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978-1-107-03948-3 - Mathematics and the Body: Material Entanglements in the Classroom
Elizabeth De Freitas and Nathalie Sinclair
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Mathematics and the Body

This book explores alternative ways to consider the relationship between mathematics and the material world. Drawing on the philosophy of Gilles Châtelet and the posthumanist materialism of Karen Barad, the authors present an 'inclusive materialist' approach to studying mathematics education. This approach offers a fresh perspective on human and non-human bodies, challenging current assumptions about the role of the senses, language and ability in teaching and learning mathematics. Each chapter provides empirical examples from the classroom that demonstrate how inclusive materialism can be applied to a wide range of concerns in the field. The authors analyse recent studies on students' gestures, expressions and drawings in order to establish a link between mathematical activity and mathematical concepts. *Mathematics and the Body* expands the landscape of research in mathematics education and will be an essential resource for teachers, students and researchers alike.

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Mathematics and the Body

Material Entanglements in the Classroom

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Series Foreword

This series for Cambridge University Press is widely known as an international forum for studies of situated learning and cognition. Innovative contributions are being made by anthropology; by cognitive, developmental and cultural psychology; by computer science; by education; and by social theory. These contributions are providing the basis for new ways of understanding the social, historical and contextual nature of learning, thinking and practice that emerges from human activity. The empirical settings of these research inquiries range from the classroom to the workplace, to the high-technology office, and to learning in the streets and in other communities of practice. The situated nature of learning and remembering through activity is a central fact. It may appear obvious that human minds develop in social situations and extend their sphere of activity and communicative competencies. But cognitive theories of knowledge representation and learning alone have not provided sufficient insight into these relationships. This series was born of the conviction that new, exciting interdisciplinary syntheses are underway, as scholars and practitioners from diverse fields seek to develop theory and empirical investigations adequate for characterizing the complex relations between social and mental life and for understanding successful learning wherever it occurs. The series invites contributions that advance our understanding of these seminal issues.

Roy Pea
Christian Heath
Lucy A. Suchman

Foreword

Brian Rotman

Elizabeth de Freitas and Nathalie Sinclair have written an admirable and provocative book. Ambitious, original and theoretically accomplished, its purpose is to develop a new materialist approach, what they call 'inclusive materialism', to the learning of mathematics – one that includes and foregrounds the activity of the body against the long-standing mentalist conception of mathematics as an activity of pure, abstract thought. Extending the current turn to materialism in philosophy and the humanities to mathematics, they reject Kantian-based epistemological schemes that understand knowledge as perception filtered through internal, *a priori* intuitions and the conceptual categories, in favour of a more Humean, empiricist approach that gives primacy to external sensation; ontologically, they reject Platonic realism, the belief that mathematical objects – points, numbers, lines and so on – are immaterial entities that exist in some Platonic heaven – 'out there', beyond time, space, matter – while mathematical activity consists of discovering truths about those objects, which is analogous to scientists studying external reality. Despite numerous critiques – the chief of which asks how material beings can make contact with things in a transcendent heaven – this metaphysical idealism remains the conventional belief, defended and widely embraced by mathematicians and others.

Mathematics and the Body is directed to mathematics educators and validates, as well as explicates, its ideas by critically examining a series of experimental classroom lessons designed by the authors and by others, which focus on fundamental mathematical concepts such as number, parallelism, circles and diagrams. As the authors observe, the issue of embodied mathematics in education is topical. In the last decade or so, a growing number of differently oriented initiatives – cognitive, phenomenological, enactive, communication-based approaches – have been devoted to examining the role played by students' bodies: their gestures; hand, eye and limb movements; their verbalizations; their drawings and diagrams; and their relation to the tokens, devices, physical objects and surfaces with which they interact. The book aims to explore the assumptions and consequences of this work. To do so, and to go beyond it, they pose and confront the fundamental question: 'How are the physical aspects of mathematical activity – be it that of students or mathematicians – transformed into the so-called abstractions and generalisations of formal mathematics?' Their answer involves formulating a new, extended notion of 'body' and correlatively a material understanding of the mathematical concepts with which such a body engages. The inclusion of mathematicians' physical activity in their question indicates a possible parallel between the creation of mathematics and its re-creation by students in the classroom – a link, that is, between the history of mathematics and the learning of it. Such is indeed the case, as is evident in their opening sentence: 'The idea for this book began as we read Gilles Châtelet's (1993/2000) stunning book on the history of mathematics, which challenges many long-standing, as well as contemporary, philosophies of mathematics.' Châtelet's book, *Figuring Space*, opens up several key moments in the historical development of the subject, demonstrating how the interrelation of gesture – resulting from 'disciplined movements of a body' – and physical diagrams operate at the heart of mathematical invention. De Freitas and Sinclair embrace Châtelet's linking of gestural bodies and formal abstractions and work to import it into the mathematics classroom.

