

Statistical Mechanics and Applications in Condensed Matter

This innovative and modular textbook combines classical topics in thermodynamics, statistical mechanics and many-body theory with the latest developments in condensed matter physics research. Written by internationally renowned experts and logically structured to cater for undergraduate and postgraduate students and researchers, it covers the underlying theoretical principles, and includes numerous problems and worked examples to put this knowledge into practice.

Three main streams provide a framework for the book: beginning with thermodynamics and classical statistical mechanics, including mean field approximation, fluctuations and the renormalization group approach to critical phenomena. The authors then examine quantum statistical mechanics, covering key topics such as normal Fermi and Luttinger liquids, superfluidity and superconductivity. Finally, they explore classical and quantum kinetics, Anderson localization and quantum interference, and disordered Fermi liquids.

Unique in providing a bridge between thermodynamics and advanced topics in condensed matter, this textbook is an invaluable resource for all students of physics.

Carlo Di Castro is a member of the Accademia Nazionale dei Lincei and Emeritus Professor of Theoretical Physics at the Sapienza University of Rome, where he has been at the forefront of teaching and research in statistical mechanics and many-body theory for over 40 years. His current interests include strongly correlated electron systems, quantum criticality, high temperature superconductivity and non-Fermi-liquid metals.

Roberto Raimondi is Associate Professor of Condensed Matter Physics at Roma Tre University, where he has made important contributions to the understanding of the transport properties of disordered and mesoscopic systems. His current research interests include spintronics, especially the spin Hall effect and topological insulators.

Cambridge University Press
978-1-107-03940-7 - Statistical Mechanics and Applications in Condensed Matter
Carlo Di Castro and Roberto Raimondi
Frontmatter
[More information](#)

Cambridge University Press
978-1-107-03940-7 - Statistical Mechanics and Applications in Condensed Matter
Carlo Di Castro and Roberto Raimondi
Frontmatter
[More information](#)

Statistical Mechanics and Applications in Condensed Matter

CARLO DI CASTRO

Università degli Studi di Roma 'La Sapienza', Italy

ROBERTO RAIMONDI

Università Roma Tre, Italy



CAMBRIDGE
UNIVERSITY PRESS

Cambridge University Press
978-1-107-03940-7 - Statistical Mechanics and Applications in Condensed Matter
Carlo Di Castro and Roberto Raimondi
Frontmatter
[More information](#)

CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org

Information on this title: www.cambridge.org/9781107039407

© Cambridge University Press 2015

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 2015

Printed in the United Kingdom by TJ International Ltd. Padstow Cornwall

A catalogue record for this publication is available from the British Library

ISBN 978-1-107-03940-7 Hardback

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication, and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.

Contents

<i>Preface</i>	<i>page xi</i>
1 Thermodynamics: a brief overview	1
1.1 Equilibrium states and the empirical temperature	1
1.2 The principles of thermodynamics	5
1.3 Thermodynamic potentials and equilibrium	11
1.4 Thermodynamic stability and availability function	14
1.5 Phase equilibrium	20
1.6 The liquid–gas transition	22
1.7 Problems	26
2 Kinetics	27
2.1 The birth of kinetic theory	27
2.2 The Boltzmann equation	29
2.3 Extension of the Boltzmann equation to quantum systems	34
2.4 The H-theorem and the approach to equilibrium	35
2.5 The Maxwell–Boltzmann distribution	38
2.6 The mean free path for Maxwell–Boltzmann statistics	39
2.7 The Kac model	40
2.8 Problems	43
3 From Boltzmann to Boltzmann–Gibbs	45
3.1 Liouville and Poincaré theorems	45
3.2 Equilibrium statistical mechanics for a gas	47
3.3 Equilibrium statistical mechanics for a generic macroscopic system	50
3.4 The microcanonical ensemble and the entropy	53
3.5 The equipartition theorem	56
3.6 The perfect gas in the microcanonical ensemble	57
3.7 Problems	59
4 More ensembles	60
4.1 The canonical ensemble	60
4.2 The Gibbs entropy	63
4.3 The grand canonical ensemble	64
4.4 Fluctuations of a gas	67
4.5 Problems	68

