Statistical Mechanics and Applications in Condensed Matter

This innovative and modular textbook combines classical topics in thermodynamics, statistical mechanics and many-body theory with the latest developments in condensed matter physics research. Written by internationally renowned experts and logically structured to cater for undergraduate and postgraduate students and researchers, it covers the underlying theoretical principles, and includes numerous problems and worked examples to put this knowledge into practice.

Three main streams provide a framework for the book: beginning with thermodynamics and classical statistical mechanics, including mean field approximation, fluctuations and the renormalization group approach to critical phenomena. The authors then examine quantum statistical mechanics, covering key topics such as normal Fermi and Luttinger liquids, superfluidity and superconductivity. Finally, they explore classical and quantum kinetics, Anderson localization and quantum interference, and disordered Fermi liquids.

Unique in providing a bridge between thermodynamics and advanced topics in condensed matter, this textbook is an invaluable resource for all students of physics.

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CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781107039407

First published 2015

Printed in the United Kingdom by TJ International Ltd. Padstow Cornwall

A catalogue record for this publication is available from the British Library

ISBN 978-1-107-03940-7 Hardback

Contents

Preface

D. Pines in his Editor's Foreword to the important series "Frontiers in Physics, a Set of Lectures" of the sixties and seventies of the past century (W. A. Benjamin, Inc.) was suggesting as a possible solution to "the problem of communicating in a coherent fashion the recent developments in the most exciting and active fields of physics" what he called "an informal monograph to connote the fact that it represents an intermediate step between lecture notes and formal monographs."

Our aim in writing this book has been to provide a coherent presentation of different topics, emphasizing those concepts which underlie recent applications of statistical mechanics to condensed matter and many-body systems, both classical and quantum. Our goal has been indeed to reach an up to date version of the book *Statistical Mechanics. A Set of Lectures* by R. P. Feynman, one of the most important monographs of the series mentioned above. We felt, however, that it would have been impossible to give to a student the full flavor of the recent topics without putting them in the classical context as a continuous evolution. For this reason we introduced the basic concepts of thermodynamics and statistical mechanics. We have also concisely covered topics that typically can be found in advanced books on many-body theory, where usually the apparatus of quantum field theory is used.

In our book we have kept the technical apparatus at the level of the density matrix with the exception of the last four chapters. Up to Chapter 17 no particular prerequisite is needed except for standard courses in Classical and Quantum Mechanics. Chapter 18 provides an introduction to statistical quantum field theory, which is used in the last chapters. Chapters 20 and 21 cover topics which, although covered in recent monographs, are not commonly found in classical many-body books.

In our book then the student will find a bridge from thermodynamics and statistical mechanics towards advanced many-body theory and its applications. In our attempt to give a coherent account of several topics of condensed matter physics, we have at the same time preserved the personal point of view of the notes of our courses. Our bibliography is for this reason far from complete and the presentation of some topics is somewhat informal and partial. Many important contributions and fundamental references have been left out.

Some fundamentals topics of modern statistical mechanics for condensed matter physics have been left out, e.g. quantum criticality (except for the two examples of Luttinger liquid and Anderson localization), the quantum Hall effect and spin glasses. For them, however, there are recent devoted books.

Suggestions on the usage of the book

Although this book is conceived and organized as an organic self-contained whole, it can also be used as a modular text. We provide here a few suggestions for planning courses

Cambridge University Press
978-1-107-03940-7 - Statistical Mechanics and Applications in Condensed Matter
Carlo Di Castro and Roberto Raimondi
Frontmatter
More information

on the great amount of the covered material at undergraduate, graduate and postgraduate level. In this way we hope also to facilitate consultation by teachers and researchers.

1. **Introductory statistical mechanics [Chapters 1–4, 6–7, (5)]**[1]
   Chapters 1–4 and 6–7 are a self-contained introduction to basic statistical mechanics. Chapter 1 is a concise review of thermodynamics in order to put statistical mechanics in context. In particular we emphasize the concepts of thermodynamic stability with respect to deviations and fluctuations from average by means of the introduction of the thermodynamic function availability as the maximum work available. The aim is to alert the reader from the start of the statistical interpretation of the law that the entropy of an isolated system can never decrease. Fluctuations play an important role in phase transitions. Both these two issues are at the heart of the foundations of statistical mechanics. The logical sequence of the thermodynamic potentials and the related pairs of conjugate variables prepares the basis of the thermodynamics for ensembles. Chapters 2 and 3 cover the basic concepts of statistical mechanics from the fundamental insights by Boltzmann to the Gibbs formulation. Chapter 4 is about the consequences of the basic axioms and discusses the relation between the different statistical ensembles. Chapter 5, which is not included in the above list, concludes the exposition of classical statistical mechanics by presenting the physical and mathematical significance of the thermodynamic limit especially with reference to the problem of phase transitions. This chapter can be used as supplementary material at undergraduate level or included in a more advanced course at graduate level.