But before they can accomplish this, they need to establish the nature of embodiment. 'When', they ask, 'does a body become a body?' A survey of the mathematical embodiment literature finds them critical of approaches that fail to escape the 'dualistic tradition of the mind/body split'; or that 'demote the body to acting merely as the vessel or container of some higher act of cognition'; or that 'centre human will or intention in the orchestrating of experience', assuming the human body to be 'the principal administrator of its own participation'. Moreover, locating knowing and agency in the individual body does not adequately address the collective social body. Where, then, are the boundaries of a body? Against the commonsense view that 'the body is an individual, discrete entity and that cognition occurs within its borders', the authors turn to posthumanist discourses of subjectivity and agency, according to which subjects are dynamic assemblages of dispersed social networks, and the 'human body itself must be conceived in terms of malleable borders and distributed networks'; that is, a body understood as a 'set of material relations that seems to structure the other material relations around it'. In the classroom, as they illustrate in their analyses of students' activities, such an assemblage-body will be composed of 'humans, writing implements, writing surfaces, texts, desks, doors, chips, as well as disciplinary forces and habits of control and capitulation'. A consequence of conceiving the body in this way is that agency and thought become distributed across multiple sources in the students' physical and psycho-social environment. Thus, analogous to Nietzsche's insistence on 'deeds without a doer', one can have 'thoughts without a thinker', in the sense that the source of thought can come from material relations outside or beside the isolated thinking self, a phenomenon that Gilles Deleuze, whose materialist ideas exert a profound effect on the authors' project, calls the 'exteriority of thought'. In short, the power and efficacy of a body in relation to mathematics must be understood as distributed across an assemblage of heterogeneous relations, a posthumanist understanding not to be identified with the capacity that is 'localized in a human body or in a collective produced (only) by human efforts'.

But how does this material body-assemblage become entangled with mathematical concepts? In what sense can we consider concepts, mathematical or otherwise, to be related to *matter*? The question goes to the theoretical heart of *Mathematics and the Body*. The authors' aim is to show how 'mathematical concepts partake of the material in operative, agential ways'. In order to accomplish this, they need to go outside a humanist conception of matter and 'materiality', as well as construct a new approach to the nature of concepts. They derive this from contemporary feminist philosophers, principally Karen Barad, but also Jane Bennett, and Diana Coole and Samantha Frost, whose common aim is to reorient how we think about 'matter' and the material world. From Barad's theory of 'agential realism', derived from Niels Bohr's explication of quantum phenomena, they take the understanding of a concept not as an immaterial mental object, but as 'a material arrangement of things' and of relations preceding, and in some sense constituting, that to which they relate, so that things are always 'intra-related', rather than interrelated. Jane Bennett's concept of 'vibrant matter', a (non-animistic) understanding that credits matter with agency, and the 'inclusive materialism' of Coole and Frost provide the wherewithall for constructing a body-concept nexus. This, along with the anthropological work of Lambros Malfouris and Bruno Latour, allows them to rethink the concept of 'mere' matter. Rejecting the Cartesian split between the active, cognizing human mind and inert, 'dead' matter – the contemporary orthodoxy underpinning the physical sciences' engagement with matter – these various thinkers urge materialisms in which the freedom and agency that Descartes restricted to the embodied human mind is opened up and dispersed across human and non-human agents. The ontology of mathematics that the authors weave from these diverse materialisms, with their insistence on the extra-human and material dimensions of thought, complements the authors' construction of the assemblage-body. With this theoretical meshing in place, de Freitas and Sinclair are ready to expand on how the two – bodies and formal mathematical concepts – might in practice become entangled. They accomplish this through the essay of Châtelet that inspired them to pursue their ambitious body-mathematics project.

Châtelet's interest is in how mathematics comes into being – its genesis, its becoming rather than 'being' – and his essay is a series of analyses of specific mathematical inventions, such as Grassmann's creation of algebras over vector fields and Cauchy's method of integrating complex functions, that reveal the physico-conceptual movements that constitute them.

His starting point is actual, physical movement. According to Châtelet, the 'amplifying abstractions' of mathematics, whatever their ultimate immaterial representation as formal constructs may be, have bodily beginnings. They originate in gestures, 'disciplined distributions of mobility', that are not signs or representations of anything prior to or outside themselves, but instead are material events that, through their actions and by the fact of their occurrence, bring new mathematical meanings into being. They are not, Châtelet insists, describable by formal languages, cannot be determined by algorithms, are not expressions of an intention (although they can be retrospectively seen as such), and are not in fact consciously produced: 'One is', he says, ' infused with the gesture before knowing it.' And they do not work through reference or signification, but rather by pointing, through allusions that – in interaction with diagrams (which are themselves responses to problems) – give rise to 'dynasties of problems' and correlative families of ever more precise allusions. A diagram, for Châtelet, is a frozen gesture, a gesture caught mid-flight in its path towards a formal abstraction: It can 'transfix a gesture, bring it to rest, long before it curls up into a sign'. Diagrams are intermediaries between bodies and mathematical objects and operations. They are, like gestures, material events. Contrary to the customary view of them, they are not depictions, illustrations or visual icons of mathematical objects or concepts (although they can be), but instead are pivotal devices in the creation of mathematical meaning – 'kinematic capturing devices', as the authors neatly describe them, 'for direct sampling that cut up space and allude to new dimensions and new structures'.