5 The thermodynamic limit and its thermodynamic stability	69
5.1 The Yang–Lee theorems	70
5.2 Proof of the first theorem	72
6 Density matrix and quantum statistical mechanics	77
6.1 Kinetics for quantum gases	77
6.2 Density matrix for pure states	80
6.3 Density matrix for systems interacting with the environment	81
6.4 Quantum statistical ensembles	82
6.5 Problem	84
7 The quantum gases	85
7.1 The statistical distribution of the quantum gases	85
7.2 The strongly degenerate Fermi gas	88
7.3 The degenerate Bose gas	90
7.4 The Bose and Fermi gases in an harmonic potential	95
7.5 The photon gas	100
7.6 The phonon gas	103
7.7 Problems	106
8 Mean-field theories and critical phenomena	107
8.1 The van der Waals equation and the classical critical indices	107
8.2 The paramagnetic–ferromagnetic transition	112
8.3 The Curie–Weiss theory and the classical critical indices for the Ising model	115
8.4 Classical linear response theory and fluctuations	121
8.5 Problems	123
9 Second quantization and the Hartree–Fock approximation	124
9.1 Occupation numbers representation	124
9.2 Creation and annihilation operators	125
9.3 Field operators	126
9.4 Basis vectors in terms of the field operators	127
9.5 Observables in second quantization	128
9.6 Equation of motion for field operators	131
9.7 Reduced density matrices	133
9.8 Hartree–Fock method for a Fermi system	135
9.9 Problems	137
10 Linear response and the fluctuation–dissipation theorem in quantum systems: equilibrium and small deviations	139
10.1 Linear response	139
10.2 The response to an electromagnetic field and gauge invariance	143

10.3	The principle of detailed balance and the fluctuation–dissipation theorem	145
10.4	Onsager’s symmetry relations	148
10.5	Generalized fluctuations theorem and minimum availability function	149
10.6	Problems	154
11	Brownian motion and transport in disordered systems	155
11.1	Einstein’s theory of Brownian motion	155
11.2	Langevin’s theory of Brownian motion	158
11.3	The Johnson–Nyquist noise	160
11.4	Transport description in the disordered Fermi gas	162
11.5	Problems	169
12	Fermi liquids	170
12.1	Temperature range of application and quasiparticles	170
12.2	Equilibrium properties	173
12.3	Transport properties	177
12.4	Problems	185
13	The Landau theory of second order phase transitions	186
13.1	Spontaneous symmetry breaking, order parameter and the classical critical indices	186
13.2	Fluctuations and Ginzburg criterion for the validity of mean-field theories	189
13.3	First order transition and tricritical point	191
13.4	Continuous symmetries and the lower critical dimensionality	195
13.5	Problem	197
14	The Landau–Wilson model for critical phenomena	198
14.1	The Gaussian transformation	198
14.2	The Gaussian approximation	201
14.3	Hydrodynamics for the isotropic ferromagnet	203
15	Superfluidity and superconductivity	205
15.1	Introduction	205
15.2	The Landau criterion	206
15.3	Phenomenology of superfluid ^4He	207
15.4	Phenomenology of superconductors	214
15.5	The condensation criterion and the order parameter	218
15.6	Order parameter and symmetry	223
15.7	The Landau–Ginzburg equations	235
15.8	The Bogoliubov model for superfluidity	242
15.9	The microscopic theory of superconductivity	245
15.10	Crossover between BCS and Bose–Einstein condensation	255

15.11 Superfluid ^3He	260
15.12 High temperature superconductivity: a brief presentation	266
15.13 Problems	273
16 Scaling theory	275
16.1 The scaling laws	275
16.2 On the existence of ODLRO at low dimension	283
16.3 Dynamical scaling	285
16.4 Problem	289
17 The renormalization group approach	290
17.1 General properties of the renormalization group	291
17.2 The Kadanoff–Wilson transformation	295
17.3 Derivation of the RG equations	299
17.4 Field-theoretic renormalization group	307
17.5 Problems	317
18 Thermal Green functions	318
18.1 The Matsubara Green function	318
18.2 The thermal Green function for Fermi and Bose gases	321
18.3 The connection with the time-dependent Green function at finite temperature	323
18.4 The physical meaning of the poles of the Green function	326
18.5 The perturbative expansion of the Matsubara Green function	331
18.6 The statistical Wick theorem	333
18.7 Diagrammatic expansion of the one-particle thermal Green function and the thermodynamic potential	335
18.8 The Dyson equation	341
18.9 Thermal density and current response functions	346
18.10 The Ward identity	348
18.11 Problems	351
19 The microscopic foundations of Fermi liquids	352
19.1 Scattering amplitudes	352
19.2 The static and dynamic limit of the truncated vertex function	355
19.3 Response functions and Fermi liquid parameters	356
20 The Luttinger liquid	358
20.1 The breakdown of the Fermi liquid in one dimension	358
20.2 The Tomonaga–Luttinger (TL) model	359
20.3 The Dyson equation, conservation laws and Ward identities	365
20.4 Remarks about renormalization	370
20.5 Problems	374

