Stochastic Processes

Theory for Applications

This definitive textbook provides a solid introduction to discrete and continuous stochastic processes, tackling a complex field in a way that instills a deep understanding of the relevant mathematical principles, and develops an intuitive grasp of the way these principles can be applied to modeling real-world systems.

- Basic underlying principles and axioms are made clear from the start, and new topics are developed as needed, encouraging and enabling students to develop an instinctive grasp of the fundamentals.
- Mathematical proofs are made easy for students to understand and remember, helping them quickly learn how to choose and apply the best possible models to real-world situations.
- Includes a review of elementary probability theory; detailed coverage of Poisson, Gaussian and Markov processes; the basic elements of queueing theory; and theory and applications of inference, hypothesis testing, detection and estimation, in addition to more advanced topics.

Written by one of the world's leading information theorists, based on his 20 years' experience of teaching stochastic processes to graduate students, this is an exceptional resource for anyone looking to develop their understanding of stochastic processes.

Robert G. Gallager is a Professor Emeritus at MIT, and one of the world's leading information theorists. He is a Member of the US National Academy of Engineering, and the US National Academy of Sciences, and his numerous awards and honors include the IEEE Medal of Honour (1990) and the Marconi Prize (2003). He was awarded the MIT Graduate Student Teaching Award in 1993, and this book is based on his 20 years' of experience of teaching this subject to students.

"The book is a wonderful exposition of the key ideas, models, and results in stochastic processes most useful for diverse applications in communications, signal processing, analysis of computer and information systems, and beyond. The text provides excellent intuition, with numerous beautifully crafted examples, and exercises. Foundations are included in a natural way that enhances clarity and the reader's ability to apply the results."

Bruce Hajek, *University of Illinois, Urbana-Champaign*

"This book provides a beautiful treatment of the fundamentals of stochastic process. Gallager's clear exposition conveys a deep and intuitive understanding of this important subject. Graduate students and researchers alike will benefit from this text, which will soon be a classic."

Randall Berry, *Northwestern University*

"In *Stochastic Processes: Theory for Applications*, Robert Gallager has produced another in his series of outstanding texts. Using a style that is very intuitive and approachable, but without sacrificing the underlying rigor of the subject matter, he has focused his treatment exactly at the level that engineers and applied scientists need to understand in order to have a working knowledge of this field. The breadth and sequencing of the coverage are also excellent. This book will be a useful resource both for students entering the field and for practitioners seeking to deepen their understanding of stochastic methods."

H. Vincent Poor, *Princeton University*

"Professor Gallager's book is the first of a plethora of textbooks on stochastic processes for engineers that strike the perfect balance between broad coverage, rigor, and motivation for applications. With a wealth of illustrative examples and challenging exercises, this book is the ideal text for graduate students in any field that applies stochastic processes."

Abbas El Gamal, *Stanford University*

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MIT

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To Marie, with thanks for her love and encouragement while I finished this book

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Preface

This text has evolved over some 20 years, starting as lecture notes for two first-year graduate subjects at MIT, namely, *Discrete Stochastic Processes* (6.262) and *Random Processes, Detection, and Estimation* (6.432). The two sets of notes are closely related and have been integrated into one text. Instructors and students can pick and choose the topics that meet their needs, and suggestions for doing this follow this preface.

These subjects originally had an application emphasis, the first on queueing and congestion in data networks and the second on modulation and detection of signals in the presence of noise. As the notes have evolved, it has become increasingly clear that the mathematical development (with minor enhancements) is applicable to a much broader set of applications in engineering, operations research, physics, biology, economics, finance, statistics, etc.

The field of stochastic processes is essentially a branch of probability theory, treating probabilistic models that evolve in time. It is best viewed as a branch of mathematics, starting with the axioms of probability and containing a rich and fascinating set of results following from those axioms. Although the results are applicable to many areas, they are best understood initially in terms of their mathematical structure and interrelationships.

Applying axiomatic probability results to a real-world area requires creating a probability model for the given area. Mathematically precise results can then be derived within the model and translated back to the real world. If the model fits the area sufficiently well, real problems can be solved by analysis within the model. However, since models are almost always simplified approximations of reality, precise results within the model become approximations in the real world.

