What Logics Mean

What do the rules of logic say about the meanings of the symbols they govern? In this book, James W. Garson examines the inferential roles of logical connectives (such as 'and', 'or', 'not', and 'if … then'), whose behavior is defined by strict rules, and proves definitive results concerning exactly what those rules express about connective truth conditions. He explores the ways in which, depending on circumstances, a system of rules may provide no interpretation of a connective at all, or the interpretation we ordinarily expect for it, or an unfamiliar or novel interpretation. He also shows how the novel interpretations thus generated may be used to help analyze philosophical problems such as vagueness and the open future. His book will be valuable for graduates and specialists in logic, philosophy of logic, and philosophy of language.

JAMES W. GARSON is Professor of Philosophy at the University of Houston. He is the author of *Modal Logic for Philosophers*, 2nd edition (Cambridge, 2013).

Cambridge University Press
978-1-107-03910-0 - What Logics Mean: From Proof Theory to Model-Theoretic Semantics
James W. Garson
Frontmatter
More information

What Logics Mean

From Proof Theory to Model-Theoretic Semantics

JAMES W. GARSON

CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

Published in the United States of America by Cambridge University Press, New York

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning, and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781107039100

First published 2013

Printed in the United Kingdom by Clays, St Ives plc

A catalog record for this publication is available from the British Library

Library of Congress Cataloging in Publication data

ISBN 978-1-107-03910-0 Hardback
ISBN 978-1-107-61196-2 Paperback

For Connie – a valiant friend

Contents

Preface

> Syntax all by itself doesn't determine semantics
>
> D. Dennett (1984, p. 28)

Where does meaning come from? There is no more compelling question in the philosophy of language. Referentialists seek an answer in a correspondence between word and object, statement and reality. Inferentialists look to an expression's deductive role, its contribution to the web of relations that determine what follows from what. Logic is the perfect test bed for assessing the merits of inferentialism. The deductive role of the connectives for a given system is defined precisely by its rules. Whether the meanings of the connectives are determined by those roles is now a question with a rigorous answer. This book proves what some of those answers are, revealing both strengths and weaknesses in an inferentialist program for logic. The results reported here are only the tip of an iceberg, but they illustrate the important contribution that metalogic can play in resolving central puzzles in the philosophy of language.

To make headway on this project, we need to explore the options in syntax, in semantics, and in ways to plausibly bridge the two. On the syntactic side, we are faced with a rich variety in the systems of logic. This book examines only intuitionistic and classical rules for propositional logic, and then briefly, rules for quantified and modal systems. So this is just a start. A second important source of syntactic variation is rule format. The details about the way the rules of a logical system are formulated affect whether that system allows unintended interpretations of its connectives. In the same way that moving from first-order to second-order languages strengthens the expressive power of the logic, so does the move from axiomatic formulations, to natural deduction systems, and to sequent calculi with multiple conclusions. Answers to questions about what logics mean depend crucially on which format is chosen. The moral is that inferentialists who claim that inferential roles fix meaning are duty bound to specify what *kind* of rules undergird those roles.

On the semantics side, we are faced with a decision concerning conceptual foundations. Exactly what vocabulary is to be used in formulating the meaning of a connective? There are two main choices: proof-theoretic semantics and model-theoretic semantics. The latter tradition follows Tarski in presuming that a semantics is a recursive definition of truth on a model. That definition allows one to delineate a corresponding notion of validity.

On the other hand, proof-theoretic semantics eschews "referential" notions such as denotation and truth. It proposes to define meaning using only syntactic concepts such as proof. It is natural for inferentialists who view the referential/inferential divide as a battleground, to opt for proof-theoretic (PT) semantics, for referential notions are perceived as the devices of their enemy. The stance of this book, however, is pluralistic. There is nothing wrong with PT semantics, but we choose to investigate model-theoretic (MT) semantics instead, for there is ample room for a model-theoretic inferentialism. Such a view holds that meaning is determined by inferential role, but that the use of model-theoretic notions in characterizing the meaning so fixed is compatible with the inferentialist project, and even useful to PT inferentialists who think of semantics entirely in proof-theoretic terms. A main concern of this book is to demonstrate by example that MT inferentialism is both interesting and viable. So henceforth by 'semantics' we will mean model-theoretic semantics, without intending to indicate a prejudice against the proof-theoretic tradition.

A last source of variation must be mentioned. Definitive answers to questions about the meanings rules express are not possible until a firm bridge between syntax and semantics is in place. We need a mathematically precise account of what a rule expresses. At least three different standards for expressive power are found in the literature, so our job is to canvass their strengths and weaknesses, and select the one that is best.