In a sense, diagrams are works in progress, never complete in themselves: '[I]f [a diagram] immobilizes a gesture in order to set down an operation, it does so by sketching a gesture that then cuts out another.' Diagrams and gestures interact, mutually presupposing each other, participating in what the authors call each other's 'provisional ontology'. Overall, the gesture-diagram nexus operates as a 'dynamic process of excavation that conjures the sensible in sensible matter'. The authors relate this conjuring to Barad's realist understanding of concepts as material arrangements. 'The concept itself', they observe, 'is entailed in the hands that gesture, the mouth that speaks, and the affect that circulates across an interaction.' They concretize this entailment through a variety of examples that range from discussing how 'the point at infinity' is cognized in projective geometry to describing at length the results of an experiment with a class of undergraduates asked to draw diagrams in response to a simple film of moving circles.

The gesture-diagram apparatus of allusions to mathematical meanings is one-half of what the authors find valuable in Châtelet's approach; the other is his deployment of the notion of the *virtual*. He takes this from Deleuze's materialist and immanentist philosophy, according to which the physical world of matter constantly comes into being – becomes – by making actual that which is virtual: 'The virtual must be defined as strictly a part of the real object – as though the object had one part of itself in the virtual into which it is plunged as though into an objective dimension.' The virtual is that which is latent in matter, the source of all that it could become, which the authors interpret as its 'mobility, vibration, potentiality and indeterminacy', and it is the link Châtelet provides between the mathematical and physical worlds.

Following Gottfried Leibniz in conceiving space as 'a flexible, folding and animated substance', Châtelet observes that the supposedly immovable objects of mathematics divorced from 'sensible matter' are, on the contrary, always in a state of potential movement and change; a geometrical point (line, circle) cannot be confined to a designated entity, the representation of a position within a fixed, absolute space. As he observes in the case of Cauchy's treatment of a singular point in the complex plane, the virtuality of a point, probed by mathematicians within 'thought-experiments', becomes the source of radically new concepts. A point is the simplest example of a diagram, but the effect is quite general. As the authors observe, 'the virtual or potentiality of a[ny] diagram consists of all the gestures and future alterations that are in some fashion "contained" in it'. Mathematical entities, then, are material objects with virtual and actual dimensions. The virtual is not so much a bridge – an interrelation between mathematics and the physical world, as if they were initially separate and then joined – as an 'intra-relation' – which Barad defines as a mutual fabrication or co-constitution, wherein the two are thoroughly entangled. This means that mathematical concepts engage in a process of becoming which binds them to the actions of mathematicians, leading to the authors' striking conclusion that '[t]he mathematical body comes into being through actualizing the virtual – through gestures, diagrams and digital networks, *we become mathematics*; we incorporate and are incorporated by mathematics" (emphasis in original).

Summarized in this way and taken in isolation, the concept of 'becoming' mathematics will doubtless strike many potential readers of *Mathematics and the Body* as a strange and counter-intuitive characterization of 'we' and of mathematics, but hopefully this will not be their experience. Throughout, de Freitas and Sinclair seem fully aware of the unfamiliarity of the ideas they mobilize and of the conceptual demands of their thesis; they go to considerable lengths to present matters as accessibly as possible. Not only does their book carefully develop the ideas of the body-as-assemblage and the body's dynamic relation with abstract concepts that forms the basis of how we become mathematics, but it also contains a wealth of material and a rich texture of connections that elaborate and contextualize their thesis. Thus, beside constantly rooting their ideas in the concrete classroom observations and experiments which feature throughout, they step back and offer a series of illuminating and provocative chapter-length discussions of key aspects of their field, ranging from the 'sensory politics of the body mathematical' and 'mapping the mathematical aesthetic' to the 'materiality of language' and the material dimension of 'inventiveness in the mathematics classroom'.

In a final reflection on what becoming mathematics might mean, both generally and in the context of the classroom, they invoke Deleuze's concept of a 'minor science', a 'minor literature', and indeed a 'minor mathematics' – forms of thought and creation which escape the constrictions of the dominant 'state', or orthodox version. They describe 'a mathematics that is not the state-sanctioned discourse of school mathematics, but that might be full of surprises, non-sense and paradox' and which, although at odds with institutional demands and the domination of a fixed curriculum, 'is likely to engage students and teachers in more expansive ways, and [their] hope is that it would engage *more* students in mathematics'. Whether or not it does remains, of course, to be seen, but in any event the minor mathematics that Elizabeth de Freitas and Nathalie Sinclair usher onto the mathematics education scene constitutes a major theoretical intervention in their field. *Mathematics and the Body* is a valuable, radical and challenging work.

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