in contact). Define the quality, x, as the mass fraction of the flowing mixture that is a gas or vapor and a void fraction, α, as the fraction of the cross-section that is occupied by the gas. For flows such as air–water, where phase changes are not occurring, x does not vary along the pipeline. For flows with a fluid pair such as steam–water, vaporization can occur, so x can change.

Define a mixture density as the ratio of the mass flow to the volume flow:

$$\rho_{\mathrm{H}} = \frac{GA}{AxGv_{\mathrm{G}} + (1-x)GAv_{\mathrm{L}}} \tag{1.45}$$

where

$$v_{\mathrm{G}} = \frac{1}{\rho_{\mathrm{G}}} \quad v_{\mathrm{L}} = \frac{1}{\rho_{\mathrm{L}}} \tag{1.46}$$

Thus

$$\frac{1}{\rho_{\mathrm{H}}} = xv_{\mathrm{G}} + (1-x)v_{\mathrm{L}} = \frac{x}{\rho_{\mathrm{G}}} + \frac{(1-x)}{\rho_{\mathrm{L}}} \tag{1.47}$$

A mixture velocity is defined as

$$u_{\mathrm{H}} = \frac{G}{\rho_{\mathrm{H}}} \tag{1.48}$$

For the condition $u_{\mathrm{G}} = u_{\mathrm{L}}$

$$\alpha = \frac{Q_{\mathrm{G}}}{Q_{\mathrm{G}} + Q_{\mathrm{L}}} = \frac{xGAv_{\mathrm{G}}}{xGAv_{\mathrm{G}} + (1-x)GAv_{\mathrm{L}}} = x\frac{v_{\mathrm{G}}}{v_{\mathrm{H}}} = x\frac{\rho_{\mathrm{H}}}{\rho_{\mathrm{G}}} \tag{1.49}$$

Also

$$\frac{\alpha}{1-\alpha} = \frac{x}{(1-x)}\frac{\rho_L}{\rho_G} \tag{1.50}$$

The momentum balance, equation (1.8), is rewritten as

$$-\frac{\rho_H}{G^2}dp + \frac{d\rho_H}{\rho_H} - \frac{\rho_H^2}{G^2}g\cos\theta_g dz - \frac{\tau_W \rho_H P}{G^2 A}dz = 0 \tag{1.51a}$$

or, alternatively, as

$$-\frac{dp}{dz} = G^2\frac{d\rho_H^{-1}}{dz} + \rho_H g\cos\theta_g + \frac{\tau_W P}{A} \tag{1.51b}$$

The conservation of energy equation (1.11) is given as

$$dh_H + \frac{1}{2}G^2 d\left(\frac{1}{\rho_H^2}\right) + g\cos\theta_g dz = \frac{q_W P}{AG}dz \tag{1.52}$$

where

$$h_H = xh^G + (1-x)h^L \tag{1.53}$$

The enthalpy of the gas, h^G, and the enthalpy of the liquid, h^L, are obtained from thermodynamic correlations.

For two-component systems such as air and water, the quality, x, does not vary along the pipeline and is fixed by the inlet conditions. The enthalpies, h^L and h^G, are fixed for a given p and ρ_H (which relate to ρ_G and ρ_L through equation (1.47)). For a single-component system, such as steam–water, the quality, x, and the densities, ρ_L and ρ_G, are fixed for a given pressure and enthalpy, if equilibrium is assumed between the phases. Thus, (1.52) can be used to provide a relation between ρ_H and p. This can be used to integrate (1.9). Equation (1.52) and thermodynamic data can be used to calculate the variation of p, ρ_H, G, x with distance along the pipe.

1.3.2 Choking

The momentum balance for a homogeneous flow can be rewritten as

$$-dp\left[1 - \frac{G^2(d\rho_H/dp)}{\rho_H^2}\right] - \frac{\rho_H^2}{G^2}g\cos\theta_g dz - \frac{\tau_W P}{A}dz = 0 \tag{1.54}$$

Choking occurs when the term in the brackets changes sign

$$\frac{1}{G_C^2} = -\frac{dv_H}{dp} \tag{1.55}$$

From (1.15), define

$$\frac{1}{c^2} = \frac{d\rho_H}{dp} = \frac{dv_H^{-1}}{dp} = -\rho_H^2\frac{dv_H}{dp} \tag{1.56}$$