21 Quantum interference effects in disordered electron systems	375
21.1 Experimental evidence of anomalous disorder effects	376
21.2 The Anderson transition and quantum interference	378
21.3 The scaling theory of the metal–insulator transition	380
21.4 The quantum theory of the disordered Fermi gas	383
21.5 A few remarks on the experiments	396
21.6 Electron–electron interaction in the presence of disorder	397
21.7 Quantum theory of the disordered Fermi liquid	406
21.8 The renormalization group flow of the disordered Fermi liquid	413
21.9 Problems	420
Appendix A The central limit theorem	422
Appendix B Some useful properties of the Euler Gamma function	424
Appendix C Proof of the second theorem of Yang and Lee	426
Appendix D The most probable distribution for the quantum gases	428
Appendix E Fermi–Dirac and Bose–Einstein integrals	430
Appendix F The Fermi gas in a uniform magnetic field: Landau diamagnetism	433
Appendix G Ising and gas-lattice models	436
Appendix H Sum over discrete Matsubara frequencies	438
Appendix I Two-fluid hydrodynamics: a few hints	439
Appendix J The Cooper problem in the theory of superconductivity	441
Appendix K Superconductive fluctuation phenomena	443
Appendix L Diagrammatic aspects of the exact solution of the Tomonaga–Luttinger model	450
Appendix M Details on the theory of the disordered Fermi liquid	458
Appendix N Answers to problems	467
<i>References</i>	511
<i>Index</i>	527

Cambridge University Press
978-1-107-03940-7 - Statistical Mechanics and Applications in Condensed Matter
Carlo Di Castro and Roberto Raimondi
Frontmatter
[More information](#)

Preface

D. Pines in his Editor's Foreword to the important series "Frontiers in Physics, a Set of Lectures" of the sixties and seventies of the past century (W. A. Benjamin, Inc.) was suggesting as a possible solution to "the problem of communicating in a coherent fashion the recent developments in the most exciting and active fields of physics" what he called "an informal monograph to connote the fact that it represents an intermediate step between lecture notes and formal monographs."

Our aim in writing this book has been to provide a coherent presentation of different topics, emphasizing those concepts which underlie recent applications of statistical mechanics to condensed matter and many-body systems, both classical and quantum. Our goal has been indeed to reach an up to date version of the book *Statistical Mechanics. A Set of Lectures* by R. P. Feynman, one of the most important monographs of the series mentioned above. We felt, however, that it would have been impossible to give to a student the full flavor of the recent topics without putting them in the classical context as a continuous evolution. For this reason we introduced the basic concepts of thermodynamics and statistical mechanics. We have also concisely covered topics that typically can be found in advanced books on many-body theory, where usually the apparatus of quantum field theory is used.

In our book we have kept the technical apparatus at the level of the density matrix with the exception of the last four chapters. Up to Chapter 17 no particular prerequisite is needed except for standard courses in Classical and Quantum Mechanics. Chapter 18 provides an introduction to statistical quantum field theory, which is used in the last chapters. Chapters 20 and 21 cover topics which, although covered in recent monographs, are not commonly found in classical many-body books.

In our book then the student will find a bridge from thermodynamics and statistical mechanics towards advanced many-body theory and its applications. In our attempt to give a coherent account of several topics of condensed matter physics, we have at the same time preserved the personal point of view of the notes of our courses. Our bibliography is for this reason far from complete and the presentation of some topics is somewhat informal and partial. Many important contributions and fundamental references have been left out.

Some fundamentals topics of modern statistical mechanics for condensed matter physics have been left out, e.g. quantum criticality (except for the two examples of Luttinger liquid and Anderson localization), the quantum Hall effect and spin glasses. For them, however, there are recent devoted books.

Suggestions on the usage of the book

Although this book is conceived and organized as an organic self-contained whole, it can also be used as a modular text. We provide here a few suggestions for planning courses

on the great amount of the covered material at undergraduate, graduate and postgraduate level. In this way we hope also to facilitate consultation by teachers and researchers.

