Introduction

The idea for this book began as we read Gilles Châtelet's (1993/2000) stunning book on the history of mathematics, which challenges many long-standing, as well as contemporary, philosophies of mathematics. His book places gestures and diagrams at the centre of mathematical inventiveness, which struck many chords for us both, not least in relation to our mutual interest in the role of these body-based and mobile devices in the teaching and learning of mathematics. We saw in Châtelet a way of better understanding how materiality might matter for mathematics, which has for so long been taken as an abstract and static discipline that resists any links with the physical world. Although we have learned a great deal from recent scholars working on the embodied nature of mathematical thinking and learning, we were unsatisfied with some of their basic philosophical assumptions about the nature of both mathematics and the body. Further, we wanted to find ways of being able to work with an embodied mathematics, while also drawing on the powerful and insightful research of colleagues pursuing a more discursive and politicised way of understanding the teaching and learning of mathematics. This latter research usefully moves beyond the essentialist assumptions of acquisitionist theories of learning and sheds considerable light on the sociocultural facets of education, but it has frequently neglected the role of the body in teaching and learning. Châtelet inspires a new kind of materialist study of mathematics and the body, allowing for new ways of exploring how mathematics partakes of the material world. In this book, we try to show how this new approach can be put to work in rethinking mathematics education.

As we read Châtelet, we began to see links with the contemporary work of 'new' materialists such as Karen Barad, as well as philosophers such as Gilles Deleuze and Brian Rotman. Their writing enabled us to see how we could talk about embodiment *and* discourse in coherent ways. They

linked us to important traditions in philosophy, feminism, history and mathematics that enabled us to contextualize and question work in mathematics education. They also gently but steadily led us into rethinking the politics and aesthetics of mathematics – issues that we had both pursued independently before and to which we were eager to return. While our own starting point was Châtelet, we begin this book by pushing on two very central, and perhaps counter-intuitive, notions that we see as underlying Châtelet's philosophy. The first is the very notion of the human body, as we unpack what it is and what it might be. The second is the nature of materiality and its relation to the human body, the social and the conceptual. Our work on these notions (body and matter), particularly in the context of mathematics education, has led us to a particular brand of materialism that we call *inclusive materialism*.

In Chapter 1, we propose a rethinking of the assumed boundaries of the body and the taken-for-granted geography of the body's interaction with the material world. Our impetus for doing so comes from questions that arose for us as we have made our way through the vast amounts of work on the body and its relation to learning. What is a body? Where does it begin and end? When does individuation of a body occur in a classroom? Must there be an interior and exterior? Is the body a bounded organism? These are questions that became more pressing as we tried to absorb the implications of Châtelet's materialist philosophy of mathematics, which refuses to see the body – any body, not just the human body – as fixed, stable or unitary. Indeed, how can we continue to speak of a bounded body if, as Rotman (2008) suggests, the proliferation of our sensory powers through techno-societal developments in the last century have us 'becoming beside ourselves'? Many current theories of embodiment do not address these questions, perhaps because they are usually oriented towards analyses of student activity, trying to account for individual processes of learning. But if we trouble this enclosed 'definition' of the body, what might be the ramifications for what is meant by activity, by learning, by embodiment? One ramification will be the idea of the body as sometimes more and sometimes less than its physical parts, as inextricably bound up with artefacts, other bodies, concepts. Another will be the decentring of the human body, that is, the acknowledgement of non-human agencies also at play in any learning situation.

The second possibly counter-intuitive notion developed in Chapter 2 is the ontological entanglement of matter and meaning. Just as we often assume that the human body is contained by the contours of its skin, so we assume that matter is inert and entirely responsive to the will of human

bodies. While some theorists have tried to accord agency to inanimate bodies as well, they have been mired in questions of intentionality – can a toaster *intend* to pop? – and much of the literature on the subject seems trapped in an ongoing dichotomizing of bodies, objects and actions. The notion of materiality that we have chosen to pursue, based largely on the work of Karen Barad, but also based on the concept of virtuality that one finds in the work of Gilles Deleuze, averts these problems by focusing on the indeterminacy of relations between various types of human and non-human agents. This allows us to extend materiality beyond the strict confines of concrete, physical objects so that meaning, discourse and concepts are also treated as material.