Choosing an appropriate probability model is an essential part of this process. Sometimes an application area will have customary choices of models, or at least structured ways of selecting them. For example, there is a well-developed taxonomy of queueing models. A sound knowledge of the application area, combined with a sound knowledge of the behavior of these queueing models, often lets one choose a suitable model for a given issue within the application area. In other cases, one can start with a particularly simple model and use the behavior of that model to gain insight about the application, and use this to iteratively guide the selection of more general models.

An important aspect of choosing a probability model for a real-world area is that a prospective choice depends heavily on prior understanding, at both an intuitive and mathematical level, of results from the range of mathematical models that might be involved. This partly explains the title of the text – *Theory for Applications*. The aim is to guide the reader in both the mathematical and intuitive understanding necessary in developing and using stochastic process models in studying application areas.

Application-oriented students often ask why it is important to understand axioms, theorems, and proofs in mathematical models when the precise results in the model become approximations in the real-world system being modeled. One answer is that a deeper understanding of the mathematics leads to the required intuition for understanding the differences between model and reality. Another answer is that theorems are transferable between applications, and thus enable insights from one application area to be transferred to another.

Given the need for precision in the theory, however, why is an axiomatic approach needed? Engineering and science students learn to use calculus, linear algebra, and undergraduate probability effectively without axioms or rigor. Why does this not work for more advanced probability and stochastic processes?

Probability theory has more than its share of apparent paradoxes, and these show up in very elementary arguments. Undergraduates are content with this, since they can postpone these questions to later study. For the more complex issues in graduate work, however, reasoning without a foundation becomes increasingly frustrating, and the axioms provide the foundation needed for sound reasoning without paradoxes.

I have tried to avoid the concise and formal proofs of pure mathematics, and instead use explanations that are longer but more intuitive while still being precise. This is partly to help students with limited exposure to pure mathematics, and partly because intuition is vital when going back and forth between a mathematical model and a real-world problem. In doing research, we grope toward results, and successful groping requires both a strong intuition and precise reasoning.

The text neither uses nor develops measure theory. Measure theory is undoubtedly important in understanding probability at a deep level, but most of the topics useful in many applications can be understood without measure theory. I believe that the level of precision here provides a good background for a later study of measure theory.

The text does require some background in probability at an undergraduate level. Chapter 1 presents this background material as a review, but it is too concentrated and deep for most students without prior background. Some exposure to linear algebra and analysis (especially concrete topics like vectors, matrices, and limits) is helpful, but the text develops the necessary results. The most important prerequisite is the mathematical maturity and patience to couple precise reasoning with intuition.

The organization of the text, after the review in Chapter 1 is as follows: Chapters 2, 3, and 4 treat three of the simplest and most important classes of stochastic processes, first Poisson processes, next Gaussian processes, and finally finite-state Markov chains. These are beautiful processes where almost everything is known, and they contribute insights, examples, and initial approaches for almost all other processes. Chapter 5 then treats renewal processes, which generalize Poisson processes and provide the foundation for the rest of the text.

Chapters 6 and 7 use renewal theory to generalize Markov chains to countable state spaces and continuous time. Chapters 8 and 10 then study decision making and estimation, which in a sense gets us out of the world of theory and back to using the theory.

Chapter 9 treats random walks, large deviations, and martingales and illustrates many of their applications.

Most results here are quite old and well established, so I have not made any effort to attribute results to investigators. My treatment of the material is indebted to the texts by Bertsekas and Tsitsiklis [2], Sheldon Ross [22] and William Feller [8] and [9].

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Suggestions for instructors and self study

The subject of stochastic processes contains many beautiful and surprising results at a relatively simple level. These results should be savored and contemplated rather than rushed. The urge to go too quickly, to sacrifice understanding for shallow bottom lines, and to cover all the most important topics should be resisted. This text covers all the material in two full term graduate subjects at MIT, plus many other topics added for enrichment, so it cannot be 'covered' in one term.