   Chapter 6 covers elementary quantum statistical mechanics. It first uses an heuristic point of view by showing how the ideas of classical kinetics can be modified to include the quantum nature of particles. Fermi and Bose statistics emerge then in a transparent way. In the rest of the chapter the basic description of a quantum system is introduced in terms of the density matrix, a tool used extensively throughout the book.

   Chapter 7 finally presents the derivation of the Bose and Fermi statistics from the Grand Canonical ensemble. The Fermi and Bose non-relativistic gases are discussed in detail. Given the current experimental relevance of the so-called cold atoms, presented in Chapter 15, we have included the discussion of Fermi and Bose gases in a confining harmonic potential. The important application of Bose statistics to photons and phonons is also included.
2. **Classical mean-field theory and critical phenomena [Chapters 8, 13, 14, 16, 17]**
   Chapters 8, 13, 14, 16, 17 offer a self-contained description of classical and modern theory of critical phenomena. Whereas in Chapter 8, two classical mean-field theories is discussed in detail (van der Waals and Curie–Weiss), in Chapter 13 we introduce the unifying description by Landau. The relevance of the fluctuations and of the symmetry properties in critical phenomena and in limiting the validity of the mean-field theories is discussed extensively. The more general Landau description is therefore presented in preparation for the scaling theory of critical phenomena, the use of renormalization group

[1] Chapters in round brackets contain supplemental topics which can be covered partially and/or optionally.

with the Gaussian approximation and the formulation of the Landau–Wilson model. The modern theory of critical phenomena develops through Chapters 14, 16 and 17. The presentation of the renormalization group approach in Chapter 17 strongly relies on physical concepts like the various aspects of the universality principle and even the discussion of the field-theoretic formulation does not require any technical knowledge of field theory. The present program is suitable for graduate students up to Chapter 17. This last chapter could be included in an advanced or postgraduate course.

A standard graduate one-year course on classical statistical mechanics and critical phenomena could be organized by assembling together modules 1 and 2.

3. **Mean-field theory for quantum systems and a unified presentation of superfluids and superconductors [Chapters 9, 12, 15, (8, 11, 21)]**
Chapter 9 is about second quantization, reduced density matrices and the Hartree–Fock approximation. It could also be included together with the the classical mean-field theory of Chapter 8, in the first module as the simplest one-body approximation of a many-particle system. Chapter 9 introduces the following Chapters 12 and 15, where the reader becomes acquainted with the normal and superfluid phases of interacting Bose and Fermi systems, starting from the phenomenology and adopting the reduced-density-matrix approach as a unifying framework at the technical level and the quasiparticle gas at the conceptual level. The last one was the paradigmatic interpretation of the whole of condensed matter physics of the last century. All together this is a comprehensive course on superfluids and on the normal and superconductive phases of fermions for graduate and postgraduate students. For the last more advanced level, after the Fermi liquid theory of Chapter 12, one could include Chapter 11 on the transport in disordered systems and the first five sections of Chapter 21 on the Anderson localization.
4. **Dissipative phenomena in classical and quantum systems [Chapters 2, 6, 10, 11, (21)]**
Chapters 2, 6, 10 and 11 may be used for a one-semester course on dissipative phenomena in classical and quantum systems. In particular, in Chapter 10 we present the theory of linear response for quantum systems and show how it reduces to the simpler classical case, already discussed in Chapter 8. The fluctuation–dissipation theorem is also generalized to non-equilibrium and its relation with the availability function and the maximum entropy production is given. Chapter 11 introduces Brownian and diffusive motion in the classical case. It also shows how these ideas can be adapted to the quantum case of a disordered Fermi gas, which plays an important role in the description of transport phenomena in condensed matter systems such as metals and semiconductors.
It is also preparatory to the discussion of Anderson localization as a phase arising from the normal Fermi liquid system in the presence of disorder, discussed in the first two sections of Chapter 21, which could also be included in this module.
5. **Modern trends in quantum statistical mechanics [Chapters 6, 7, 9, 12, 15 (20, 11, 21)]**
The last two modules (3 and 4) can be included in a larger one-year, graduate or postgraduate, course on quantum statistical mechanics and the density matrix (Chapter 6), quantum gases (Chapter 7), second quantization and the Hartree–Fock method