While Barad's materialism provides a compelling basis for rethinking meaning and matter, it stops short of being able to account for the seemingly abstract and immaterial nature of mathematics. This is where Châtelet comes in, providing a way of interpreting new materialist approaches, such as Barad's, in the case of mathematics. To be sure, his mathematics may not be a very familiar one because of the particular examples he selects, but also because he is proposing a radically different approach to the question of the ontological status of mathematical concepts. It certainly challenges the major philosophies of mathematics, but it might also cast mathematics in terms that are hard to reconcile with school mathematics. Nevertheless, this is the challenge we have taken on in this book, mapping this new approach onto school mathematics, situated within a post-humanist, materialist perspective that we call *inclusive materialism*. More generally, we approach mathematics from within an empiricist tradition by looking closely at the material specificities of mathematical experiences.[1] We pursue questions such as: What are the concrete material actions that constitute the activity of doing mathematics? What are the relations of exteriority – the relations between material parts – that comprise the corporeal habits of this cultural practice? Thus we position ourselves within a tradition in which abstract thought and materiality are assumed to be entwined. According to phenomenological currents within this tradition, thinking and reasoning – and

[1] The literature in mathematics education can be separated into two broadly conceived groups: the first abides by a Kantian-inspired theory of learning, in which it is argued that cognitive faculties synthesize sense perception; and the second aligns with a Humean-inspired approach, in which it is argued that conceptual categories are constituted through perceptual routine habits and material interactions. Unlike the Kantian tradition, which assumes that our experiences of the world are structured through internal categories or concepts that we impose on the material world of phenomena (Delanda, 2006), the Humean tradition is an empiricist one that lends itself to the study of emergent material habits and emergent cognitive structures.

any other related cognitive constructs – are always external or located in the 'flesh': 'Thinking is not a process that takes place "behind" or "underneath" bodily activity, but *is* the bodily activity itself' (Ferrara and Nemirovsky, 2005, p. 139). Given that empiricism comes in many different flavours, any claim must be seen as reflecting a particular set of cultural and scientific practices, practices that shift with history and context. This book problematizes any universal concept of the 'empirical' by drawing on historical arguments and emphasizing the ways in which research assumptions – both tacit and explicit – inform what we are able to see in our data.

We have not been exhaustive in this exploration; indeed, we have sometimes been opportunistic in choosing the particular aspects of mathematics education that we have worked on before, either individually or together. This is the case, for example, for our investigation of creativity and language use in the classroom, as well as for our re-examination of the mathematical aesthetic. At other times we have followed more closely some of the constructs that have been put forward by Barad and Châtelet, such as assemblages and virtuality, each of which is brought to bear on the concerns of mathematics education. One unexpected line of flight took us into the literature on disability, from which we learned a tremendous amount and in which we found kindred attempts to shift perspective along post-humanist materialist lines.

This book reflects out attempt to work through the implications of inclusive materialism in relation to mathematical activity. Our aim is to expand on Châtelet's sometimes poetic and difficult writing and to show through example and application how his insights are highly relevant to mathematics education. Much of the literature we draw on may be new to readers in mathematics education, but we hope that we will whet the appetite of those readers and perhaps spur others on in pursuing some of these ideas in their research. The idea of post-humanism that we develop is a difficult one to embrace – it seems to go against a common-sense inclination to centre the human subject in the study of teaching and learning. The idea of a body with unstable contours only provisionally individuated is also difficult to grasp given our life experiences and the seemingly definitive end to those experiences that occurs at the time of death. But we ask that the reader enter into a thought experiment – a 'what if' exercise – where such common-sense beliefs are set aside not only for the sake of thinking differently, but for the possibility of learning more about how mathematics and the human body consort. We offer this book as an invitation to explore what we deem to be fruitful ramifications from this particular way of thinking differently. Each chapter is meant to provoke and push the reader to