My conviction is that if a student acquires a deep understanding of any, say, 20% of the material, then that student will be able to read and understand the rest with relative ease at a later time. Better still, a full appreciation of that 20% will make most students eager to learn more. In other words, instructors have a good deal of freedom, subject to a prerequisite structure, to choose topics of interest to them and their students to cover in a one term course.

One of the two MIT courses leading to this text covers Chapters 1, 2, 4, 5, 6, 7, and 9, skipping many of the more detailed parts of the latter five chapters. The other course covers Chapters 1, 3, 8, and 10, again omitting many topics. The first course is largely discrete and the second largely continuous, and a different mix is probably more appropriate for a student taking only one subject.

The topics in Chapter 1 are largely covered in good elementary probability subjects, but students are usually better at doing plug and chug exercises on these topics than having the depth of understanding required by the subsequent topics. Thus instructors should spend some time reviewing these topics.

It is difficult to be precise about the extent to which one topic is a prerequisites of another. The table below lists the prerequisites of each section. Most sections have only one prerequisite, but that recursively includes the prerequisites of the prerequisite. Instructors and students are encouraged to use their own judgement here.

Sect.	Prereq.	Sect.	Prereq.	Sect.	Prereq.	Sect.	Prereq.	Sect.	Prereq.
1.2	1.1	3.4	3.3	5.6	5.5	7.5	7.2	9.7	9.6
1.3	1.2	3.5	3.4	5.7	5.6	7.6	7.5	9.8	9.7
1.4	1.3	3.6	3.5	5.8	5.7	7.7	7.6	9.9	9.8
1.5	1.4	3.7	3.5	6.1	5.5, 4.3	7.8	7.2	9.10	9.9
1.6	1.5	4.1	1.5	6.2	6.1	8.1	1.8	9.11	9.9
1.7	1.6	4.2	4.1	6.3	6.2	8.2	8.1, 3.3	10.1	1.8, 3.5
1.8	1.7	4.3	4.2	6.4	6.3	8.3	8.2	10.2	10.1
2.1	1.5	4.4	4.3	6.5	6.4	8.4	8.3	10.3	10.2
2.2	2.1	4.5	4.4	6.6	6.5	8.5	8.4	10.4	10.3
2.3	2.2	4.6	4.5	6.7	6.3	9.1	5.5	10.5	10.4, 3.7
2.4	2.3	5.1	1.7, 2.2	6.8	6.3	9.2	9.1	10.6	10.3
2.5	1.2	5.2	5.1	7.1	6.3	9.3	9.2	10.7	10.3
3.1	1.7	5.3	5.2	7.2	7.1	9.4	9.3	10.8	10.3
3.2	3.1	5.4	5.3	7.3	7.2	9.5	9.4, 8.2		
3.3	3.2	5.5	5.4	7.4	7.2	9.6	9.5		

Acknowledgements

This book has its roots in a book called *Discrete Stochastic Processes* that I wrote back in 1996, some lecture notes on continuous random processes from about the same time, and lecture notes that I have been writing at MIT from 2007 to 2012 for a subject also entitled Discrete Stochastic Processes.

I am deeply grateful to Professors John Wyatt, John Tsitsiklis, and Lizhong Zheng who have used these notes in teaching the Discrete Stochastic Process course in recent years. Their many general observations about the value and teachability of various topics and the suggestions of alternative approaches have been invaluable. Their ability (particularly in John Tsitsiklis' case) to catch minor flaws in proofs and suggest cleaner approaches has saved me from many errors. They are also responsible for a number of excellent new exercises.

Natasha Blitvic, Mina Karzand, and Fabian Kozynski who were teaching assistants in the course for the last five years, were also very helpful both in creating and improving the wording in a number of exercises, but also in explaining why students were having difficulty and how to improve the presentation.

I am also indebted to a number of people in the MIT community who have been helpful in the evolution of this book. Professors Dimitri Berksekas and Yuri Polyansky have helped in discussing topics for the book and in reading various sections. Shan-Yuan Ho has been enormously helpful in reading the entire manuscript and catching many things, ranging from typos to poorly presented concepts. She also wrote the solution manual for the old book and has been helpful through the whole evolution of this project.

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