(Chapter 9), normal Fermi liquid (Chapter 12), superfluidity and superconductivity (Chapter 15), the Luttinger liquid (20 (the first three sections)) as an example of non-Fermi-liquid behavior, diffusive motion and the metal–insulator transition (Chapters 11 and 21 (the first three sections)).

6. **Advanced topics and techniques in quantum statistical mechanics [Chapters 10, 18, 19, 20, 21]**
Chapters 10, 18, 19, 20 and 21 may be used, together with related appendices for an advanced course in quantum statistical mechanics. In particular, in Chapter 18 we provide a concise self-contained introduction to the thermal Green function and its diagrammatic associated perturbation expansion technique, which is the tool most used by researchers when applying statistical mechanics to quantum systems. As a first application, Chapter 19 shows how to set a microscopic foundation for the phenomenological theory of Fermi liquids of Chapter 12. Chapter 20 uses the Green function technique to study the renormalization group treatment of the one-dimensional Tomonoga–Luttinger model, which provides the first example of non-Fermi-liquid behavior. This model, introduced in the nineteen-fifties mostly as an interesting theoretical model, has found an increasingly widespread application in the description of several systems of experimental interest. Its theoretical interest was recently rekindled by the non-Fermi-liquid behavior of the normal phase of the high temperature superconductors discussed in Chapter 15. Finally, Chapter 21 deals with the problem of disordered interacting Fermi gases and liquids and the so-called Anderson metal–insulator transition and its generalization to include interaction among the electrons. The last two chapters describe also a non-trivial application of the renormalization group approach to quantum systems somehow differing from its use in critical phenomena. In the last case the symmetry properties of the system (gauge invariance) have been exploited to select the proper renormalizations to be carried out in this most complex problem. In the Luttinger one-dimensional model, singularities plague the calculation of the physical response functions in perturbation theory. Singularities cannot be present in the final measurable responses of a stable liquid phase and must disappear in the final finite answers instead of being resummed to a power behavior as in critical phenomena. The renormalization group together with the proper symmetries of this one-dimensional system keeps the singularities under control and provides the exact final answer.

We would like also to indicate a few general references, which we have found useful when preparing our lecture notes and writing the institutional part of each chapter. Below we group them in correspondence with chapters.

- Pippard (1957) for Chapter 1.
- Huang (1963); Thompson (1972); Landau and Lifshitz (1959) for Chapters 2–5, 8, 13.
- Feynman (1972) for Chapters 6, 15.
- Rammer (2007); Peliti (2011) for Chapters 10, 11.
- Nozières and Pines (1966); Nozières (1964) for Chapters 12, 19.
- Patashinskij and Pokrovskij (1979); Ma (1976); Chaikin and Lubensky (1995); Amit and Martín-Mayor (2005) for Chapters 13, 14, 16, 17.

- London (1959); Atkins (1959); Feynman (1972); De Gennes (1966); Tinkham (1975); Schrieffer (1999); Leggett (2006) for Chapter 15.
- Abrikosov et al. (1963); Fetter and Walecka (1971); Mahan (2000); Rammer (2007) for Chapters 18, 20, 21.

Acknowledgments

This volume is based on a series of lectures we have given along the decades at the undergraduate, graduate, postgraduate and postdoctoral level. We therefore first of all thank all our students who have been the primary actors of this work. We should like to express our gratitude to our colleagues G. Jona Lasinio, C. Castellani, W. Metzner, S. Caprara, L. Benfatto, P. Schwab, C. Gorini and G. Vignale for continuous stimulating discussions on the topics presented in this volume and in particular to M. Grilli and A. Varlamov who also read parts of it, suggesting important corrections, and to J. Lorenzana who tested one of the suggested modules providing us with an important feedback. One of us (R. R.) would also like to thank his present students J. Borge and D. Guerci for "testing" parts of the book during its development.

Last but not least, we are tremendously grateful to E. Gasbarro for a careful check of all the bibliographical records.