consider alternative ways of conceptualizing the relationship between mathematics and matter. Thus the writing style is at times more evocative and poetic than what one usually finds in mathematics education literature. But we believe that this kind of speculative and creative work is extremely important in our field. Such work pushes the field into new uncharted terrain and allows for new conjectures about teaching and learning. Our aim in each chapter has been to show how ideas from new materialism can be put to work in opening up the landscape of research in mathematics education. Of course, no book can create an exhaustive map, and this has not been our objective. We have tried, however, to link our ideas with others who work in mathematics education, teasing out some of the at times subtle differences between various approaches. In each chapter, we pushed ourselves to perturb our own assumptions about the fundamental constructs that are often taken for granted in the field. We chose not to use just our own data throughout the book; instead, we discuss diverse kinds of examples that can be found in other publications and video resources, so that readers might access this material on their own.

Outline of the book

Chapter 1 is concerned with outlining the main philosophical assumptions and theoretical constructs about the body that are used in this book. We begin by examining the ways in which embodiment is currently conceptualized in mathematics education and the critiques that have been made of current conceptions of the body, both within mathematics education but also from without, by learning scientists, sociologists and post-humanist philosophers. We then pursue answers to some of the previously listed questions by proposing that the body need not be delimited by the container of its skin, nor should the concept of the body be exclusively reserved for sentient beings. We draw on feminist philosophy, and its own evolving quest to understand the nature of the female body, to better articulate how this new body might operate and to elaborate further on Rotman's sense of the body as that which is always becoming. This take on the body resonates powerfully with new materialism, which seeks to rethink the nature and role of matter and, in so doing, perturbs existing assumptions about how human beings interact with matter. Thus we argue that the body be conceived as an assemblage of diverse materialities in motion.

What is at stake in new materialism is ontology – what matter *is*. This is the focus of Chapter 2. The ontological questions we are posing are reminiscent of those posed by Warren McCullough (1965, p. 7) when he

asked 'What is a number that Man may know it? And what is Man that he may know number?' But we try to shift the ground of this question so that knowing and becoming are more entangled. Chapter 1 provides the tools we need to see how McCullough's 'Man' and 'number' are involved in an ongoing dynamic such that it makes more sense to speak of man-number as a relationship than of either man or number as an independent, discrete entity. In other words, 'Man' and 'number' are part of a mutually constitutive material assemblage that has fewer boundaries and less fixity than McCullough's questions imply. Karen Barad's attempts to develop an approach to matter and concept in terms of *intra-activity* (a term she coins to avoid the assumption of distinct bodies acting with each other that the word interaction suggests) and an epistem-ontology are useful in this respect.

In Chapter 2, we build on the work of Barad to argue that theoretical concepts are inextricably material and that matter is intrinsically indeterminate. She elaborates her materialism by relying heavily on the work and writing of the physicist Niels Bohr. Indeed, she uses developments in twentieth-century physics to show how knowing and becoming are entangled. Although she does not write explicitly about mathematics or about learners, her focus is on the material nature of theoretical concepts, and thus we found her work highly pertinent to mathematics. But even if one acknowledges that concepts from physics – like electron, force or string – partake of the material world (hence making their materiality seem both natural and reasonable), it is likely harder to swallow the idea of the materiality of mathematical concepts. We can easily think of the way in which a concept like number *connects* or *applies* to the physical world, as in counting apples and measuring heights, but in what sense can it be said that number *partakes* of the material world? Theories of embodiment have argued that the sensorimotor experiences of humans – which are said to occur within the physical world – enable humans to create, understand and learn concepts like number. But such theories say next to nothing about what numbers are, let alone how mathematical concepts such as number can inhabit the physical world rather than some metaphysical, Platonic realm. We suspect that some of the research on embodiment may in fact serve to entrench further an image of mathematical concepts as universal, static and pre-given, despite the new focus on the role of the body in learning, in part because the mathematics is not adequately historicized. Our hope is that the new kinds of materialisms that we explore in this book will supply us with a set of theoretical tools to study more thoroughly mathematical concepts as partaking of the physical world.