1. **Introductory statistical mechanics [Chapters 1–4, 6–7, (5)]¹**

Chapters 1–4 and 6–7 are a self-contained introduction to basic statistical mechanics. Chapter 1 is a concise review of thermodynamics in order to put statistical mechanics in context. In particular we emphasize the concepts of thermodynamic stability with respect to deviations and fluctuations from average by means of the introduction of the thermodynamic function availability as the maximum work available. The aim is to alert the reader from the start of the statistical interpretation of the law that the entropy of an isolated system can never decrease. Fluctuations play an important role in phase transitions. Both these two issues are at the heart of the foundations of statistical mechanics. The logical sequence of the thermodynamic potentials and the related pairs of conjugate variables prepares the basis of the thermodynamics for ensembles. Chapters 2 and 3 cover the basic concepts of statistical mechanics from the fundamental insights by Boltzmann to the Gibbs formulation. Chapter 4 is about the consequences of the basic axioms and discusses the relation between the different statistical ensembles. Chapter 5, which is not included in the above list, concludes the exposition of classical statistical mechanics by presenting the physical and mathematical significance of the thermodynamic limit especially with reference to the problem of phase transitions. This chapter can be used as supplementary material at undergraduate level or included in a more advanced course at graduate level.

Chapter 6 covers elementary quantum statistical mechanics. It first uses an heuristic point of view by showing how the ideas of classical kinetics can be modified to include the quantum nature of particles. Fermi and Bose statistics emerge then in a transparent way. In the rest of the chapter the basic description of a quantum system is introduced in terms of the density matrix, a tool used extensively throughout the book.

Chapter 7 finally presents the derivation of the Bose and Fermi statistics from the Grand Canonical ensemble. The Fermi and Bose non-relativistic gases are discussed in detail. Given the current experimental relevance of the so-called cold atoms, presented in Chapter 15, we have included the discussion of Fermi and Bose gases in a confining harmonic potential. The important application of Bose statistics to photons and phonons is also included.

2. **Classical mean-field theory and critical phenomena [Chapters 8, 13, 14, 16, 17]**

Chapters 8, 13, 14, 16, 17 offer a self-contained description of classical and modern theory of critical phenomena. Whereas in Chapter 8, two classical mean-field theories is discussed in detail (van der Waals and Curie–Weiss), in Chapter 13 we introduce the unifying description by Landau. The relevance of the fluctuations and of the symmetry properties in critical phenomena and in limiting the validity of the mean-field theories is discussed extensively. The more general Landau description is therefore presented in preparation for the scaling theory of critical phenomena, the use of renormalization group

¹ Chapters in round brackets contain supplemental topics which can be covered partially and/or optionally.

with the Gaussian approximation and the formulation of the Landau–Wilson model. The modern theory of critical phenomena develops through Chapters 14, 16 and 17. The presentation of the renormalization group approach in Chapter 17 strongly relies on physical concepts like the various aspects of the universality principle and even the discussion of the field-theoretic formulation does not require any technical knowledge of field theory. The present program is suitable for graduate students up to Chapter 17. This last chapter could be included in an advanced or postgraduate course.

A standard graduate one-year course on classical statistical mechanics and critical phenomena could be organized by assembling together modules 1 and 2.

3. Mean-field theory for quantum systems and a unified presentation of superfluids and superconductors [Chapters 9, 12, 15, (8, 11, 21)]

Chapter 9 is about second quantization, reduced density matrices and the Hartree–Fock approximation. It could also be included together with the classical mean-field theory of Chapter 8, in the first module as the simplest one-body approximation of a many-particle system. Chapter 9 introduces the following Chapters 12 and 15, where the reader becomes acquainted with the normal and superfluid phases of interacting Bose and Fermi systems, starting from the phenomenology and adopting the reduced-density-matrix approach as a unifying framework at the technical level and the quasiparticle gas at the conceptual level. The last one was the paradigmatic interpretation of the whole of condensed matter physics of the last century. All together this is a comprehensive course on superfluids and on the normal and superconductive phases of fermions for graduate and postgraduate students. For the last more advanced level, after the Fermi liquid theory of Chapter 12, one could include Chapter 11 on the transport in disordered systems and the first five sections of Chapter 21 on the Anderson localization.

4. Dissipative phenomena in classical and quantum systems [Chapters 2, 6, 10, 11, (21)]

Chapters 2, 6, 10 and 11 may be used for a one-semester course on dissipative phenomena in classical and quantum systems. In particular, in Chapter 10 we present the theory of linear response for quantum systems and show how it reduces to the simpler classical case, already discussed in Chapter 8. The fluctuation–dissipation theorem is also generalized to non-equilibrium and its relation with the availability function and the maximum entropy production is given. Chapter 11 introduces Brownian and diffusive motion in the classical case. It also shows how these ideas can be adapted to the quantum case of a disordered Fermi gas, which plays an important role in the description of transport phenomena in condensed matter systems such as metals and semiconductors.