The idea of the expressive power of a rule is a generalization of an idea that will be familiar from model theory. In the language of predicate logic, there are well-formed formulas that express a variety of conditions on the domain of quantification. For example, $\exists x \exists y \sim x=y$ expresses that there are at least two objects in the domain. That means that a model satisfies that formula iff its domain meets that condition. By analogy, a rule R should express a condition C on models exactly when a model satisfies R iff it obeys C. But what does it mean to say that a model satisfies a rule? A model satisfies a sentence iff it makes the sentence true. What is the corresponding honorific in the case of rules? This book argues that the correct choice is preservation of validity, and that alternative choices face serious problems.

What are the outcomes given the options chosen here? Given the very negative conclusions of the work of Quine (1960, Section 12) and Davidson (Lepore and Ludwig, 2005, Chapter 15) on underdetermination of meaning in natural languages, and Dennett's summary pronouncement that "syntax all by itself doesn't determine semantics" (Dennett, 1984, p. 28), one might expect that functional role radically underdetermines meaning in logic, and that rules never determine a semantics. This appraisal appears to be supported by a well-known collection of negative results for propositional logic (Carnap, 1943, pp. 81ff.; McCawley, 1993, pp. 107ff.; Shoesmith and Smiley, 1978, p. 3; Belnap and Massey, 1990). So it looks bad for model-theoretic inferentialism. However, it is argued here that this wholesale underdetermination is the result of poor choices in rule format and in the definition of what rules express. A more optimistic assessment plays out in the chapters of this book.

Chapter 1 lays out the whole project more intelligibly than this preface can manage. Chapters 2 and 3 examine and dismiss two alternative accounts of what rules express. Chapter 4 develops the notion of expression based on preservation of validity in detail, and defines natural semantics as the semantics so expressed. Since Kripke's intuitionistic semantics plays a central role in this book, Chapter 5 presents that semantics and illustrates how to define an isomorphism to a natural semantics. The next few chapters report results on natural semantics for conditionals (6), disjunction (7), and negation (8). We learn here that the rules for the conditional and for intuitionistic negation express exactly their readings in Kripke semantics. These results will hearten inferentialists of an intuitionistic persuasion. However, an unfamiliar condition is expressed by the rules for disjunction and worries about its legitimacy are explored in detail. Furthermore, there are concerns about classical negation to face as well, although in the classical setting some of the problems with disjunction are resolved. It is then argued that the classical natural deduction rules for propositional logic express a variant of an intuitionist semantics ||PL|| that is entirely acceptable.

The odd outcome, however, is that the classical rules for negation express an intuitionistic reading. Supervaluations and ||PL|| show interesting similarities and differences, which are explored in Chapter 9. Chapter 10 is a philosophical interlude, showing how ||PL|| may be deployed as a logic for an open future. Chapter 11 lays out results for logics in sequent format with multiple conclusions where classical rather than intuitionist semantics is expressed. Chapter 12 shows how completeness results may be obtained for systems with respect to their natural semantics. Chapter 13 demonstrates that natural semantics can be helpful in vindicating notions of harmony found in the proof-theoretic tradition. It also shows that natural semantics can be transformed into a useful proof-theoretic version. Chapter 14 describes the natural semantics for the quantifiers, which is essentially intensional, and differs from both the objectual and substitution interpretations. Furthermore, it fails to support the presumption that terms of the language denote objects. Chapter 15 applies natural semantics for standard predicate logic to the problem of vagueness. The final chapter provides a brief account of some results in modal logic. The book ends with a summary of what has been accomplished, and offers a defense of model-theoretic inferentialism in the face of some objections.

Acknowledgements

Even if I could remember what I have forgotten about their contributions, it would not be possible to acknowledge everyone who contributed to this book. But some people deserve special thanks as sources of inspiration:

to my wife, who supports me through thick and thin, and to whom the book is dedicated,

to my teacher Nuel Belnap, who showed me by example how much fun logic can be,

to Joshua D. K. Brown, whose kindness, support, and intellectual energy have brought me to life,

to Will Langley and Brandon Tinklenberg, who talked me into giving them a course on this book, which I profited from and thoroughly enjoyed,

to Jack Woods, who made many valuable suggestions and who always gets me thinking,

to Lloyd Humberstone, who read most of a first draft, headed off many errors, and made valuable suggestions,

to Thomas Wright, who spotted errors in a draft of Chapter 15,

to Hilary Gaskin at Cambridge University Press, who helped get this project on its way, and to anonymous readers who offered kind support and excellent suggestions for improvement.