In Chapter 3, we extend the discussion of the materiality of mathematical concepts with the help of Châtelet's work on the role of gestures and diagrams in mathematical invention. Gestures and diagrams both have received increased attention in mathematics education, especially within the recent emergence of embodied and semiotic perspectives. Châtelet's work, however, seeks to show, through examples from the history of mathematics, how gestures and diagrams play a pivotal role in mathematical invention. He wants to show how the material mobility of the human body comes to produce formal mathematics. Diagrams are essential clues for him because they provide a trace of the moving hand, while also enabling – on the surface of the paper – the exploration and creation of new objects and dimensions. Châtelet thus brings together two hitherto distinct areas of research – on gestures and diagrams, respectively – in pursuing his non-representational, non-dualistic account of mathematical thinking. The diagram, argues Châtelet, is by its very nature never complete, and the gesture is never just the enactment of an intention. The two participate in each other's provisional ontology. In contrast to current work around gestures on the one hand and diagrams on the other, Châtelet insists that extracting one from the other is both awkward and possibly misleading. He argues that the gestural and the diagrammatic are pivotal sources of mathematical meaning, mutually presupposing each other and sharing a similar mobility and potentiality.

To exemplify this claim, Châtelet selects certain episodes in the history of mathematics and physics to show how particular diagrams – what he terms 'cutting-out gestures' – have been deployed during inventive thought experiments to bring forth new mathematical concepts. Châtelet is careful, however, to analyse the way the mathematical concepts are folded into the material activity during the event, rather than simply ascribing the invention to cognitive ability or human discernment. In other words, he uses these historical episodes to explore ontological questions about the relationship between the mathematical and the physical, as well as cultural questions about what it means to do mathematics. He argues that the study of such diagram-gestures helps us undo some of the troubling consequences of the Aristotelian division between movable matter and immovable mathematics. We look at his examples – Archimedes, Oresme and Cauchy – and present his theory of mathematical inventiveness. In the last section of the chapter, we discuss a teaching experiment in which students watched a Nicolet stop-action film of a mobile circle, and then were prompted to diagram what they saw. We show examples of their diagrams and discuss how one might interpret their work in terms of Châtelet's ideas on the coupling of diagram and gesture. We use this

experiment as a way to exemplify Châtelet's approach in the context of more modest mathematical breakthroughs and also as a way to illustrate how matter is always at play in the relationship between mathematics and the learner.

In Chapter 4, we examine some classroom episodes through this Châtelet-inspired materialist approach. In particular, we want to see how mathematical inventiveness in the classroom can be described not only in terms of virtuality, but also in terms of an assemblage of materialities (including children, teachers, computers, projectors, hands, arms, etc.) that operates more as a system of 'impersonal' forces and less as a reflection of individuated human agency. The two classroom examples discussed in this chapter are not meant to be prescriptions for classroom teaching and learning; instead, they are meant, once again, to underscore the entanglement of gestures, diagrams and words, and to put this concept of the virtual that we have adopted from Châtelet to work. However, within this pedagogical context, the diagrams involved will be the dynamic ones of the computer screen, which not only put mathematical objects into motion, but also give rise to new gestures and words. We are interested in how particular virtualities that might otherwise remain dormant are actualized through the use of the technology of the computer screen. More importantly, our materialist approach to these examples aims to focus on the role of movement and affect in mathematical activity and to resist a reading of mathematical behaviour that relies too centrally on language use.

Chapter 5 seeks to pursue our dedication to materiality by probing and stretching our readings of language use in the mathematics classroom. While Châtelet's philosophy of mathematics draws attention to the crucial role of gestures and diagrams in mathematics, offering powerful insights into the materiality of mathematics itself, he does not attend to the role of language in mathematics explicitly. This absence is problematic, because language plays a significant role in how the body comes to matter as a sociocultural entity, and it plays an equally important role in learning mathematics. While the study of materiality in teaching and learning mathematics helps us resist the logocentric or language-centric reading of mathematical behaviour found in many contemporary discursive approaches, one clearly still needs to attend to language use in studying mathematics teaching and learning. The challenge is to use semiotic and linguistic tools to study activity without blinding the researcher to that which might not lend itself to such an analysis. In Chapter 5, we experiment with ways of doing so, studying language use as part of a collective process of material *in(ter)vention*, not a translation of thought or only (or