It is also preparatory to the discussion of Anderson localization as a phase arising from the normal Fermi liquid system in the presence of disorder, discussed in the first two sections of Chapter 21, which could also be included in this module.

5. Modern trends in quantum statistical mechanics [Chapters 6, 7, 9, 12, 15 (20, 11, 21)]

The last two modules (3 and 4) can be included in a larger one-year, graduate or postgraduate, course on quantum statistical mechanics and the density matrix (Chapter 6), quantum gases (Chapter 7), second quantization and the Hartree–Fock method

(Chapter 9), normal Fermi liquid (Chapter 12), superfluidity and superconductivity (Chapter 15), the Luttinger liquid (20 (the first three sections)) as an example of non-Fermi-liquid behavior, diffusive motion and the metal–insulator transition (Chapters 11 and 21 (the first three sections)).

6. Advanced topics and techniques in quantum statistical mechanics [Chapters 10, 18, 19, 20, 21]

Chapters 10, 18, 19, 20 and 21 may be used, together with related appendices for an advanced course in quantum statistical mechanics. In particular, in Chapter 18 we provide a concise self-contained introduction to the thermal Green function and its diagrammatic associated perturbation expansion technique, which is the tool most used by researchers when applying statistical mechanics to quantum systems. As a first application, Chapter 19 shows how to set a microscopic foundation for the phenomenological theory of Fermi liquids of Chapter 12. Chapter 20 uses the Green function technique to study the renormalization group treatment of the one-dimensional Tomonaga–Luttinger model, which provides the first example of non-Fermi-liquid behavior. This model, introduced in the nineteen-fifties mostly as an interesting theoretical model, has found an increasingly widespread application in the description of several systems of experimental interest. Its theoretical interest was recently rekindled by the non-Fermi-liquid behavior of the normal phase of the high temperature superconductors discussed in Chapter 15. Finally, Chapter 21 deals with the problem of disordered interacting Fermi gases and liquids and the so-called Anderson metal–insulator transition and its generalization to include interaction among the electrons. The last two chapters describe also a non-trivial application of the renormalization group approach to quantum systems somehow differing from its use in critical phenomena. In the last case the symmetry properties of the system (gauge invariance) have been exploited to select the proper renormalizations to be carried out in this most complex problem. In the Luttinger one-dimensional model, singularities plague the calculation of the physical response functions in perturbation theory. Singularities cannot be present in the final measurable responses of a stable liquid phase and must disappear in the final finite answers instead of being resummed to a power behavior as in critical phenomena. The renormalization group together with the proper symmetries of this one-dimensional system keeps the singularities under control and provides the exact final answer.

We would like also to indicate a few general references, which we have found useful when preparing our lecture notes and writing the institutional part of each chapter. Below we group them in correspondence with chapters.

- Pippard (1957) for Chapter 1.
- Huang (1963); Thompson (1972); Landau and Lifshitz (1959) for Chapters 2–5, 8, 13.
- Feynman (1972) for Chapters 6, 15.
- Rammer (2007); Peliti (2011) for Chapters 10, 11.
- Nozières and Pines (1966); Nozières (1964) for Chapters 12, 19.
- Patashinskij and Pokrovskij (1979); Ma (1976); Chaikin and Lubensky (1995); Amit and Martín-Mayor (2005) for Chapters 13, 14, 16, 17.

- London (1959); Atkins (1959); Feynman (1972); De Gennes (1966); Tinkham (1975); Schrieffer (1999); Leggett (2006) for Chapter 15.
- Abrikosov et al. (1963); Fetter and Walecka (1971); Mahan (2000); Rammer (2007) for Chapters 18, 20, 21.

Acknowledgments

This volume is based on a series of lectures we have given along the decades at the undergraduate, graduate, postgraduate and postdoctoral level. We therefore first of all thank all our students who have been the primary actors of this work. We should like to express our gratitude to our colleagues G. Jona Lasinio, C. Castellani, W. Metzner, S. Caprara, L. Benfatto, P. Schwab, C. Gorini and G. Vignale for continuous stimulating discussions on the topics presented in this volume and in particular to M. Grilli and A. Varlamov who also read parts of it, suggesting important corrections, and to J. Lorenzana who tested one of the suggested modules providing us with an important feedback. One of us (R. R.) would also like to thank his present students J. Borge and D. Guerci for “testing” parts of the book during its development.

Last but not least, we are tremendously grateful to E. Gasbarro for a careful check of all the bibliographical records.