mostly) a semiotic act of representation. We propose this definition of language use in terms of in(ter)vention – rather than communication – to draw attention to both the materiality and inventiveness of language. In offering this material reading of language production, we hope to show how a *micropolitics* of classroom discourse attends carefully to the production of meaning while continuing, as in Chapter 4, to decentre the rational thinking subject as the source of meaning. This chapter pays tribute to the extensive and enlightening work that has been done by scholars of discourse analysis, such as Michael Halliday and Norman Fairclough, and shows how our materialist approach is not necessarily at odds with some of their basic orientations. Then, again drawing on Barad, but also on other contemporary philosophers, we dwell on the materiality of speech – the 'auditory gestures', as Rotman calls them – as a way of motivating the move from language as communication to language as in(ter)vention. We experiment with an alternative research methodology by which meanings in the mathematics classroom emerge in the pauses, accelerations, fallings-away and other bodily encounters that produce sounds, rather than merely in the discrete sonic units of spoken words. By contrasting this analysis with a more traditional discursive one, we hope to draw attention to some of the traps into which current transcription practices lead mathematics education researchers, as well as some of the openings for richer accounts of the motions and emotions of classroom activity.

In Chapter 6, we look at how the correlation of the 'true' and the 'sensible' (which are coupled in Rancière's political aesthetic approach) has served to produce Western conceptions of mathematical ability. Current theories of embodiment privilege the body in a way that has never before happened in the history of mathematics. If Western culture historically invested in a disembodied vision of mathematics, how will the stakes change if we embrace a more embodied one? In particular, what are the political ramifications of taking embodiment so seriously? We argue that the senses have played a unique role in conceptualizing mathematical ability, and we show how images of embodied (dis)ability operate in our educational practices. We tell one particular historical story, tracing how the concept of intuition is tied to particular Western assumptions about perception. Our aim is to show how post-humanist approaches to the body demand that we interrogate this notion of intuition, as well as the assumption that a body possesses a set of sense organs of pre-given capacity. Following a Deleuzian approach to the body as assemblage, the sensory organs are but one configuration of an unstable collective that may also include walking sticks, rings, cylinders and moving points. Sense organs are provisional and open to new

configurations. We survey some of the increasingly critical literature on disability, which disputes disability as an assignation to the individual of a physical deficit.

We are sensitive to the ways in which this work on (dis)ability is polemical, but we believe that the arguments are worth studying for how they teach us about the body and how they make visible taken-for-granted exclusionary practices within education. We see in this work an attempt to move away from Kantian conceptions of perception-as-synthesis towards a study of the relational, highly variable and responsive rhythmic vibrations that are the foundation of sensation. In this sixth chapter, we explore the proposal that the body is thus constituted through thresholds or levels of resonance, and sensation is dislocated and only provisionally situated in a perception. Indeed, this is a body with potentially different perceptual capabilities from those which are currently considered normal. Hearing, touching, tasting, seeing, smelling and any other modality are temporary – if persistent – kinds of perception, but our future may entail entirely different calibrations of sensation. This post-humanist approach to sensation allows the body to break free from the confines of current perceptual organisation and demands that we recognize the human body in all its potentiality, even in our current classrooms, where bodies can be seen as differently abled and differently (organ)ised rather than disabled or distracted. This is not to dismiss the reality of those with disabilities, but rather to help researchers and educators think differently about the processes by which (dis)ability comes to be recognized in classrooms. We open the chapter with an example of a young girl interacting with a touchscreen application that shows how we can understand her various visual, auditory and haptic sensations as part of a rhythmic play beneath her ability to form concept-producing judgements.

Chapter 7 seeks to contribute to the political discussions around mathematics education more broadly. We recognize that any theory of embodiment must adequately address the sociocultural structuring of experience. The challenge is to attend to the materiality of experience, within the phenomenological tradition and contemporary post-humanist materialisms, while also recognizing the macropolitical and economic forces that in part shape it. Many political discussions within education hitherto, for the most part, have been informed and inspired by the critical discourse-focused approach of Michel Foucault. They inherit from Foucault an unwillingness to consider phenomenological approaches to the body – like that found in the work of Maurice Merleau-Ponty – because these seemed to essentialise and universalise the body and 'lived experience'. However, through the